Determination of Rock Strength and Deformability of Intact Rocks

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ABSTRACT

The paper presents the results of laboratory tests performed on a large number of intact sedimentary rock specimens (limestone, sandstone, siltstone) and metamorphic rocks from Greece. The physical properties (porosity, dry density), mechanical properties (uniaxial compressive strength), Young's modulus, point load index, Schmidt hardness) and dynamic properties (wave velocity) were determined. Furthermore, laboratory results of intact rock properties were collected and evaluated. From the statistical analysis of the data, regression equations were established amongst intact rock parameters. The ratio of rock strength and deformation modulus was investigated and its range was determined for all rock types. A comparison was made between existing empirical equations for the correlation of uniaxial strength, Schmidt hardness and dry density (σ_c-SHV-ρ_d). This comparison proved that Deere - Miller's (1966) chart yields the better prediction of the uniaxial compressive strength. Additionally, the correlation between uniaxial compressive strength, σ_c, and point load strength, Is_{50}, shows that an exponential fit gives a significantly better fit than a linear one. Finally, a multivariable analysis was done between tangent Young's modulus, uniaxial compressive strength and wave velocity (E_t-σ_c-V_p) and an empirical correlation is proposed.

KEYWORDS: Intact rock, Index properties, Point load strength, Uniaxial compressive strength, Modulus of elasticity, Statistical relationships

INTRODUCTION

Rock engineering properties is considered to be the most important parameters in the design of ground works and in the classification of rock for engineering purposes. The expression of correlations between the engineering properties of intact rock has always been the scope of experimental research, driven by the need to depict the actual behavior of rock and to calculate most accurately the design parameters.

At the present study investigation of physical, mechanical and dynamic properties was performed, correlations were expressed and compared by the ones proposed in literature. The strength and deformability of intact rock was investigated thoroughly. In order to accomplish that, a number of laboratory tests on limestone, sandstone and siltstone were conducted and a large database, containing results on laboratory tests on intact rock, was created.

LABORATORY TESTING

A large number of rock mechanics laboratory tests were conducted on intact rock specimens of sedimentary rocks originated from central and southern Greece (Tziallas, 2008). The following tests were performed: determination of porosity and dry density (n, ρ_d), uniaxial compression test (σ_c) with determination of tangential Young modulus (E_t), point load test ($Is_{(50)}$), Schmidt hammer test (SHV) and ultrasonic wave propagation test (Vp, Vs). For the determination of E_t, L.V.D.T. was used and a Schmidt hammer type L for the corresponding test.

In order to propose correlations of rock properties, a database (containing more than 5000 records) was created with data from literature as well as from the Central Laboratory of Public Works. The database contains laboratory tests on 37 different sedimentary, igneous and metamorphic rocks. Both the laboratory tests and this large database resulted in more than 80 reliable correlations (with $r^2>0.5$), and a number of conclusions as presented here after.

ROCK STRENGTH AND DEFORMABILITY

Range of E_t and σ_c ratio

The range of Young modulus in relation to the uniaxial compressive strength was determined. The correlation between these mechanical properties can only produce the boundary lines as the scatter is high. The slope of the boundary lines represent the modulus ratio, MR (eq.1) which was introduced by Deere (1968) and expresses the ratio of Young's modulus E_i and the uniaxial compressive strength of intact rock σ_{ci}:

$$MR = \frac{E_i}{\sigma_{ci}} \tag{1}$$

The range of the ratio for the sedimentary, metamorphic rocks types is shown in figure 1.

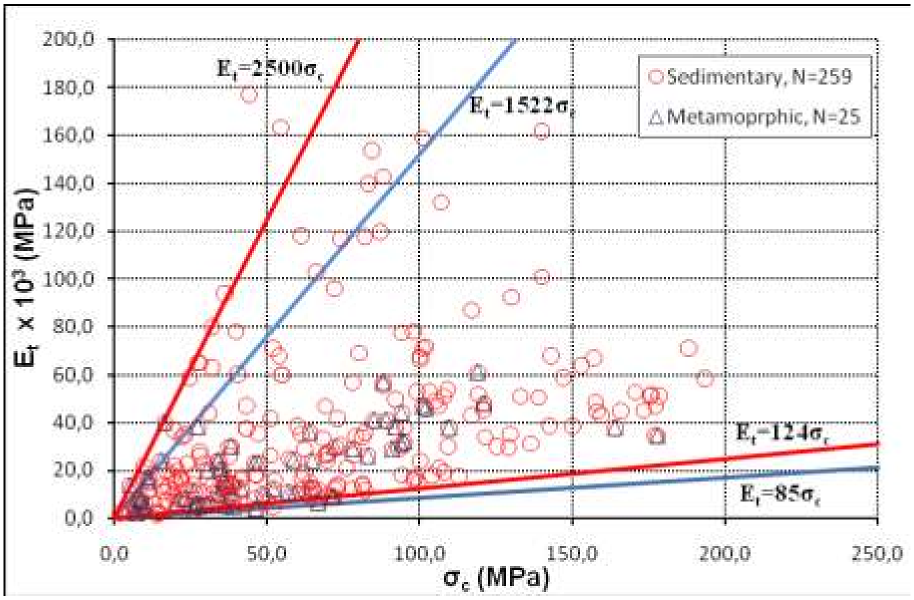

Figure 1: Relation of Et/σ_{ci} for sedimentary and metamorphic rocks

Metamorphic rocks present a narrower range than the sedimentary ones. However, the range of MR for igneous rocks is so limited that an exponential equation (eq.2) can be expressed, with very high regression coefficient (r^2=0.95):

$$\mathbf{E_t = 3576.5e^{0.016\sigma_{ci}}} \qquad (2)$$

where E_t and σ_{ci} in MPa

Determination of modulus ratio MR

A detailed analysis was conducted for the range of MR for various rock types originated from Greece.

With statistical analysis methods, the range of modulus ratio MR was defined for each rock type and the comparison was made with the proposed values of Hoek & Diederichs (2006) as shown in Table 1. Additionally, the minimum, the maximum values of MR and the number of the specimens of each rock type are presented.

It is obvious that the proposed MR values, are lower and the range is smaller than that proposed by Hoek & Diederichs (2006) except of the sedimentary rocks where the values are similar. For the metamorphic and igneous rocks, the difference is significant where in some rocks (e.g. marble) the range is lower than this proposed by Hoek & Diederichs (2006). The lower value of MR, represents either lower Young modulus or higher unconfined strength σ_c for each rock. Therefore, the MR values proposed here signify rocks of lower stiffness (higher deformability), which can sustain higher deformation before fracture.

Table 1: Number of the specimens, minimum, maximum and the range of MR

Rock	N	Min	Max	MR	MR (Hoek & Diederichs, 2006)
Sandstone	17	158	1157	200-400	200-350
Crystalline Limestone	68	160	1122	300-500	400-600
Dolomite	5	350	467	300-400	300-500
Marble	17	313	648	300-500	700-1000
Schist	39	84	919	100-800	250-1100
Granite	19	228	309	200-350	300-550
Andesite	14	133	202	150-250	300-500
Basalt	4	173	199	150-200	300-500
Agglomerate	17	124	1183	100-200	400-600
Tuff	6	97	239	100-250	200-400

Furthermore, the modulus ratio of schist appears widely scattered due to foliation, as Hoek & Diederichs (2006) have mentioned, where the MR value is highly dependent on whether the laboratory tests were conducted parallel or perpendicular to the foliation planes. The range of MR in limestone shows that a significant number of data fall between 500 and 700, which represents the case of more brittle limestones or limestones with lower uniaxial compressive strength.

CORRELATION BETWEEN E_t, σ_C AND V_P

A multivariable correlation was done for two mechanical parameters, Young modulus E_t and uniaxial compressive strength, σ_{ci}, with longitudinal wave velocity, V_p, for all categories of rocks. Saroglou (2007) has proposed similar correlations of mechanical properties of anisotropic intact rock. Such a correlation is very useful in order to determine Young's modulus, the measurement of which is generally expensive and time consuming, whereas the determination of the other two parameters is rather easy and cheap.

The correlation refers to sedimentary rocks (limestone, sandstone and dolomite), metamorphic rocks (gneiss, marble and schist) and igneous rocks (granite and peridotite). Using Statsoft Statistica® software, the proposed correlation is as follows:

$$\boldsymbol{E_t = 0.051(\sigma_{ci} \cdot Vp)^{0.541} - 0.27\sigma_{ci}}, \quad \mathbf{r^2=0.77} \tag{3}$$

where E_t in GPa, σ_{ci} in MPa and V_P in m/sec

This correlation can be represented as a surface on a three-axis plot or as a monogram in a two-dimensional plot as shown in fig. 2 and fig. 3 respectively.

The curves of lower Vp terminate at a certain value of σ_{ci} as there is no intact rock, which will display significant uniaxial compression strength with such a low longitudinal wave velocity.

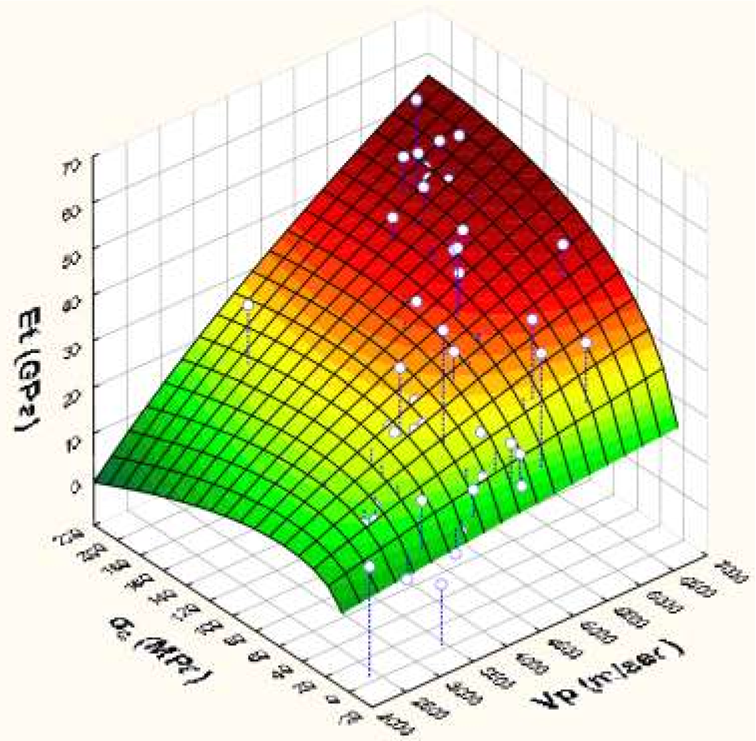

Figure 2: Correlation of E_t, σ_c and V_P on a three-axis plot

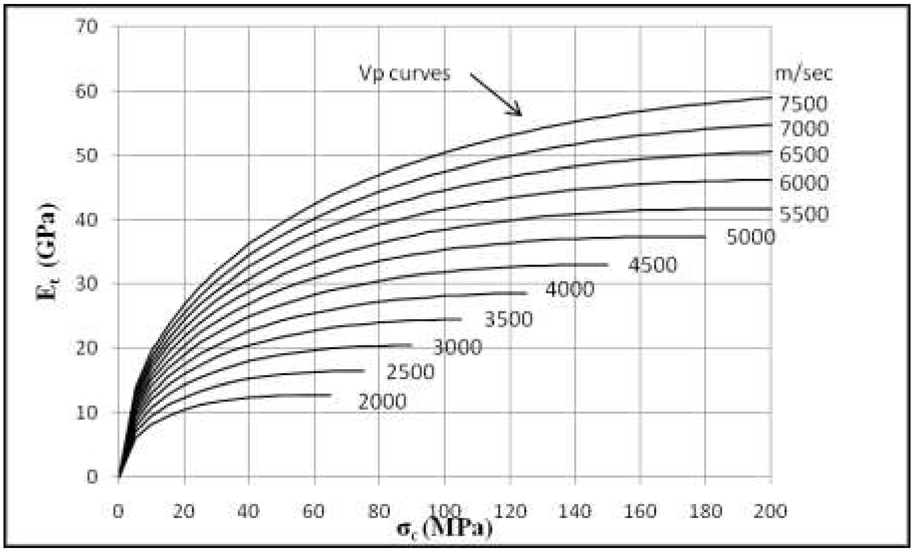

Figure 3: Correlation of E_t, σ_c and Vp as a monogram

The conclusion that can be drawn from the above diagram is that as the longitudinal wave velocity increases, so does the Young's modulus, which is expected, since higher values of V_P imply strong interparticle forces between the rock grains, less microcracks and lower degree of anisotropy, thus lower values of deformability (resulting in higher E_t).

DETERMINATION OF σ_{CI}, SHV AND ρ_D CORRELATION

The correlations proposed in literature between unconfined compressive strength σ_{ci}, Schmidt hammer rebound value SHV, and dry density ρ_d were investigated in order to define the effectiveness of each one in predicting rock strength. The laboratory determined values from the database were compared with the estimated by each method for various rocks. The correlations examined are the following:

Table 2: Empirical equations correlating σ_{ci}, SHV and ρ_d

Author	Equation
Deree & Miller (1966)	Chart only
Aufmuth (1973)	σ_{ci}=6.9γ^2SHV-1.14*10^6
Beverly et al. (1979)	σ_{ci} =12.74exp(0.185γSHV)
Kidybinski (1980)	σ_{ci} =0.447exp[0.045(SHV+3.5)+γ]
Kahraman et al. (1996)	σ_{ci} =4.5*10^{-4}(SHVγ)$^{2.46}$

Deere & Miller’s chart is not described with an equation. Additionally, the ones proposed in literature for this purpose are not representative. In the present study, the following descriptive equation of the diagram is given:

$$\sigma_{ci} = 10^{(0.0009\times\rho_d\times SHV+1)} \tag{4}$$

$$d_1 = \pm\frac{\log(\sigma_{ci}) - 0.25}{0.05} \text{ for } \sigma_{ci} < 100MPa \tag{4.1}$$

$$d_2 = \pm\frac{\sigma_{ci} - 48.1}{1.48} \text{ for } \sigma_{ci} > 100 \text{ MPa} \tag{4.2}$$

Where ρ_d in kN/m^3, σ_{ci} in MPa and d_1, d_2 the correction proposed in the present study.

It is obvious that the more accurate correlation is that from Deere & Miller. Prediction by Beverly et al. (1979) overestimates the strength while Aufmuth’s (1973) proposal underestimates it (Fig. 5). On the other hand Kidybinski’s (1980) and Kahraman et al. (1996) equations have good prediction for high values of σ_{ci} (for igneous rocks) but for lower values the scatter is significant.

Deere & Miller’s (1966) σ_{ci} chart is generally used without taking into account the correction proposed by the authors. Illustrating both the corrected values by the equation proposed in the present study (Eq. 4.1 and 4.2) and the uncorrected values of σ_{ci}, the difference is very obvious.

The sign (positive or negative) in eq. 4.1 and 4.2 is determined accordingly. For higher values of uncorrected estimated uniaxial compressive strength, the correction should be subtracted by the estimated value.

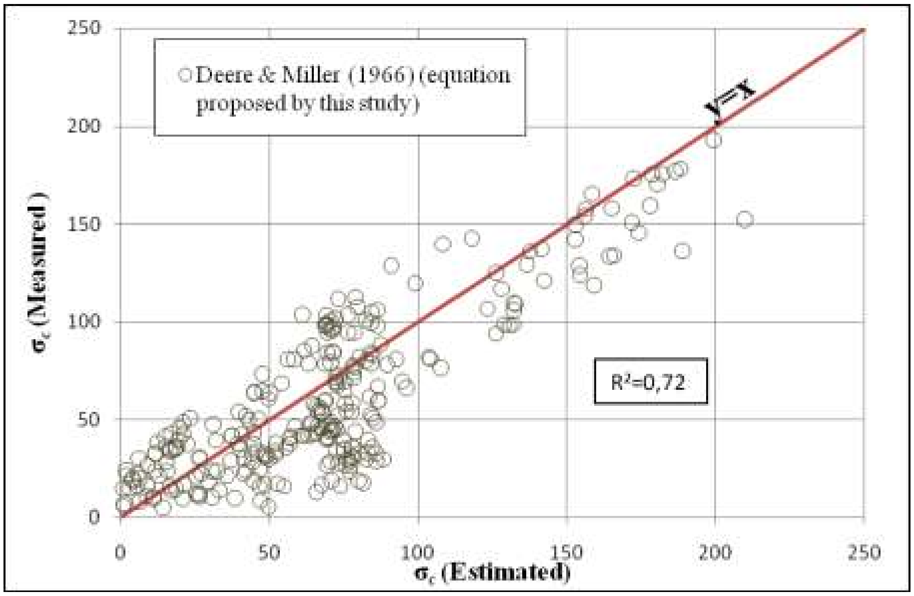

Figure 4: Accuracy of Deere & Miller (1966) correlation using the equations proposed in the present study

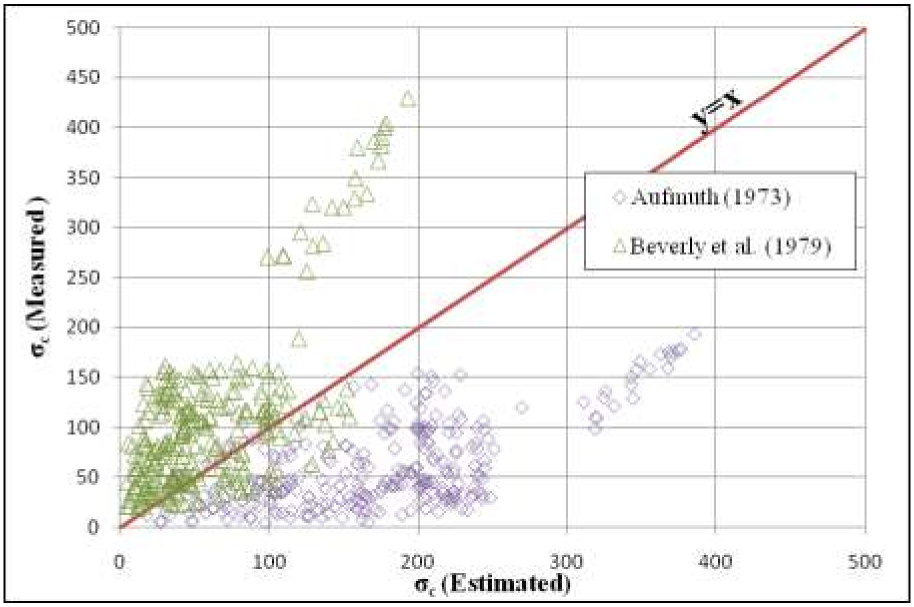

Figure 5: Accuracy of the empirical equations proposed by Aufmuth (1973) and Beverly et al. (1979)

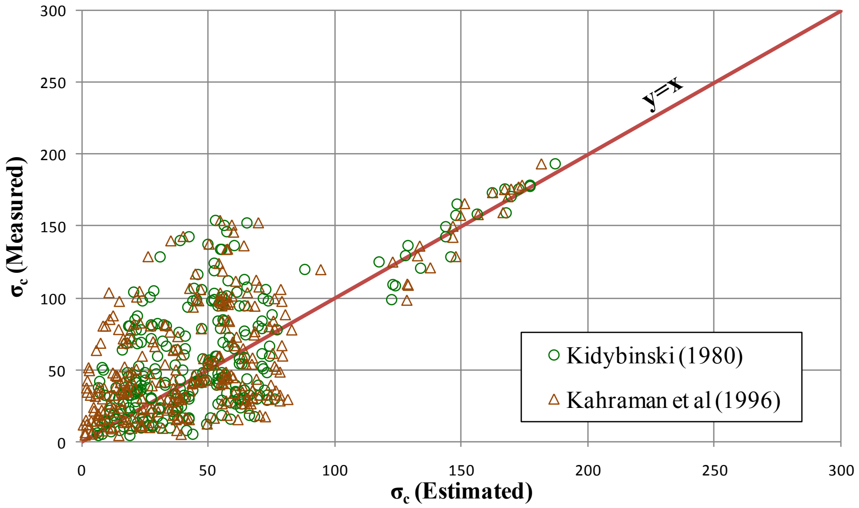

Figure 6: Accuracy of the empirical equations proposed by Kidybinski (1980) and Kahraman *et al.* (1996)

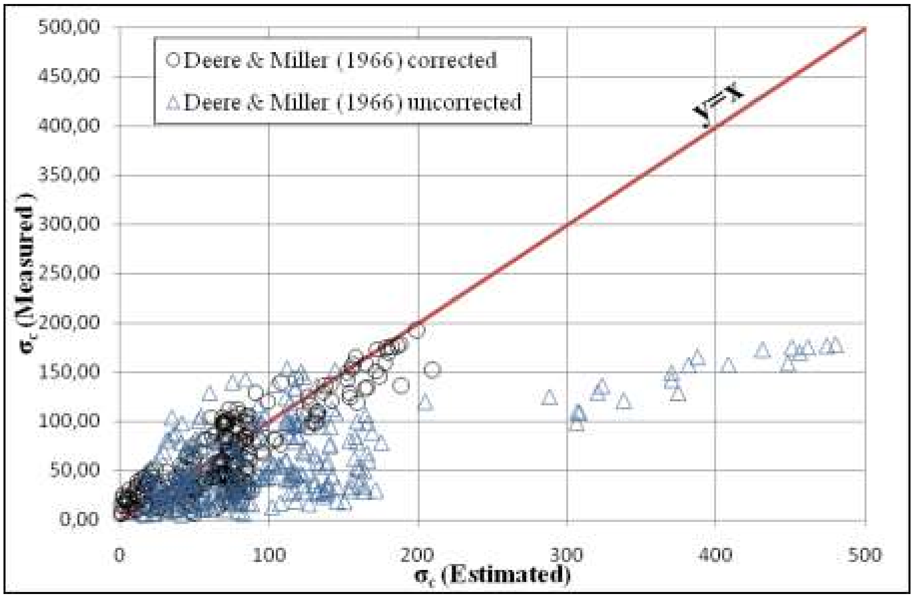

Figure 7: Corrected and uncorrected values of σ_{ci} (Deere & Miller, 1966)

CORRELATION OF σ_C AND $Is_{(50)}$

Generally, the uniaxial compressive strength and the point load index are correlated using linear empirical equations. Initially Hawkins (1998) proposed coefficient “k” which falls in the range of 15-25 and 10-24 depending on the range of point load index and the condition of specimen (dry or saturated) for sandstone, limestone and igneous rocks.

A significant number of correlations for σ_{ci} and $Is_{(50)}$ exists in literature as presented in Kahraman (2001).

Tsiambaos & Sabatakakis (2004) have proposed exponential equation (eq. 5) for sedimentary rocks (limestone, sandstones, marlstone)

$$\sigma_{ci} = 7.3 Is_{(50)}^{1.71} \quad , \quad r^2 = 0.82 \qquad (5)$$

Based on Hawkin's categorization of $Is_{(50)}$, Sabatakakis et al. (2008) proposed the following linear equations (eq. 5.1, 5.2 and 5.3) derived by the exponential equation (eq. 5):

$$\sigma_{ci} = 13 Is_{(50)} \text{ , for } Is_{(50)} < 2 \qquad (5.1)$$

$$\sigma_{ci} = 24 Is_{(50)} \text{ , for } 2 < Is_{(50)} < 5 \qquad (5.2)$$

$$\sigma_{ci} = 28 Is_{(50)} \text{ , for } Is_{(50)} > 5 \qquad (5.3)$$

Based on the data from literature, uniaxial compressive strength was correlated with $Is_{(50)}$ using both linear and exponential regression equations for the following rock categories: all rock types, sedimentary, metamorphic, igneous and sandstone (shown in fig. 8 and 9).

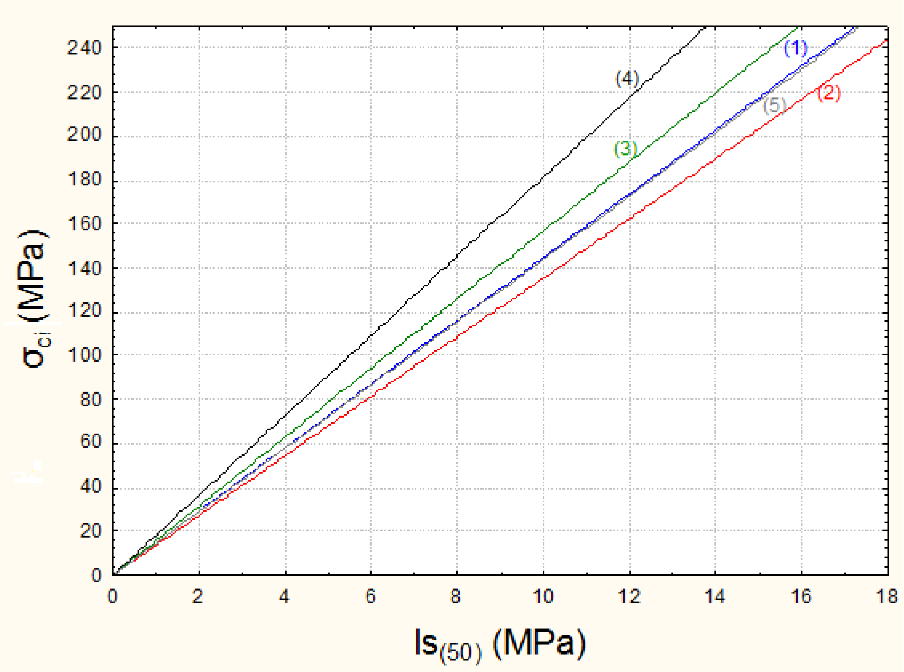

Figure 8: Linear fit for the correlation of σ_{ci} and $Is_{(50)}$ for various rock types.

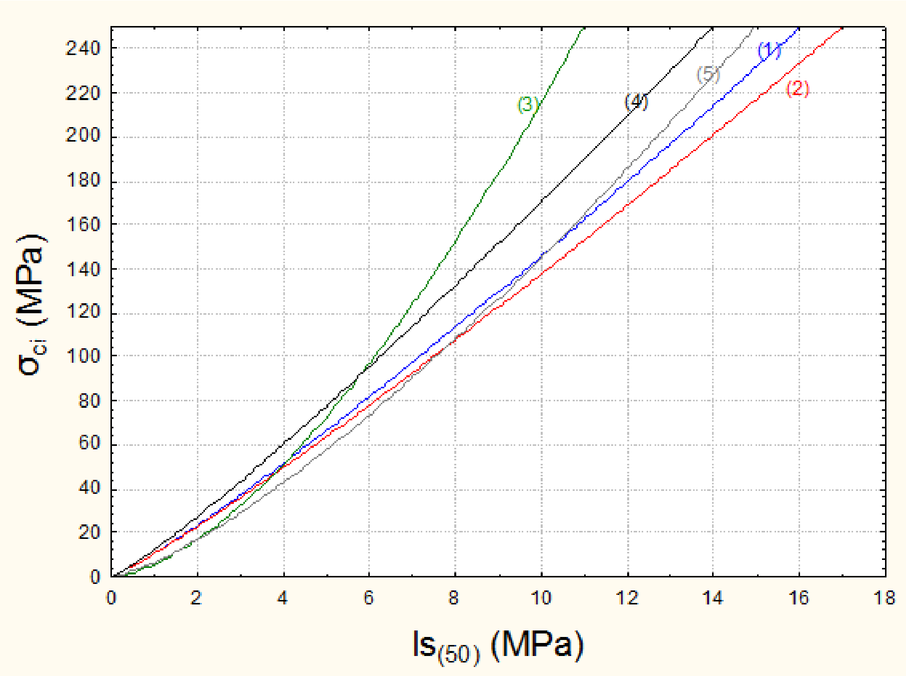

Figure 9: Exponential fit for the correlation of σ_{ci} and $Is_{(50)}$ for various rock types.

Table 3: Regression equations and coefficients for both linear and exponential fit

	Rock Type	N	Equation	r^2
═ **(1)**	All rock types	323	$\sigma_c = 14.49\ Is_{(50)}$ $\sigma_c = 10.58\ Is_{(50)}^{1.14}$	0.62 0.66
═ **(2)**	Sedimentary	257	$\sigma_c = 13.55\ Is_{(50)}$ $\sigma_c = 10.46\ Is_{(50)}^{1.12}$	0.48 0.61
═ **(3)**	Sandstone	61	$\sigma_c = 15.70\ Is_{(50)}$ $\sigma_c = 5.69\ Is_{(50)}^{1.58}$	0.62 0.79
═ **(4)**	Metamorphic	25	$\sigma_c = 18.15\ Is_{(50)}$ $\sigma_c = 12.36\ Is_{(50)}^{1.14}$	0.78 0.75
═ **(5)**	Igneous	34	$\sigma_c = 14.40\ Is_{(50)}$ $\sigma_c = 6.65\ Is_{(50)}^{1.34}$	0.88 0.91

The considerable difference of the regression coefficient of each rock category is obvious. The exponential equation is more accurate than the linear one for almost all rock types except from the metamorphic rocks, where the linear equation is more accurate. The correlation of uniaxial compressive strength with point load index is summarized in Table 3.

CONCLUSIONS

In the present study, original correlations of intact rock properties have been proposed in order to determine indirectly the uniaxial compressive strength and modulus of elasticity of intact rock.

The determination of the modulus of elasticity is achieved through: a) the uniaxial compressive strength (σ_{ci}), b) the uniaxial compressive strength (σ_{ci}) and the longitudinal wave velocity (V_P).

The ratio of the modulus of elasticity and uniaxial strength (MR) has been determined for a number of rock types and compared with that proposed by Hoek & Diederichs (2006). It was found that MR is different than that proposed by Hoek & Diederichs (2006) and that may be attributed to a number of reasons (rock texture, magnitude of inter-particle forces especially in sedimentary rocks, degree of weathering, anisotropic nature of metamorphic rocks). Additionally, the geological history that rocks have undergone in a certain geographic area can affect significantly the strength and deformability of particular rock types (mainly sedimentary rocks like limestones and sandstones but also some metamorphic rocks, like marble).

An original multi-variable equation for the prediction of modulus of elasticity, using the uniaxial compressive strength and the longitudinal wave velocity, was proposed. The prediction for the laboratory data was very good and thus chart, which was based on this equation, was proposed. This chart can be used for all rock types.

The proposed indirect methods for the determination of uniaxial compressive strength are using: (*a*) the Schmidt hardness (SHV) and the dry density (ρ_d) and (*b*) the point load index ($Is_{(50)}$).

An equation describing the Deere & Miller's (1966) chart was proposed and applied to the laboratory data. It was shown that Deere & Miller's equation for determining the strength of rock, when taking into account the proposed correction, is much more accurate than the ones proposed in literature. However, it should be used with the corrected values of uniaxial compressive strength to avoid wrong estimation.

Finally, the relation between uniaxial compressive strength and point load index was determined for all the rock categories. It was concluded that the exponential regression yields a higher correlation degree than the linear one.

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