Discussion on Two Simplified Swelling Pressure Models for Expansive Soils

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ABSTRACT

The swelling pressure model developed by former researchers based on the Gouy-Chapmann interacting diffuse double layer theory and osmotic pressure theory is effective in simulating swelling pressure of expansive soils. But it is difficult to combine equations of the model to calculate swelling pressure directly from void ratio because one of the equations is elliptic integral. So some researchers proposed the best-fit linear equation for the parameters *u* and *Kd* in the elliptic integral and then presented simplified swelling pressure model composed of the best-fit linear equation and other equations. However, when the range of *Kd* is big, the linear relationship does not fit *u*-log(*Kd*) well. Here the second order exponential function is applied to fit *u*-*Kd* relationship, and the results indicate that it fits *u*-*Kd* exactly without the influences of the range of *Kd* and other factors. At last, based on the *u* and *Kd* values obtained by numerical integration of the interacting double layer model and the second order exponential function for *u*-*Kd*, the feasibility to use the single double layer theory to establish swelling pressure model is discussed.

KEYWORDS: Expansive soils, Swelling pressure model, *u*-*Kd* relationship, Second order exponential function, Single double layer theory

INTRODUCTION

It is very important to determine the swelling pressure of expansive soils in many situations concerned with stability problems of foundations, retaining walls, and nuclear waste disposal systems. The expandable mineral montmorillonite mainly contributes to the swelling of expansive soils. Because of isomorphous substitutions in the crystal lattice, in general the montmorillonite particles carry negative charges at the surfaces of the platelets. In dry clay, adsorbed cations are tightly held by the negatively charged clay surfaces to neutralize the electro-negativity of the clay particles. When the clay is placed in water the cations diffuse into solutions between the particle layers. Consequently, the negative surface and the distributed cations in the adjacent layers are together termed the diffuse double layer (Mitchell 1976; Brady 1996; van Olphen 1977). Several theories have been proposed for the description of ion distributions adjacent to charged surfaces in colloids. The Gouy-Chapmann theory of the diffuse double layer has received the greatest attention, and it has been applied to the behavior of clays with varying degrees of success (Mitchell 1976). Based on the Gouy-Chapmann interacting diffuse double layer theory and osmotic pressure theory, the equations developed by Bolt (1956), van Olphen (1977), Sridharan (1982), to determine swelling pressure are as follows:

$$e_0 = G\rho_w Sd \tag{1}$$

$$\left(\frac{dy}{d\xi}\right)_{x=0} = \frac{B}{S}\frac{1}{\sqrt{2n_0\varepsilon kT}} \tag{2}$$

$$(2\cosh z - 2\cosh u)^{\frac{1}{2}} = -\left(\frac{dy}{d\xi}\right)_{x=0} \tag{3}$$

$$\int_z^u (2\cosh y - 2\cosh u)^{-1/2}\, dy = -Kd \tag{4}$$

$$p = 2n_0 kT(\cosh u - 1) \tag{5}$$

where K is the double layer parameter given as:

$$K = \sqrt{\frac{2n_0 v^2 e^2}{\varepsilon kT}} \tag{6}$$

and where x is the distance from the clay surface, d is half the distance between parallel clay platelets, B is capacity of exchangeable cations(C/g), S is the specific surface area of soil(m^2/g), ε is the dielectric constant of the pore fluid(is the product of permittivity of vacuum ε_0=8.854×10^{-12} $C^2J^{-1}m^{-1}$ multiplies relative dielectric constant ε_r =80.4), k is Boltzmann's constant(=1.38×10^{-23} JK^{-1}), T is the absolute temperature, v is the valance of cations, e is the elementary electric charge(=1.602×10^{-19} C), n_0 is the molar concentration of ions in bulk fluid, p is the swelling pressure, e_0 is the void ratio, G is the specific gravity of soil solids, ρ_w is the density of water, u is the non-dimensional mid-plane potential, z is the non-dimensional potential at the clay surface, y is the non-dimensional potential at x, and ξ is the distance function (= Kx).

For any given pore fluid medium, determination of the swelling pressure using eq. (5) requires the non-dimensional mid-plane potential u. Since equation (4) is an elliptic integral with the unknown integration domain, determination of u is an indirect and time-consuming process, so it is difficult to combine above equations to calculate swelling pressure from e_0 directly. Hence a relationship between u and the non-dimensional distance function, Kd, must be established to determine u for any given values of Kd. At present, best-fit linear equations for the parameters u and Kd in the elliptic integral are proposed (Sridharan 1982, 2002; Tripathy 2004) and then simplified swelling pressure models composed of best-fit linear equations and other equations, i.e. eq. (1), (5) and (6), are presented. However, when the values of Kd are relative big, the linear relationship does not fit u and Kd well. Here the second order exponential function is applied to fit u-Kd relationship, and the results indicate that it fits u-Kd exactly without the influence of the values of Kd and other factors.

The equations for the single double layer model are as follows (Mitchell 1976; van Olphen 1977; Komine 1996):

$$y = 4 * \tanh^{-1}[\exp(-Kx) * \tanh(z/4)] \tag{7}$$

$$z = 2 * \sinh^{-1}\left[\frac{1}{2} * \left(\frac{dy}{d\xi}\right)_{x=0}\right] \tag{8}$$

where all terms are previously defined. van Olphen (1977) has showed that when the interaction between two parallel layers is weak, that is, the value of *Kd* is big, the mid-plane potential can be taken as the sum of the double layer potentials at distance *d* based on the solutions for a single plate, that is,

$$u = 2y_d = 8 * \tanh^{-1}\left[\exp(-Kd) * \tanh(z/4)\right] \tag{9}$$

So the simplified model based on the single double layer model could be established with eq. (1), (5) and (6) (Komine 1996, 2004). However, the feasibility of applying the simplified model is confused. Hence, It is significant to discuss, with the help of numerical values of *u* and *Kd* obtained from the interacting double layer model and the second order exponential best-fit function for *u-Kd*, how to use the swelling pressure model based on the single double layer model.

THE SECOND ORDER EXPONENTIAL BEST-FIT FUNCTION FOR *U-KD*

To establish the relationship between *u* and *Kd*, equations (2)-(6) are used for this purpose. For any pressure, *u* can be found from eq. (5). For known *B*, *S*, and *u* values, $(dy/d\xi)_{x=0}$ and then *z* can be found from eq. (2) and (3). From eq. (4), for known *u* and *z* values, *Kd* can be found by numerical integration. Then the best-fit equations for *u-Kd* could be got. For any void ratio, e_0, knowing *K* from eq. (6) and *d* from eq. (1), *Kd* can be found. Then the *u* value for the corresponding *Kd* value can be determined from established *u–Kd* relationship (Sridharan 2002; Tripathy 2004). In this paper, the authors introduce another method, which is described later, based on the single double layer model to establish the best-fit equation for *u-Kd*.

From above description for determining the *u-Kd* relationship, we can conclude that $(dy/d\xi)_{x=0}$ is a dominant factor to determining the *u-Kd* relationship. Olphen (1977) and Sridharan (1982) have provided numerous values of *u* and *Kd* for various $(dy/d\xi)_{x=0}$. The *u-Kd* values calculated by them are showed in Figure 1 and Figure 2 respectively, and the best-fit linear equations for *u-Kd* calculated by Sridharan (1982) are listed in Table 1.

In Figure 1, diffusion of *u-Kd* in semi-logarithm coordinate system is essentially linear when *Kd* are smaller than 3. When $(dy/d\xi)_{x=0}$=50 and 100, however, the relationships of *u-Kd* are not exactly linear, where the relevant coefficient R^2 don't reach 0.99. In Figure 2, especially, when *Kd* are larger than 3, the deviations from linear curve occur. We can conclude that the linear equation does not fit *u*-log(*Kd*) well, especially when *Kd*>3.

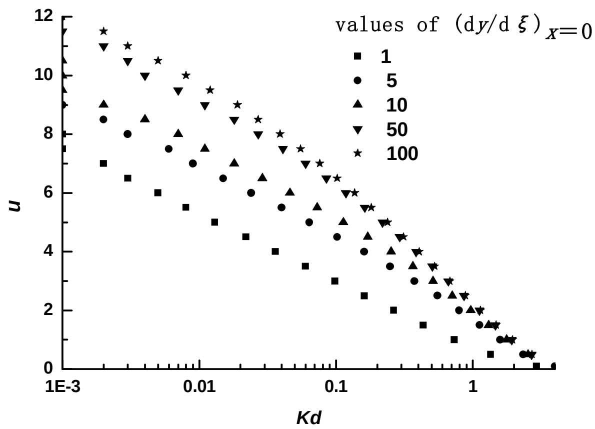

Figure 1: Diffusion of *u*-*Kd* in semi-logarithm coordinate system (Sridharan 1982)

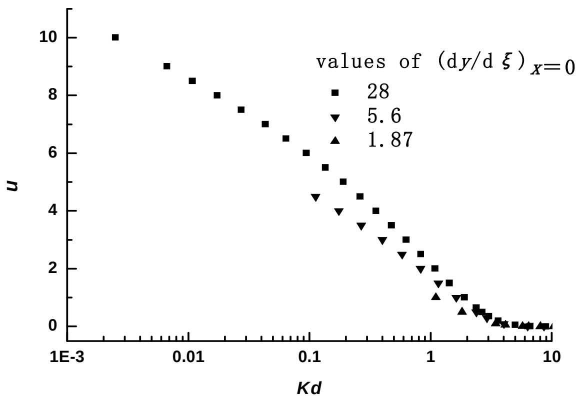

Figure 2: Diffusion of *u*-*Kd* in semi-logarithm coordinate system (Olphen 1977)

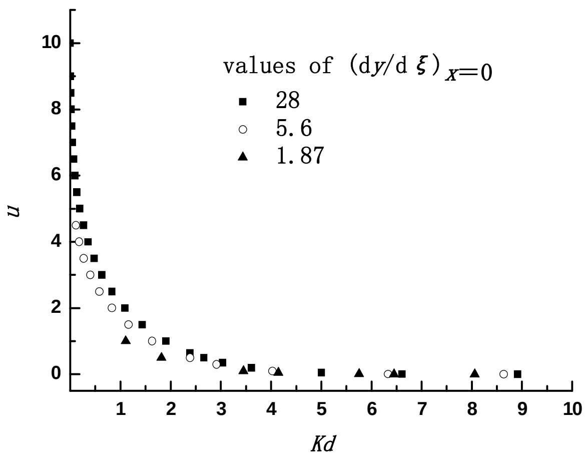

Figure 3: Diffusion of *u*-*Kd* in linear coordinate system (Olphen 1977)

In Figure 3, *u* and *Kd* values are plotted in linear coordinate system and show exponential relationship. So the values of *u* and *Kd* are fitted by the second order exponential function, listed in Table 2. The results indicate that they fit *u*-*Kd* exactly without the influence of the values of *Kd* and other factors. So it is workable to replace equation (2), (3), (4) by the second order exponential function fitting for *u*-*Kd* to establish the simplified swelling pressure model with a high accuracy.

Table 1: The best-fit linear equations for *u*-log(*Kd*) of Sridharan (1982)

$(dy/d\xi)_{x=0}$	Best-fit linear equations	R^2
1	$u=0.774-2.279*\log_{10}(Kd)$	0.996
5	$u=1.767-2.542*\log_{10}(Kd)$	0.995
10	$u=2.071-2.717*\log_{10}(Kd)$	0.991
50	$u=2.470-3.270*\log_{10}(Kd)$	0.989
100	$u=2.541-3.513*\log_{10}(Kd)$	0.986

Table 2: The best-fit results for *u*-*Kd* using the second order exponential function

$(dy/d\xi)_{x=0}$	Second order exponential equations to fit *u*-*Kd*	R^2
1	$u=3.940*\exp(-Kd/0.304)+3.739*\exp(-Kd/0.007)+0.384$	0.99284
5	$u=4.874*\exp(-Kd/0.716)+3.723*\exp(-Kd/0.016)+0.295$	0.99537
10	$u=5.607*\exp(-Kd/0.703)+4.096*\exp(-Kd/0.012)+0.377$	0.99171
50	$u=6.797*\exp(-Kd/0.635)+4.356*\exp(-Kd/0.013)+0.517$	0.99329
100	$u=6.807*\exp(-Kd/0.694)+4.429*\exp(-Kd/0.024)+0.431$	0.99549
28	$u=3.699*\exp(-Kd/0.032)+6.083*\exp(-Kd/0.896)+0.112$	0.99737
5.6	$u=4.175*\exp(-Kd/1.132)+1.787*\exp(-Kd/0.124)-0.005$	0.99998
1.87	$u=1.474*\exp(-Kd/1.025)+1.474*\exp(-Kd/1.025)-0.0004$	0.99999
1.87	$u=1.474*\exp(-Kd/1.025)+1.474*\exp(-Kd/1.025)-0.0004$	0.99999

DISCUSSION ON THE SWELLING PRESSURE MODEL BASED ON THE SINGLE DOUBLE LAYER THERORY

From previous authors' analysis on the single double layer theory, it is known that only when *Kd* is big enough the theory could be used to establish the swelling pressure model. Although Komine (1996, 2004) has applied the single double layer to a swelling pressure model, the feasibility of the model has not been discussed. Immediately, we will make a discussion on it with the numerical solutions of *u* and *Kd* obtained from the interacting double layer model and the second order exponential function for *u*-*Kd*.

Table 3: Some parameters of bentonite used in Komine (1996)

S	388.8 m^2/g
B	70.64 C/g
v	1.5
T	295K
ε	$80\times8.854\times10\text{-}12\ C^2\cdot J^{-1}\cdot m^{-1}$
n_0	0.02 mol/l

Table 4: True values of *u* and *Kd* calculated by interacting double layer model and *u* values calculated by the second order exponential function and the single double layer model

$z^{(1)}$	$u^{(2)}$	$Kd^{(3)}$	$u^{(4)}$	$u^{(5)}$	$u^{(2)}/u^{(5)}$
6.1950	2.7447	0.7064	2.7348	3.8796	0.7075
6.1900	2.5728	0.7782	2.5735	3.5719	0.7203
6.1840	2.3187	0.8967	2.3274	3.1276	0.7414
6.1790	2.0429	1.0447	2.0522	2.6619	0.7674
6.1750	1.7478	1.2297	1.7525	2.1871	0.7992
6.1730	1.5571	1.3671	1.5579	1.8948	0.8218
6.1720	1.4447	1.4560	1.4433	1.7283	0.8359
6.1712	1.3435	1.5417	1.3406	1.5825	0.8490
6.1709	1.3023	1.5782	1.2990	1.5244	0.8543
6.1700	1.1655	1.7077	1.1613	1.3355	0.8727
6.1697	1.1144	1.7596	1.1101	1.2668	0.8797
6.1694	1.0598	1.8173	1.0557	1.1947	0.8871
6.1691	1.0012	1.8824	0.9974	1.1184	0.8953
6.1688	0.9377	1.9567	0.9346	1.0373	0.9040
6.1685	0.8683	2.0433	0.8661	0.9504	0.9135
6.1682	0.7911	2.1470	0.7903	0.8561	0.9242
6.1679	0.7038	2.2758	0.7049	0.7519	0.9359
6.1676	0.6016	2.4460	0.6051	0.6337	0.9492
6.1673	0.4749	2.6977	0.4811	0.4923	0.9647
6.1670	0.2935	3.1976	0.2999	0.2984	0.9837
6.1669	0.2502	3.3609	0.2552	0.2534	0.9875
6.1669	0.2455	3.3805	0.2503	0.2485	0.9879
6.1669	0.2254	3.4673	0.2292	0.2278	0.9895
6.1669	0.1974	3.6019	0.1995	0.1991	0.9917
6.1668	0.1573	3.8315	0.1558	0.1582	0.9943
6.1668	0.1235	4.0755	0.1176	0.1240	0.9963
6.1668	0.1025	4.2633	0.0931	0.1027	0.9973

(1) Values of *z* postulated for interacting double layer model
(2) True values of *u*
(3) True values of *Kd*
(4) Values of *u* calculated by eq. (10)
(5) Values of *u* calculated by eq. (9)

Some parameters of the bentonite used in Komine(1996) are given in Table 3. The valence in Table 3 is a mean of Na^+ and Ca^{2+}. The method building the second order exponential best-fit equation based on the single double layer theory is introduced as follows: $(dy/d\xi)_{x=0}$=21.7869 and z=6.16682 could be obtained from equation (2) and (8). And then a series of values of *z* in interacting double layer model can be postulated by taking the values larger than the single double layer *z*. Known *z* of interacting double layer model, *u* and *Kd* can be found from equation (3) and (4), and then be listed in Table 4. The values of *u* calculated by eq. (3) and the values of *Kd* calculated by the numerical integration of eq. (4) here are called true values of *u* and *Kd* respectively.

The best-fit equation fitted by the second order exponential function for true values of u and Kd in Table 4 can be derived as following equation:

$$u = 2.505 * \exp(-Kd/1.204) + 2.505 * \exp(-Kd/1.204) - 0.052 \qquad (10)$$

R^2=0.99996. Substitute true values of *Kd* into above equation, the best-fit values of *u* can then be obtained.

Values of u calculated by various methods are listed in Table 4, and observes from Table 4 indicate that:

1) The differences between best-fit values fitted by the second order exponential function and true values of *u* are negligible, so it is workable to apply the second order exponential function to fit *u-Kd*.

2) The curve of the single double layer model get closer to the true values of *u* with the increasing of *Kd* values, shown in Figure 4. If the ratio of true value *u* to single double layer model *u* needs to reach 0.8, the *Kd* should be 1.2297 at least. However when the model based on the single double layer theory built by Komine (1996) was using, the largest value of *Kd* is 0.9, according to which, the ratio of true value *u* to single double layer *u* is only 0.74.

3) The curves of equation (9) and (10) are also plotted in Figure 4. There is a big gap between two curves when *Kd* is smaller than 1, and only when *Kd* is larger than 1, two curves are closer.

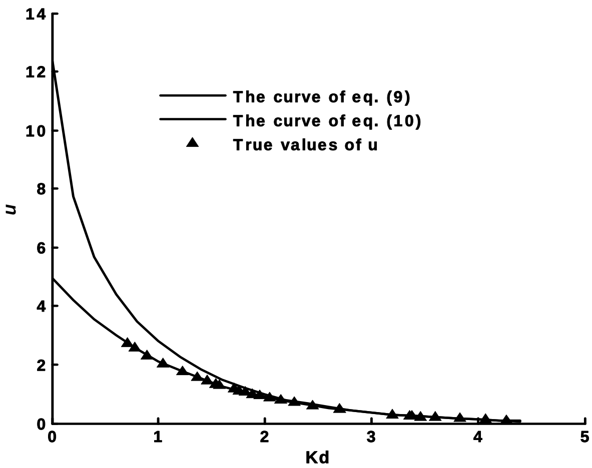

Figure 4: Various relationships of *u-Kd* obtained by various methods

CONCLUSIONS

From the results presented in this paper it is could be concluded that:

(1) Despite the values of (dy/dξ)x=0 and Kd, the second order exponential functions fit u-Kd well and the relevant coefficients R2 are bigger than 0.99. The fitted results using the second order exponential functions are better than those using the linear functions. So the second order exponential function could be used to build the simplified swelling pressure model, which could derive the swelling pressure p directly from e0.

(2) Only when the values of Kd is relative large, the curve of the single double layer model are close to the true values of u, and then the single double layer could be used to established the swelling pressure model.

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