Face Stability Analysis of Tunnel with Pipe Roof Reinforcement Based on Limit Analysis

Haitao Wang
PhD candidate, State Key Lab. of Coastal and Offshore Eng.,
Dalian University of Technology, Dalian, China
e-mail:wht_dalian@163.com.

Jinqing Jia
Professor, State Key Lab. of Coastal and Offshore Eng.,
Dalian University of Technology, Dalian, China
e-mail: keyknown@163.net

ABSTRACT

Based on the kinematic method of limit analysis and the shear strength reduction technique, the three-dimensional model for expressing the tunnel face stability with pipe roof reinforcement was established. For a typical example, the solutions computed by the proposed approach were compared with the results given by wedge model, trapezoid wedge model and centrifugal-model test. The proposed approach was also employed to study how the cover depth of tunnel, tunnel diameter and soil parameters affect the support pressure and safety factor of tunnel face. It is shown that the calculation results of limit analysis is smaller when compared with that by wedge model and trapezoid wedge model, and also more closer to the results of centrifugal-model test. It is also found that for the relative depth investigated ($H/D=0.5\sim 8$), cover depth has a little influence on the stability of tunnel face for a given diameter, but a good linear relationship exits between safety factor and tunnel diameter. The linear and quadratic formulas are able to account for the influence of cohesion and friction angle of soil on limited support pressure of tunnel face, respectively. The analytical results of tunnel face stability also reveal that the safety factor of tunnel face with pipe roof reinforcement is about 1.05~1.20 times of that without pipe roof reinforcement. The effect of pipe roof reinforcement on tunnel face stability is not significant.

KEYWORDS: pipe roof reinforcement; tunnel face stability; limit analysis; strength reduction technique

INTRODUCTION

The construction of tunnel in weak ground can give rise to some problems including instability of the tunnel, damage to already existing structures, and so forth. In order to overcome these problems, some auxiliary techniques are adopted in tunneling. One of such methods is the pipe roof reinforcement method. This method consists on installing, prior to the excavation of a length of tunnel, a series of pipes, either parallel to the tunnel axis or at a certain angle with it. By injecting grout through the pipes, the ground in between the pipes is stiffened and the pipes are connected, creating a kind of 'umbrella' above the area to be excavated. This arrangement creates a stiff layer of ground and allows safe excavation even in poor ground conditions.

In recent years, there have been numerous studies into the behaviors of pipe roof reinforcement. Ibrahim (2008) reported the field monitoring results from a tunneling project involving pipe roof reinforcement technique. They found that with the use of pipe roof reinforcement, the stability of the tunnel face in a poor geotechnical condition is improved and the ground movement can be reduced effectively. Chang (1999) implemented the analytical design method for the pipe roof reinforcement based on Winkler elastic foundation beam theory. Considering the delay effect of initial lining and revising the Winkler elastic foundation model, Wang and Jia (2008) investigated the mechanical behaviors of pipe roof reinforcement based on Pasternak elastic foundation beam theory. The calculation results prove that the Pasternak model is a better way to understand the reinforcement mechanism and improve design practice. Yoo (2002) conducted a parametric study on the effect of reinforcing layouts on the deformation behavior of the tunnel face and drew a conclusion that there existed an optimum reinforcing layout to reduce the deformation of the tunnel for a given tunnel geometry and ground condition. Hisatake and Ohno (2008) conducted a series of centrifuge tests to clarify the effects of pipe roof supports and the excavation method on the displacements above a tunnel face. The results of the model tests show the quantitative effects of the ring-cut excavation method and clarify that the method decreases the ground displacements remarkably. The model tests also indicate that the maximum settlement of a ground excavated by the full-face excavation method with pipe roof supports is one fourth of that without them. Recently, Shin et al. (2008) performed a large scale model testing for the pipe-reinforced tunnel heading in a granular soil to understand reinforcing mechanism and improve design practice. They found that the pipe reinforcement of heading increases longitudinal load transfer to an unexcavated area, and consequently decreases deformation and increases face stability.

Overview of the previous studies indicates that there are few attempts to evaluate the tunnel face stability with pipe roof reinforcement quantitatively. In this study, based on the kinematic method of limit analysis and the shear strength reduction, the three-dimensional model for expressing the tunnel face stability with pipe roof reinforcement is established. For a typical example, the solutions computed by the proposed approach are compared with the results given by wedge model, trapezoid wedge model and centrifugal-model test to verify the reasonability of the method. The proposed approach is also employed to study how the cover depth of tunnel, tunnel diameter and soil parameters affect the support pressure and safety factor.

LIMIT ANALYSIS FOR TUNNEL FACE STABILITY

Three-dimensional analytical model

Leca and Dormieux (1990) used the limit analysis concept to evaluate the stability of a tunnel face driven in frictional soil and compared these results with centrifuge tests performed by Chambon and Corte (1994). A reasonable agreement was found between the theoretical upper bound estimates and the face pressures measured at failure from the tests. Therefore, by modifying the upper bound solution suggested by Leca and Dormieux (1990), the face stability of tunnel with pipe roof reinforcement was evaluated quantitatively in this study. To establish the three-dimensional analytical model, some key assumptions were necessary and they were listed as follows:

1. The soil is modeled as a Mohr-Coulomb material, characterized by its cohesion c and its friction angle φ throughout this study;

2. Failure mechanism was assumed to involve the movement of solid conical blocks with circular cross-sections as shown in Fig. 1. Two cones were considered, C_1 (apex Ω_1, axis Δ_1) of which the base $\sum_1$ is in the same plane as the tunnel face; and C_2 (apex Ω_2, axis Δ_2) of which the base $\sum_{12}$ is in plane π (C_1 and C_2 have the same geometric properties). The first moving block B_1 corresponds to the portion of C_1 located below plane π and B_2 is the portion of C_2 located below the lower surface of the pipe roof reinforcement.

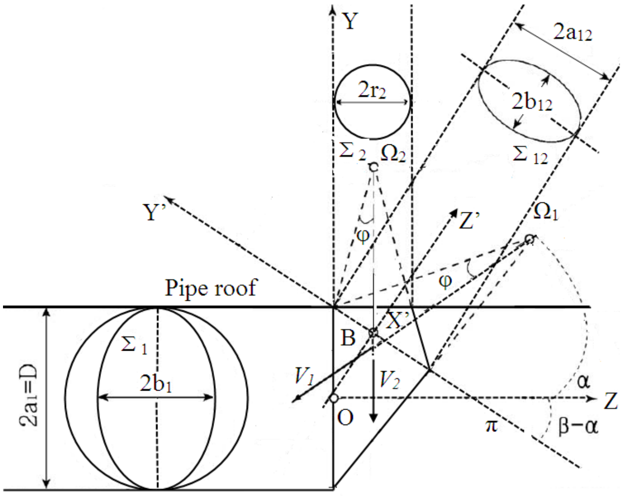


Figure 1: Analytical model for face stability of tunnel with pipe roof reinforcement

3. According to different overburden pressure above the tunnel face, two kinds of the analytical models were established for tunnel with pipe roof reinforcement and without pipe roof reinforcement. The overburden pressure above the tunnel face for both kinds was calculated as follows:

Tunnel without pipe roof reinforcement: The overburden pressure above the tunnel face can be calculated by Terzaghi earth pressure theory. The mechanical model was shown in Fig.2 (a);

Tunnel with pipe roof reinforcement: The overburden pressure above the tunnel face is the counterforce of foundation, which can be obtained from reference 3. The mechanical model was shown in Fig.2 (b)

Upper-bound solutions for the face stability of tunnel

In order to satisfy the stability condition of the tunnel face according to the upper bound theorem, the following relation was considered:

$$P_e \leq P_v \tag{1}$$

Where P_e represents the power of the external loads, which is the sum of three components, P_T , P_S, and P_γ. P_T is the power of the support pressure σ_T , P_S is the power of the surcharge σ_S and P_γ is the power of the soil weight γ . P_v denotes the dissipation power. The derivations for each component of the upper bound theorem have been omitted.

The upper bound solution for the collapse mechanism of two conical blocks is described following Leca and Dormieux (1990) as:

$$\begin{aligned} P_e &= P_T + P_S + P_\gamma \\ &= \frac{\pi D^2}{4}\frac{\cos^2(\alpha+\varphi)}{\cos^2\varphi}\left[\frac{\mathrm{R}_E^2}{\cos^2(\alpha+\varphi)}\sigma_S - \frac{\cos\alpha}{\mathrm{R}_C^2}R_A\sigma_T + \left(\sin\alpha\frac{R_A R_B}{\mathrm{R}_C^2} + \frac{\cos\alpha\cos\varphi\cos(\beta+\varphi)}{2\sin\varphi\sin(\beta+\varphi)}R_C - \frac{\mathrm{R}_E^3}{2\sin\varphi\cos^2(\alpha+\varphi)}\right)\frac{\gamma D}{3}\right]V_2 \end{aligned} \tag{2}$$

where

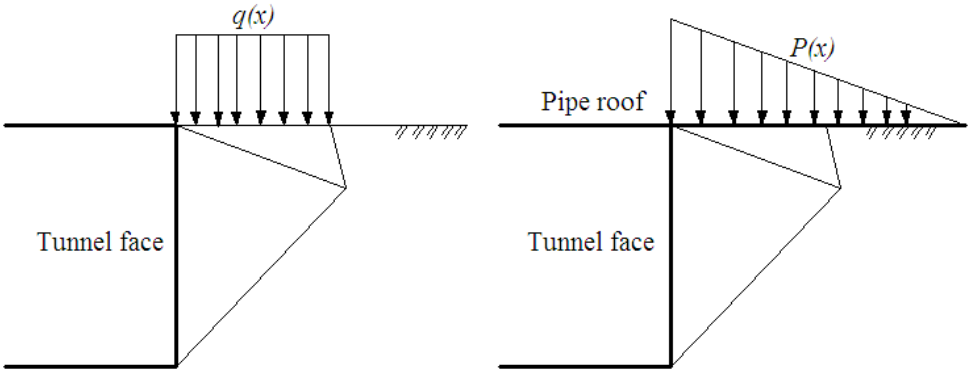
$$R_A = \frac{\sqrt{\cos(\alpha-\varphi)\cos(\alpha+\varphi)}}{\cos\varphi} \tag{3}$$

$$R_B = \frac{\cos(\alpha-\varphi)\cos(\alpha+\varphi)}{\sin 2\varphi} \tag{4}$$

$$R_C = \frac{\cos(\alpha+\varphi)}{\cos\varphi}\left[\frac{\sin(\beta-\varphi)}{\sin(\beta+\varphi)}\right]^{1/2} \tag{5}$$

$$R_D = \frac{\sin\beta}{\sin\varphi\sin(\beta+\varphi)} \tag{6}$$

$$R_E = \frac{\sin(\beta-\varphi)\cos(\alpha+\varphi)}{\sin(\beta+\varphi)} \tag{7}$$



(a)Tunnel without pipe roof reinforcement (b) Tunnel with pipe roof reinforcement

Figure 2: Calculating chart of overburden pressure above tunnel face

The energy is dissipated along the lateral surfaces of the moving block B_1 and B_2 and the base Σ_{12}. The dissipation power can be expressed as equation (8).

$$\begin{aligned} P_v &= P_{1v} + P_{2v} + P_{12v} \\ &= \frac{\pi D^2}{4}\frac{\cos^2(\alpha+\varphi)}{\cos^2\varphi}\left[\frac{\cos\alpha}{\sin\varphi}\frac{R_A}{R_C^2} - \frac{R_E^2}{\sin\varphi\cos^2(\alpha+\varphi)}\right] c\cos\varphi V_2 \end{aligned} \tag{8}$$

where, P_{1v}, P_{2v}, P_{12v} are the respective contributions of B_1 and B_2 and $\sum_{12}$.

The upper bound solution for the face stability of the tunnel with pipe roof reinforcement is obtained by substituting equation (2) and equation (8) in equation (1), resulting in:

$$N_S\left[(K_P-1)\frac{\sigma_S}{\sigma_C}+1\right] + N_\gamma(K_P-1)\frac{\gamma D}{\sigma_C} \le (K_P-1)\frac{\sigma_T}{\sigma_C}+1 \tag{9}$$

where σ_C is unconfined compression strength of soil, K_P is Rankine's passive earth pressure coefficient, N_S and N_γ are weighting coefficients.

$$\sigma_C = 2\frac{c\cos\varphi}{1-\sin\varphi} \tag{10}$$

$$K_P = \frac{1+\sin\varphi}{1-\sin\varphi} \tag{11}$$

$$N_S = \frac{1}{\cos\alpha\cos^2\varphi}\frac{\sin(\beta-\varphi)}{\sin(\beta+\varphi)}\frac{R_E^2}{R_A} \tag{12}$$

$$N_\gamma = \frac{1}{3}\left[\tan\alpha R_B + \frac{\cos\varphi\cos(\beta+\varphi)}{2\sin\varphi\sin(\beta+\varphi)}\frac{R_C^3}{R_A} - \frac{1}{2\sin\varphi\cos\alpha\cos^2\varphi}\frac{\sin(\beta-\varphi)}{\sin(\beta+\varphi)}\frac{R_E^3}{R_A}\right] \tag{13}$$

N_S and N_γ are the functions of the friction angle φ. Leca and Dormieux (1990) found that the optimal values N_s^c and N_γ^c were always obtained for essentially the same value α^c of parameter $\alpha : \alpha^c = 49^\circ - \varphi/2$. This means that in the plane of symmetry of the tunnel, the angle of the critical failure surface to the horizontal is $49^\circ + \varphi/2$,which is larger than the angle of active failure in plane strain conditions, $45^\circ + \varphi/2$. Therefore the area in front of the tunnel which is influenced by the collapse is more limited than in the case of a long open cut. This could be seen as a stabilizing effect due to three-dimensional equilibrium conditions around the tunnel face. Since plane π is chosen such that Δ_2 is vertical, we have between α and β the relation: $2\beta - \alpha = \pi / 2$.

Safety factor of tunnel face

The strength reduction technique for finite element analysis of slope stability has been applied successfully by many authors (Huang, 2009; Wei, 2009). This method allowed finding the safety factor of a slope by initiating a systematic reduction sequence for the available shear strength parameters c and φ to just cause the slope to fail. The reduction values of shear strength parameters c_m and φ_m were defined as:

$$c_m = \frac{c}{K}, \qquad \tan\varphi_m = \frac{\tan\varphi}{K} \tag{14}$$

in which the K is the strength reduction factor. The safety factor of the slope, K_S, is the value of K to bring the slope to failure.

In this study, the assumption was that when σ_T equal to zero, the tunnel face was in limit equilibrium status. As σ_T is the non-linear function of the c and φ, using the iterative method to reduce the shear strength parameters until the σ_T to zero, the safety factor of the tunnel face, K_S, is the value of strength reduction factor.

CALCULATION CASE

There have been many methods to calculate the support pressure of tunnel face and to evaluate the tunnel face stability. Chambon and Corte (1994) carried out centrifuge tests to study the face stability of tunnels in sands. Qin (2005) using wedge model to calculate the support pressure of tunnel face. Wei and He (2007) modified the wedge model and assumed the up side of sliding block as a trapezoid prism, using the Terzaghi loose earth pressure theory to calculate limited support pressure of tunnel face based on the principle of sliding block entire force balance. In this study, the solutions computed by the proposed approach were compared with the results given by wedge model, trapezoid wedge model and centrifugal-model test to verify the reasonability of the method. The results obtained from the four methods were summarized in Table 1.

Table 1: Comparison of the limited support pressures

D/ m	H/ m	c/ kPa	γ/ kN·m-3	Limited support pressures (kPa)			
				Centrifuge test	Wedge model	Trapezoid wedge model	Limit analysis approach
5	2.5	0	16.1	3.6	9.262	7.766	6.524
5	5	0	16.1	3.3	10.623	8.287	6.805
5	10	0	16.1	4	11.280	8.404	7.266
10	10	0	16.0	7.4	21.115	16.472	13.526
10	20	0	16.0	8.0	22.420	16.705	13.561
10	40	0	16.0	8.2	22.606	16.713	13.629

As seen from Table 1, it is apparent that the limited support pressures calculated by theoretical approach is larger than the results of centrifuge tests, but the calculation results of limit analysis are in close agreement with test results. In sandy soil, the progression of failure of tunnel face is a gradual process. When the cover depth is large enough, failure does not reach the ground surface, so the earth pressure calculated by Terzaghi earth pressure theory is usually larger than actual value, this maybe the reason to explain why the support pressures calculated by theoretical approach is larger than the results of centrifuge tests.

Other similarities between upper bound solutions and experimental results were shown in Fig.3, in which the failure zone observed in the centrifuge along the tunnel centre plane was represented for the case of a loose sand for H/D =1.0. The critical geometry associated with the upper bound solution was shown with dashed lines. Even though it does not extend in the vertical direction as much as the actual failure area, it coincides almost perfectly with the observed surface in front of the tunnel.

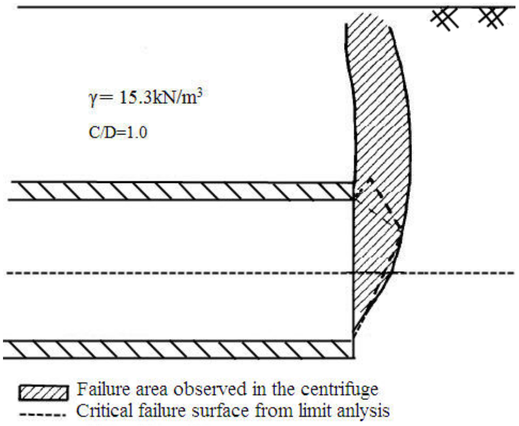


Figure 3: Comparison between theoretical critical surface and observed failure area

PARAMETRIC STUDY

In this study, to optimize design and construction of tunnel with auxiliary techniques, the proposed approach was employed to study how the tunnel cover depth, tunnel diameter and soil parameters affect the support pressure and safety factor. Two kinds of the analytical models were considered, tunnel with pipe roof reinforcement and without pipe roof reinforcement. Steel pipes used for pipe roof reinforcement have outer diameter of 108mm, length of 30m and thickness of 6 mm. The distance between the pipes is 40 cm and overlap length of pipe is 1.5m. A unit weight of $\gamma = 18$ kN/m^3 was assumed for soil with the cohesion of 20kPa and the friction angle of 35°.

Influence of cover depth of tunnel

It is assumed that cover depth of tunnel ranges from 2.5m to 40m. In this study, tunnel diameter is 5m, therefore, H/D ranges from 0.5 to 8. The relation between face support pressure (σ_T), safety factor (K_S) and cover depth of tunnel (H) were shown in Fig.4 and Fig.5, respectively.

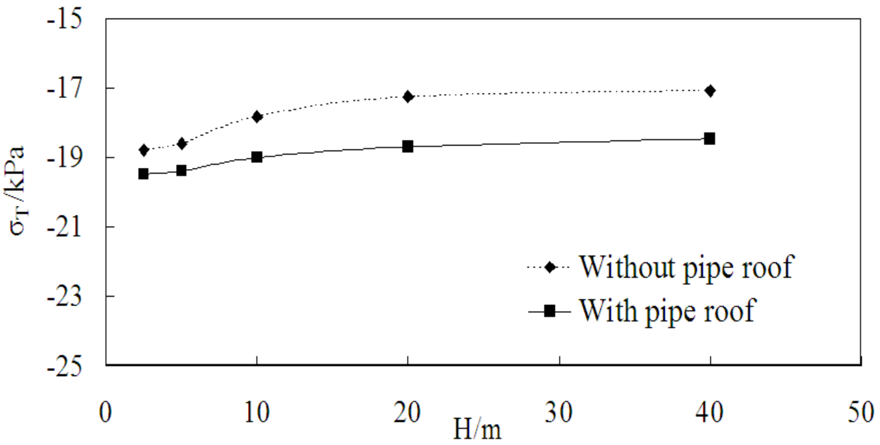


Figure 4: Relation between σ_T and H

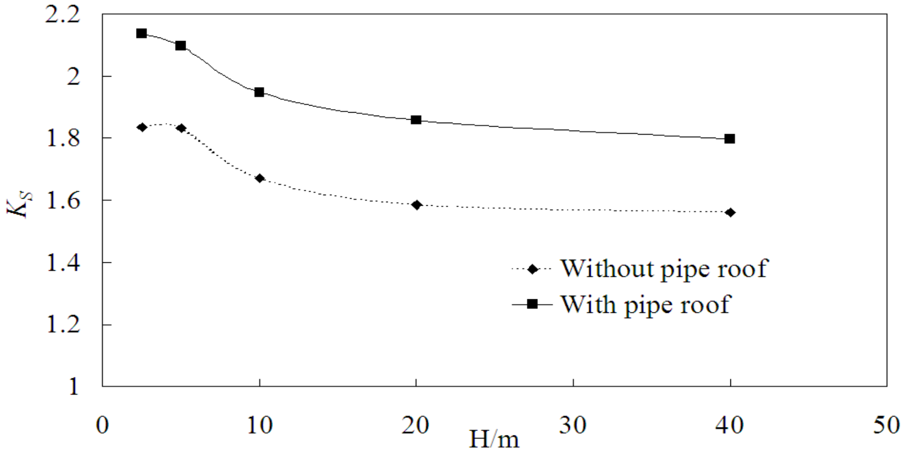


Figure 5: Relation between K_s and H

It can be seen in Fig. 4 that the support pressures of tunnel face are minus. These mean that the tunnel face can maintain stability and doesn't need any auxiliary techniques. For the relative depth investigated ($H/D = 0.5 \sim 8$), depth does not have a large influence on the limited support pressure of tunnel face and safety factor for a given diameter. Such conclusion is well supported by the results of centrifuge tests reported by Chambon and Corte (1994). The effect of pipe roof reinforcement on the stability of tunnel face can also be seen from Fig.4 and Fig.5. When H is 2.5m, the K_S of tunnel face for reinforced case is 2.137, and 1.835 for unreinforced case; When H is 40m, K_S of tunnel face for reinforced case and unreinforced case are 1.797 and 1.561, respectively. It can be understood that the safety factor of tunnel face with pipe roof reinforcement is about 1.15 times of that without pipe roof reinforcement.

Influence of tunnel diameter

It is assumed that tunnel diameter ranges from 5m to 15m. In this study, the cover depth of tunnel was 20m. The relation between face support pressure (σ_T), safety factor (K_S) and tunnel diameter (D) were shown in Fig.6 and Fig.7, respectively.

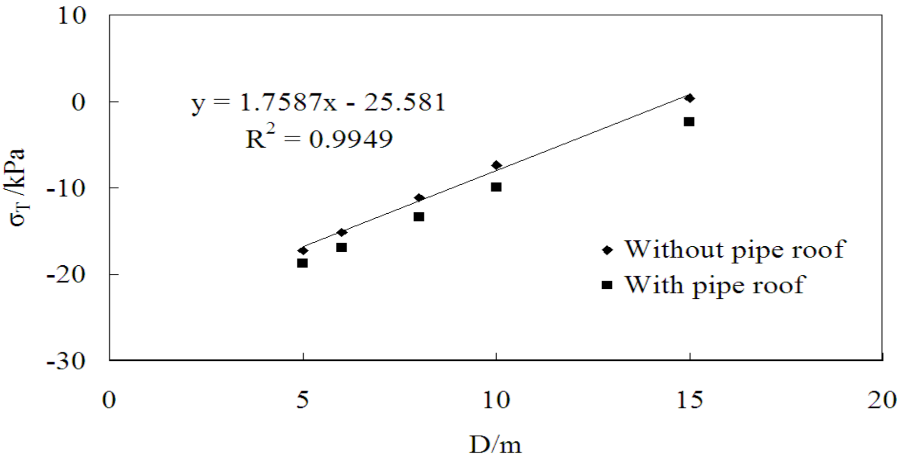


Figure 6: Relation between σ_T and *D*

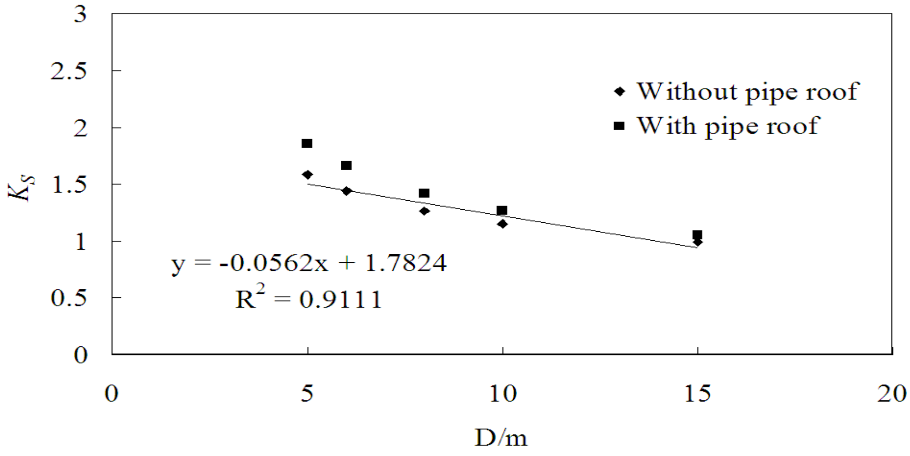


Figure 7: Relation between K_S and D

From Fig.6 and Fig.7, it appears that the limited support pressure of tunnel face increase linearly and safety factor decrease linearly in according with the tunnel diameter. The relationship between σ_T , K_S and γD can be described as following formula:

$$\sigma_T = 0.098\gamma D - 25.581 \tag{15}$$

$$K_s = -0.003\gamma D + 1.782 \tag{16}$$

When *D* is 5m, the K_s of tunnel face for reinforced case is 1.858, and 1.585 for unreinforced case; When *D* is 15m, K_s of tunnel face for reinforced case and unreinforced case are 1.055 and 0.955, respectively. These indicate that although pipe roof reinforcement can be expected to support the soil pressure induced by the loosening area ahead of the face, it may have limited value on face stability in large excavation span conditions.

Influence of soil parameters

In order to evaluate the tunnel face stability with different soil parameters, a systematic parametric study was conducted. It is assumed that cohesion of soil ranges from 5kPa to 80kPa, friction angle of soil ranges from 20° to 40°. The relation between safety factor (K_s) and soil parameters were listed in Table 2, and the relation between face support pressures (σ_T) and soil parameters were shown in Fig.8 and Fig.9, respectively.

Table 2: Relation between K_S and soil parameters

K_S	c / kPa					φ/°				
	5	10	20	50	80	20°	25°	30°	35°	40°
Unreinforced	0.824	1.113	1.585	3.271	6.801	0.978	1.168	1.368	1.585	1.817
Reinforced	0.866	1.231	1.858	4.021	7.607	1.238	1.439	1.643	1.858	2.075

From Table 2, it can be noted that the stability of the tunnel face is improved with increasing c and φ. It indicates that the improvements of soil conditions can greatly improve the stability of tunnel face. It was also found that the safety factor of tunnel face with pipe roof reinforcement is about 1.05~1.20 times of that without pipe roof reinforcement. The effect of pipe roof reinforcement on tunnel face stability is not significant. As presented in Fig.8, it was found that a good linear relationship exists between cohesion of soil (c) and face support pressure (σ_T). From Fig. 9, it appears that the quadratic formula is able to account for the influence of friction angle of soil (φ) on face support pressure (σ_T).The relationship between c, φ and σ_T can be described as following formula:

$$\sigma_T = -1.425c + 11.237 \tag{17}$$

$$\sigma_T = 0.084\varphi^2 - 5.927\varphi + 86.415 \tag{18}$$

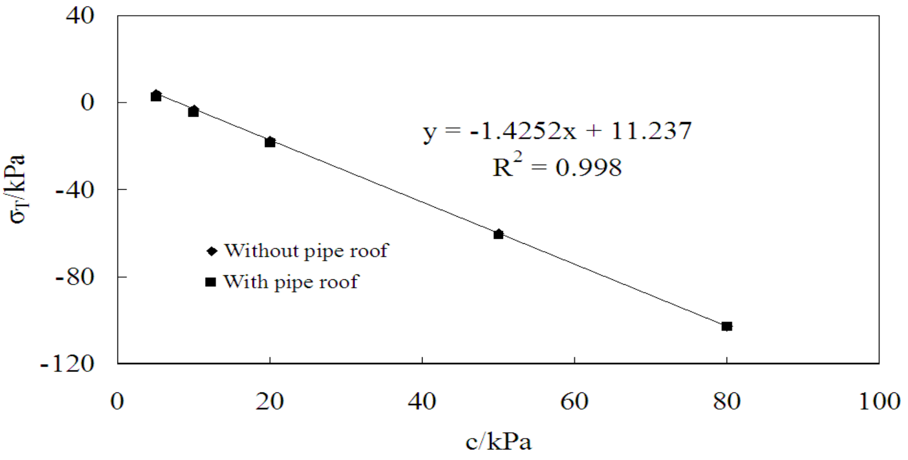


Figure 8: Relation between σ_T and c

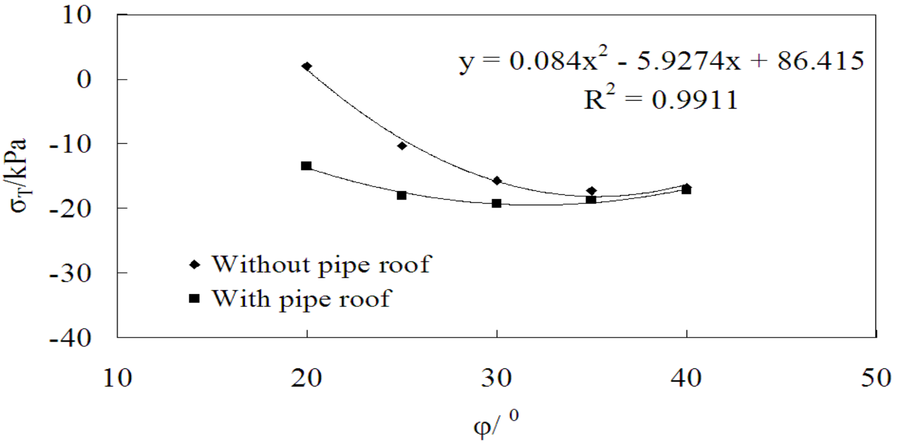


Figure 9: Relation between σ_T and φ

CONCLUSIONS

(1) Based on the kinematic method of limit analysis and the shear strength reduction technique, the three-dimensional model for expressing the tunnel face stability with pipe roof reinforcement is established. This method can be employed to define the safety factor and its corresponding critical failure mechanism for a given tunnel.

(2) For a typical example, the solutions computed by the proposed approach were compared with the results given by wedge model, trapezoid wedge model and centrifugal-model test. The results indicate that limit analysis approach can provide reasonable estimates of tunnel face stability.

(3) It is found that for the relative depth investigated ($H / D = 0.5 \sim 8$), depth does not have a large influence on the safety factor of tunnel face for a given diameter, but a good linear relationship exists between safety factor of tunnel face and tunnel diameter. It is also found that the linear and quadratic formulas are able to account for the influence of cohesion and friction angle of soil on limited support pressure of tunnel face, respectively.

(4) The safety factor of tunnel face with pipe roof reinforcement is about 1.05~1.20 times of that without pipe roof reinforcement. The effect of pipe roof reinforcement on tunnel face stability is not significant.

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