A Parametric Study Leading to Software Analysis Revealing an Equation for the Preliminary Design of Reinforced Earth Embankments

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ABSTRACT

Once appropriate soil parameters i.e. physical and mechanical parameters, pore water pressure, slope geometry are established, slope stability checks need to be performed to ensure passive forces are greater than the active forces tending to cause a slope failure. The designer should be able to ensure that the slope is stable by assigning a certain Factor of Safety (FOS). Two methods have been recommended for limit equilibrium analyses of reinforced slopes. These two major traditional methods are based on allowed and ultimate values. Sometimes the analysis using these equations is performed for the non-geotextile condition, and a critical slip circle and minimum factor of safety is obtained. A driving moment M_D and soil resistance moment M_R are determined. If the Factor of Safety without geotextiles is inadequate, then an additional reinforcement resistance moment can be computed. It has been found that by using these equations for designing the reinforcement required to maintain a specific stability they underestimate the values of required reinforcement compared to the results obtained from the GTS CADBUILD (GEO5) software. In this paper back-analyzed data from GEO5 software has been used to create an empirical equation for preliminary design of the required reinforcement to maintain a specific FOS. The results derived using this equation is compared with data derived using traditional equations.

KEYWORDS: embankment, reinforcement, Factor of safety, critical slip circle parameters, limit equilibrium, slopes

INTRODUCTION

Two methods have been used for limit state equilibrium analyses of reinforced slopes based on allowed reinforcing forces and a specified ultimate reinforcing force [1]. These two methods yield the same amount of required force to achieve a Factor of Safety of unity but differ on other values of FOS. A global Factor of Safety should be multiplied by the allowed tensile strength to get the ultimate strength required. Usually a computer program is used to achieve a target value of factor of safety by conducting repeated trials by varying applied forces.

Centrifuge tests by [2], [3] have clearly confirmed that limit equilibrium analyses provide valid indications of factor of safety and failure mechanism. At the same time when using Limit equilibrium methods (LEM) it should be borne in mind that the analyzed systems are assumed to be on the verge of failure; therefore it is important to understand the residual tensile force at anytime [4]. Logically the factors causing uncertainty in reinforcing elements are not the same as those causing uncertainty in soil strength therefore each element in reinforced embankment should be factored separately. Finally and changing of critical slip circle parameters have a huge effect on estimating amount of required reinforcement. These issues need to be addressed on designing reinforced slopes. Mwasha [5] has demonstrated that currently methods used in limit equilibrium analyses for designing reinforced slopes should be modified accordingly so as to predict long-term behaviour of reinforced embankments on soft soil. In this paper the author will use back-analysis method to design an embankment on the soft soil. According to [1] and Palmeira [6], the back analyzed method has been justifiably used to analyze slopes to ensure long-term stability of the slope. Back-analysis was performed using GEO5 [7]. GEO5 has been validated by [8]. The validation ensured that the software functioned correctly and was reliable. A parametric has been conducted yielding more than 2000 slope stability analyses. These analyses were used to simulate the situation at the end of construction. An empirical equation has been created which can be used for preliminary designing and assessment of slope stability.

FACTOR OF SAFETY

The Factor of safety can be defined with respect to shear strength, load and moment. These different types of definition lead to different values of Factor of safety. The most frequently used definition of FOS is one based on moments. In this case the factor of safety is defined as ratio the ratio of the available resisting moment divided by the actual driving moment as shown in Equation 1

$$FOS = \frac{available\ \ resisting\ \ moment}{actual\ \ driving\ \ momentd} = \frac{M_R}{M_D} \tag{1}$$

The equilibrium Equation shows the balance between the driving moment (MD) and the pseudo-resisting moment which is developed by the available resisting moment (MR) reduced by the factor of safety. If the resisting moment is due entirely to the shear strength of the soil alone, then the factor of safety applied to the resisting moment is the same as the factor of safety defined with respect to shear strength.

A PARAMETRIC STUDY

A parametric study was conducted in order to review typical engineering properties for existing free drained embankment and soft clay soils. The result of this analysis, 4 types of slopes and mechanical properties for free drained embankments and foundation soils were tabulated as shown in Table 1.

These data are to be used for back-analysis the amount of required reinforcement to achieve a specific FOS.

Table 1: Typical value of the relevant parameters extracted from filed data.

	Typical steepest slope (V: H)	Slope range chosen for analysis V: H
Embankment	1:1 to 1:5	1:2 to1:5
	Typical shear strength parameters	Selected shear strength parameters
	$c' = 0(kN/m^2)$, $\phi' = 35^o$ to 41^o	$c'(kN/m^2) = 0$, $\phi' = 35^o$ and 41^o
	Range of bulk unit weight	Selected bulk unit weight
	18 to $20(kN/m^3)$	$18(kN/m^3)$
Soft soil	Typical shear strength parameters	Selected shear strength parameters
	$c' = 0$, $\phi' = 14^o$ to 26^o	$c' = 0$, $\phi' = 14^o$ to 26^o
	Range of bulk unit weight	Selected bulk unit weight
	15 to $20(kN/m^3)$	15 to $22(kN/m^3)$

REINFORCED EMBANKMENT ON THE SOFT GROUND

Depending on how the reinforcement force M_{TR} is incorporated in Equation 1 and whether the factor of safety is applied to a resisting or disturbing moments the results calculated using these traditional methods give different results except if factor of safety is unity.

$$M_D = \frac{M_R}{FOS_G} + \frac{M_{TR}}{FOS_G} \tag{2}$$

$$M_D = \frac{M_R}{FOS_G} + M_{TR} \tag{3}$$

If global FOS is unity then both equations can be represented by Equation 3

$$M_D = M_R + M_{TR} \tag{4}$$

Equation 4 can be rewritten as shown in Equation 5

$$FOS_G = FOS_U + \frac{Y\,T_{RM}}{R\sum W \sin\alpha} \tag{5}$$

Therefore the Global Factor of Safety (FOS_G) can be considered to comprise two components, i.e. that due to the soil shear strength alone (FOSU) and Factor of Safety due to reinforcement (FOSTR) Equation 6.

$$FOS_G = FOS_U + FOS_{TR} \tag{6}$$

and

$$\frac{Y}{R}\frac{1}{\sum W \sin\alpha} = A^* \tag{7}$$

In Equation 7 the value of A* depends on the driving moment and the critical slip circle parameters Y.

Then expression 5 can be re-written as shown in Equation 8, Where TRM is the mobilized tensile strength (Tensile strength at given global FOS)

$$1 = \frac{FOS_U}{FOS_G} + T_{RM} A^* \tag{8}$$

In order to achieve a specific FOS both soil shear strength and tensile strength from the incorporated reinforcements should work together to generate a global Factor of Safety. Since the addition of reinforcement to a slope can change the position of the critical circle (from that of an unreinforced slope FOSU) it is useful to define a parameter (FOSSR), which is the contribution to Factor of Safety, of the tensile force within a reinforced soil. Therefore Equation 8 can be written in the more general form as Equation 9

$$1 = \frac{FOS_{SR}}{FOS_G} + \frac{T_R}{FOS_G} A^* \tag{9}$$

Relation between different types of factor of Safety used is shown in Equation 10

If T_R=0 and FOS_G=1, then

$$FOS_{SR} = FOS_U \tag{10}$$

Back-analysis method

Equation 9 can be used effectively if the critical slip circle parameter A* for reinforced slope is known. A computer program GEO5 was used to analyze simple self drain slopes erected on homogenous soft soil. Slopes having Vertical: Height (V:H) = 1:2, 1:3, 1:4. 1:5 were analyzed based on data from parametric study.. For demonstration the foundation depth (D) was 3m /to an embankment height (H_e) of 3m. The investigation was conducted at the end of construction. The pore pressure at this time was estimated as suggested by [8]. Effective angles of internal friction for foundation (ϕ'f) varied from 15, 20, 23 and 26 degrees and for an embankment (ϕ 'e) varied from 35 to 41 degrees.

The back-analysis process was conducted using GEO5 [7] by incorporating reinforcement parameters at the base of an embankment. A method of trial and error was formulated in order to estimate the tensile strength required to achieve specific FOS. The global Factors of Safety estimated were 1.0, 1.2, 1.5 and 2.0.

Table 2: Estimation of required reinforcement to achieve specific FOS using back analysis methods

SLOPES V:H	Embankment And Foundation parameters	FOS	FOS_{SR}	F_P (kN/m)	F_A (kN/m)	R(m)	T_R (kN/m)	T_R (kN/m)	T_R (kN/m)	T_R (kN/m)
1:2	γ_e = 18kN/m^3 γ_f = 20kN/m^3 , ϕ'_f = 15^o $\phi'e$ = 41^o C' = 0	**0.51**	**0.51**	**52.23**	**102.84**	**5.50**	Un reinforced			
		1.00	**0.66**	92.20	139.03	6,21	90			
		1.20	**0.68**	93.96	139.03	6.21		140.00		
		1.50	**0.69**	96.48	139.03	6,21			215.00	
		2.00	**0.72**	100.10	139.03	6.21				348.00
1:3		**0.56**	**0.58**	**36.35**	**62.17**	**5.47**	Un reinforced			
		1.00	**0.73**	112.82	154.77	8.50	65			
		1.20	**0.74**	115.08	154.79	8.50		109		
		1.50	**0.76**	117.96	154.77	8.50			175	
		2.00	**0.79**	121.88	154.81	8.50				290
1:4		**0.71**	**0.71**	**42.04**	**59,54**	**6.89**	Un reinforced			
		1.00	**0.83**	126.61	152.67	10.10	36			
		1.20	**0.83**	136.02	155.25	10.19		72.00		
		1.50	**0.87**	136.57	156.54	10.37			137.00	
		2.00	**0.90**	140.67	156.54	10.37				243.40
1:5		**0.84**	**0.84**	**50.74**	**60.62**	**4.65**	Un reinforced			
		1.00	**0.88**	139.54	157.97	13.74	23			
		1.20	**0.89**	151.92	170.70	14,00		50		
		1.50	**0.94**	159.91	170.77	14.42			121.00	
		2.00	**0.96**	164.17	170.78	14.92				221.00

Where:

R (m) – Radius of critical slip surface in meters

TR (kN/m) – Tensile strength required to achieve a specified global FOS in kN per meter.

FP (kN/m) –Passive force kilo Newton/meter

FA (kN/m) –Active Force kilo Newton/Meter

When the calculated FOSG corresponds to the required FOSG the resultant output data gives disturbing and resisting moments as well as critical slip circle parameters and values of A* were computed Table 2.

The stages followed when conducting the trial and error analysis were:

1. Select a value of global Factor of Safety (FOSG) for the case to be analyzed

2. Input slope geometry, foundation and embankment soil parameters and run the program to find the minimum Factor of Safety (FOSU) with no reinforcement.

3. If the Factor of Safety determined was less than FOSG required then incorporate an assumed value of force provided by horizontal reinforcement. Re-run the program and repeat the analysis to find the new minimum FOS. If this value is not equal to FOSG then a new value of reinforcement force is assumed and the slope is reanalyzed.

4. The iteration process is repeated until the calculated Factor of Safety is on the range FOSG±0.005

When the calculated FOSG corresponds to the required FOSG the resultant output data gives disturbing and resisting moments as well as the critical slip circle parameters and values of A* were calculated

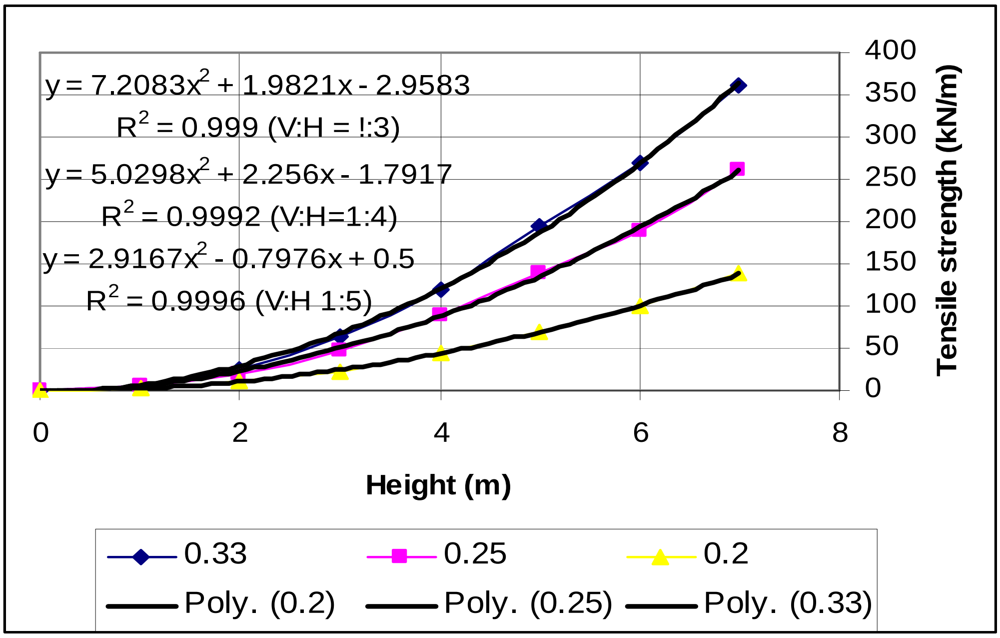

Figure 1: The effects of embankment height and foundation depth on amount of reinforcement required to achieve a global FOS of unity.

Using a back analysis method the amount of required reinforcement for D/H values of 0.5, 1, 1.5, 2 and 2, 5 for slopes V: H 1:3, 1:4 and 1:5 were investigated. It was found that on increasing embankment height and foundation depth there was exponential increase in amount of reinforcement required to achieve a global FOS of unity. A chart for slopes 1:3, 1:4 and 1:5 having D/H ratio of unity is shown in Figure 1.

For Slope V:H = 1:3

$$T_R = 7.21\ He^2 + 1.98He - 2.96 \qquad (11)$$

R^2= 0.9990

For Slope V:H = 1:4

$$T_R = 5.03\ He^2 + 2.26He - 1.79 \qquad (12)$$

R^2= 0.9992

For Slope V:H = 1:5

$$T_R = 2.92\ He^2 + 0.79He\ 0.50 \qquad (13)$$

R^2= 0.9996

Displayed R- squared values for chart on equations were relatively high. Data extracted from this chart compares well with work done by Ingold.[9]. Critical slip circle parameters were further integrated into Equation 7 to creation of parameter A*

INVESTIGATING THE EFFECTS OF A* ON SLOPE PARAMETERS

A* was further analyzed by varying it with foundation and embankment parameters.

It was found that the value of A* varies with both slope angle and effective angle of internal friction. The values of A* increased as the slopes become flatter but for each slope the value of A* was unaffected by FOS_G as shown in Table 3. The exception was for slope V: H=1:5 where there was a small variation. Since the later variation is small the author decided to assume a constant value for all slopes. If the value of FOS_G is unity the values of A* per unity T_R were 0.0037, 0.0042, 0.0046 and 0.0050 for slopes V: H =1:2, 1:3 1:4 and 1:5 respectively.

Table 3: Values of A*

FOS_G	Value of A* at unity T_R			
	V:H=1:2	V:H=1:3	V:H=1:4	V:H=1:5
1.00	**0.0037**	**0.0042**	**0.0046**	**0.0050**
1.20	0.0037	0.0042	0.0046	0.0046
1.50	0.0037	0.0042	0.0046	0.0049
2.00	0.0037	0.0042	0.0046	0.0047

The values of A* was found to increase exponentially as the slope becomes flatter and could be represented by Eq. 14.

$$A^{*} = 0.0058 - 0.0041\beta^{0.95} \tag{14}$$

Where β is the slope angle.

Therefore Eq. 8 can be rewritten to Eq. 15

FOSG = FOSTR +(0.0058-0.0041β0.95)TR (15)

The value of A* was found to vary with the angle of internal friction as shown in Table

2 The values of A* increase as effective angle of internal friction increases for all slopes investigated. During the back-analysis process it was found that for slopes V:H = 1:4 and 1:5 no reinforcement is required to achieve global FOS of unity if the angle of internal friction for the foundation soil is equal or more than 20°. For slope V:H = 1:3 no reinforcement is required to achieve global FOS of unity if ϕ′ is equal or more than 23 degrees therefore the values of A* are omitted in Table 4.

Table 4: Effects of effective angle of internal friction on values of A*

ϕ′	Value of A* for the reinforced slopes and Tv=0.00			
	V:H=1:2	V:H=1:3	V:H=1:4	V:H=1:5
15	0.0037	0.0042	0.0046	0.0050
20	0.0051	0.0055		
23	0.0059	0.0061		
26	0.0064			

It was found that values of FOS_{SR} increased almost linearly with increasing effective angle of internal friction of foundation soil Table 4. The relationship has been represented using Λ

(FOS_{SR} verses tanϕ′ graph) and ξ (is the intercept on the FOS_{SR} axis) as shown in Equation 11

The results of this investigation showed that the value of Λ increased and values of ξ decreased. The results are shown in Table 5.

Table 5: Values of Λ on varying slope angle

	Values of Λ and ξ for slopes(Tv=0.00)			
	V:H=1:2	V:H=1:3	V:H=1:4	V:H=1:5
Λ	1.20	1.78	2.50	2.88
ξ	0.34	0.25	0.17	0.12

$$FOS_{SR} = \xi + \Lambda \tan\phi' \tag{16}$$

Substituting Equation 9 into Equation 16 creates an Equation 17, which represents the variation of global FOS with slope angle, effective angle of internal friction of foundation soil and the tensile force required to achieve a specific FOS at the end of constructing an embankment of the soft soil:

$$FOS_G=\xi+\Lambda\tan\phi'+A^* T_{R0} \tag{17}$$

Predicted values of tensile force are compared against values obtained by analysis using GEO5 as shown in Table 6 . There is good correlation between the two sets of values.

$$T_{R0} = \frac{FOS_G - \xi - \Lambda \tan\phi'}{A^*} \tag{18}$$

Equation 18 can be used during the preliminary design stage of an embankment on soft clay to estimate required tensile force (and hence select suitable reinforcement) required to achieve a specific FOS at Tv = 0, if the slope and effective angle of internal friction of the foundation soil is known.

Table 6: Predicted reinforcement required to achieve specific FOS_G of unity on varying the slope angle and the effective angle of internal friction of the foundation soil

ϕ`	Predicted and back-analyzed tensile force T_R(kN/m)) ϕ_e`= 41 and Tv=0.00			
	V:H=1:2	V:H=1:3	V:H=1:4	V:H=1:5
	predicted	predicted	predicted	predicted
15°	92.0	65.00	35.00	22.00
20°	48.50	23.00	0	0
23°	31.00	5.00	0	0
26°	18.00	0	0	0

Table 7: Comparing different methods of estimating amount of required reinforcement at given FOS_G

Global FOS (FOS_G)	Prediction of reinforcement for the given slopes (V:H) to achieve given global FOS (kN/m) ϕ_e`= 41 and $\phi$`$_f$=15							
	V:H=1:5		**V:H=1:4**		**V:H=1:3**		**V:H=1:2**	
	A	**B**	**A**	**B**	**A**	**B**	**A**	**B**
1.00	**22.0**	12.0	**35.0**	22.0	**65.0**	35.0	**92.0**	73.0
1.20	**62.0**	26.0	**78.0**	36.0	**112.6**	52.0	**146.0**	103.0
1.50	**122.0**	48.0	**143.0**	59.0	**184.0**	78.0	**227.0**	148.0
2.00	**222.0**	84.0	**252.0**	96.0	**303.0**	121.0	**362.0**	236.0

A- Estimated reinforcement using Equation 13; B- Estimated reinforcement using traditional method.

Comparing with traditional methods

Table 7 contains a summary of tensile force predicted using traditional methods [6] [7]. The results are also compared with the results of back-analysis using computer program GEO5. The data for these analyses were obtained after conducting back-analysis using computer program GEO5.

It was found that the amount of required reinforcement to achieve specific global FOS is highly underestimated. The reasons for this underestimation may be caused by these factors:

- FOSSR is higher than FOSU (Factor of Safety for unreinforced soil) therefore FOSTR is overestimated. In this case the amount of reinforcement is underestimated since the higher the FOS the lower the amount of reinforcement are required to achieve stability. Equation 19

$$FOS_G - FOS_{SR} = FOS_{TR} < FOS_G - FOS_U = FOS^*_{TR} \quad (19)$$

- The active force is usually higher for reinforced slope than for unreinforced slope. (Table 2) Therefore on using lower value of active force in traditional methods results in lower value of reinforcement.

- Critical slip surface parameters such as radius and lever arm do vary from unreinforced slope as compared with reinforced slopes [10]. The ratio between the radius and lever arm (Y/R) for unreinforced soil is higher as compared with that of reinforced slope. The high value of (Y/R) could contribute to lower value of reinforcement required as exhibited on using traditional method.

As shown in Table 8, the predicted amount of reinforcement required marginally exceed those found using back-analysis method.

Table 8: Tensile force estimation to achieve specific global FOS

Authors	Reinforcement required T_R (kN/m) at given Global FOS_G V:H 1:2			
	1.00	1.20	1.50	2.00
Proposed Equation 13	92.00	146.00	227.00	362.00
USA-Technical Manual TM 5-818-8 (1995) [11] Duncan and Wright (2005)[1]	30.00	37.69	46.37	55.27
Morris(1998)[12]; Duncan and Wright (2005)[1]	30.00	45.23	69.56	110.54
Computer program GEO5 Back analyses[7]	90	140	215.00	348

Example

A reinforced embankment V:H 1:3 is erected on soft soil. The effective angles of internal friction for foundation and embankment soils are 15 and 41 degrees respectively. Bulk density for foundation and slope are 20kN/m^3 and 18kN/m^3. Using equations and charts provided find amount of reinforcement required to achieve a specific FOS of unity. Compare results with chart the proposed chart.

1. Using Equation 14 find coefficient A*

$$A^* = 0.0058 - 0.0041\beta^{0.95}$$

Where β is the slope angle = 0 333 therefore A* = 0.0042

2. Find FOS_{SR} using Equation 15

$$FOS_{SR} = FOS_G - (0.0058\text{-}0.0041\beta^{0.95})T_R \quad \text{or } FOS_{SR} = 1 - 0.0042\, T_R$$

Using Equation 16, $FOS_{SR} = \xi + \Lambda \tan\phi'$

From Table 3 find values for Λ and ξ which are 1.78 and 0.23 respectfully;

$FOS_{SR} = 0.71$

3. Using Equation 18 find amount of reinforcement required to achieve a specific FOS of unity T_{R0} = 68 kN/m

CONCLUSION AND RECOMMENDATIONS

It has been found that two traditional equations used for estimating amount of required reinforcement to achieve a specific FOS underestimate the value of required reinforcement by a large amount. The proposed equations can be used for preliminary assessment of reinforcement required to achieve a specific FOS. It is strongly recommended that the future work on this topic to be conducted by varying foundation depth as well as embankment height for different soil parameters. These different parameters will be used to create wider applicable solution for this problem.

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