

# An Improved Method for Foundation Modulus in Highway Engineering

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## ABSTRACT

Though there are various methods available for determining a foundation modulus, we found that the calculation results of modulus using Biot and Vesic procedures differ greatly with the change of foundation width, height and Poisson's ratio. A new method is presented in this paper to determine the foundation modulus under different conditions which are obtained through establishing a matrix, which is a function of highway's and site's types. The analysis on the reflection of various factors to foundation modulus indicates that the presented method is suitable for the calculation of foundation modulus in highway foundation design.

**KEYWORDS:** highways, site types, foundation modulus

## PREFACE

With the rapid development of China's urbanization and industrialization, and the continuous improvement of living standards, the demand for land has been rising unabatedly. People have to carry out constructions on increasingly difficult foundation-soil conditions. How to accurately evaluate the engineering properties of the foundation-soil, especially the load carrying capacity of foundation-soil has become more and more significant.

For analysis of beams and slabs resting on a soil medium, engineers have been using a classical mathematical model called the Winkler model, where the behavior of the soil is simplified by means of fictitious springs placed continuously underneath the structure. The corresponding spring constant  $k$  is called the modulus of subgrade reaction of the soil, or foundation modulus. Based on this concept, many computer codes have been developed by engineers for the analysis of beams and slabs on an elastic foundation; the user of the code has to determine a suitable value of  $k$  to represent the soil. There is no easy way to determine this value of  $k$  because its value is not unique for a given type of soil.

So far, many researchers have made great efforts to deal with the problem.

Biot (1937) proposed the foundation modulus' computing formula through theoretical analysis [1]. Vesic (1963) tried to develop a value for  $k$ , except, instead of matching bending moments, he matched the maximum displacements of the beam in both models [2].

Liu etc. (2000) dealt with the static analysis of homogenous isotropic rectangular plates on Winkler foundation on the basis of shear deformation theory [3]. Daloglu *et al.* (2000) developed a method to evaluate an equivalent modulus of subgrade reaction  $k$  used in the Winkler model [4].

Fischer etc. (2008) analyzed beams on a foundation as rails on the ground based on Winkler's concept and introduced a bedding coefficient  $k$  [5]. Jayachandran *et al.* (2008) analyzed plates resting on an elastic medium in a simplified way with the linear Winkler foundation approach [6].

Moreover, some researchers considered the Biot's result as the lower limit of the foundation modulus while the Vesic's as the upper limit [7]-[9]. However, the calculated results from the two methods are significantly different such that it is difficult to select the method to employ for an actual foundation.

Therefore, a new method for foundation modulus is presented through the analysis to the two methods combined with the using functions of buildings and site types.

## THE PROBLEM OF THE TWO METHODS

The well-known Winkler's foundation model can be expressed as a equation  $p = ky$ , where  $p$  is compressive stress in the foundation,  $y$  settling amount,  $k$  foundation modulus.

Biot<sup>[5]</sup>(1937) has developed an empirical formula for  $k$ :

$$k = \frac{0.65E_s}{1-\nu^2} \sqrt[12]{\frac{E_s b^4}{EI}} \quad (1)$$

and Vesic<sup>[6]</sup>(1963) improved Biot's formula as:

$$k = \frac{0.95E_s}{(1-\nu^2)} \left( \frac{E_s b^4}{(1-\nu^2)EI} \right)^{0.108} \quad (2)$$

where  $E_s$  = elastic modulus of foundation soil,  $\nu$  = Poisson's ratio of foundation soil,  $b$  = foundation width,  $E$  = elastic modulus of beam,  $I$  = moment of inertia of the cross section of the beam.

The difference between the two methods are analyzed shown as the following figures, Fig. 1 to Fig. 5, where  $k_b$  expresses foundation modulus from Biot's method,  $k_v$  that from Vesic's method, and  $h$  height of beam. For convenient comparison of the change trend of  $k_b$  and  $k_v$  curves, the  $k_b$  curve is moved parallel to  $k_v$ ' curve, which intersects with  $k_v$  at the left endpoint.

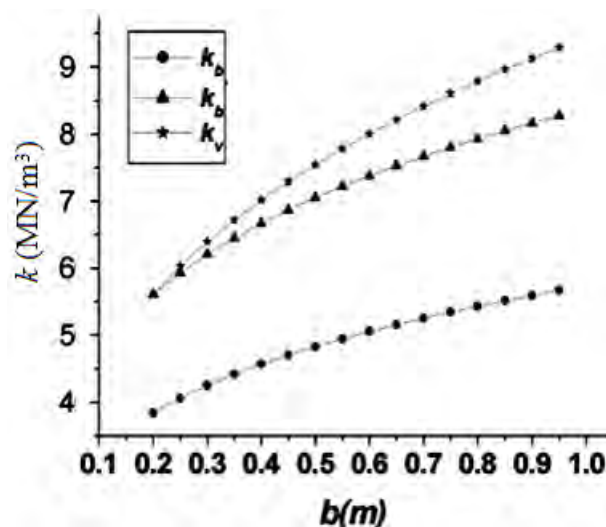


Figure 1:  $k - b$  curves

(note: 1 MN = MegaNewton = 1 million Newtons = 1 thousand kiloNewtons)

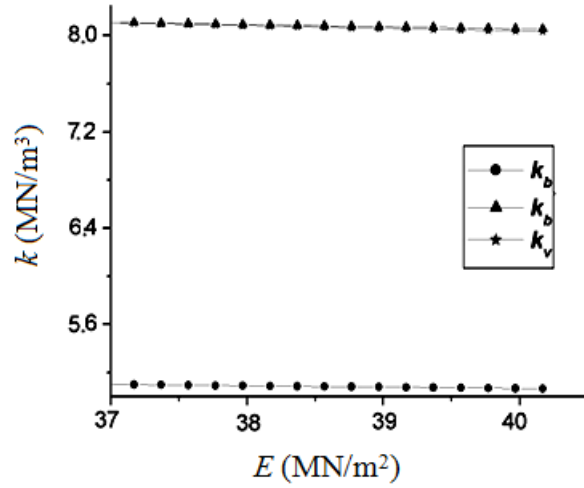


Figure 2:  $k - E$  curves

$E_s$  (MN/m²)

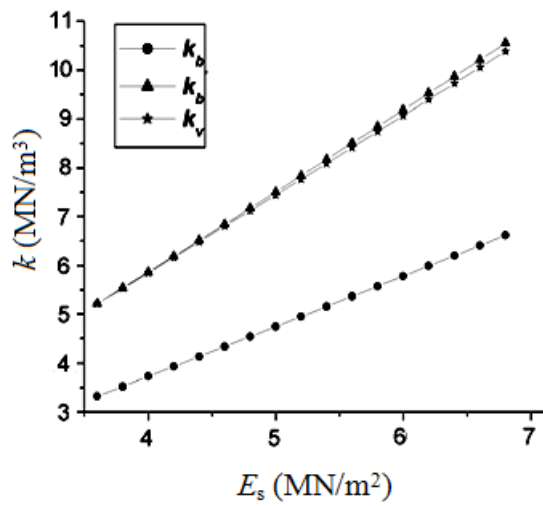


Figure 3:  $k - E_s$  curves

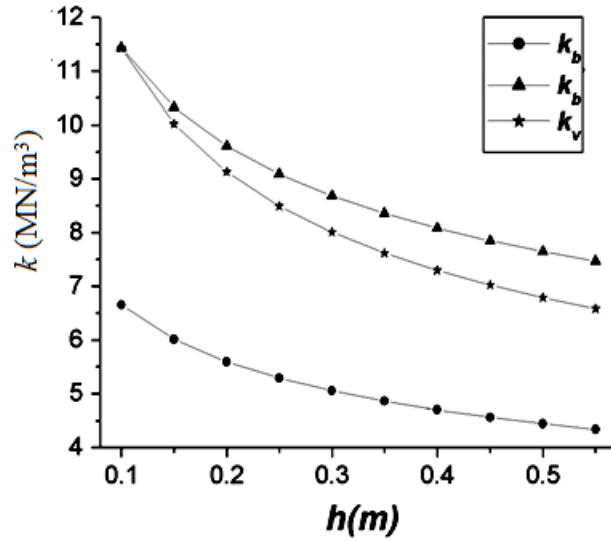


Figure 4:  $k-h$  curves

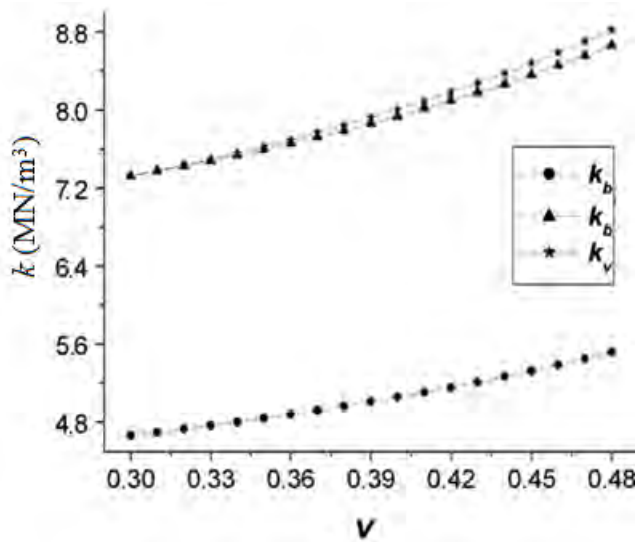


Figure 5:  $k-v$  curves

Fig.1 through Fig.5 indicate that the calculating results from Biot's and Vesic's methods are quite different and the change trends of  $k$  with  $b$ ,  $h$  and  $v$  are different. These differences bring great uncertainty to the calculations of foundation bearing capacity and deformation. Consequently, an improvement to these methods in order to make it convenient for application is essential.

## ESTABLISHMENT OF THE MATRIX OF USING FUNCTIONS TO SITE TYPES

The highways are classified into five groups in China based on their using functions: freeway (denoted as Group A), arterial highway (denoted as Group B), secondary road (denoted as Group C), and tertiary road (denoted as Group D), and quartus road (denoted as Group E) [10].

It's known from Winkler's foundation model that the settlement is inversely proportional to foundation modulus, so the more important a highway is, the greater is the importance of the foundation modulus.

In China the construction sites are classified into four types based on equivalent shear wave velocity and cover thickness of soil [11]: I (solid soil or rock), II (medium-hard soil), III (medium-soft soil), IV (soft soil). Generally, the softer foundation is, the smaller foundation modulus is.

So foundation modulus should be as great as possible to the important highways on hard foundation, while to the less important highways on soft foundation, foundation modulus should be, on the contrary, as small as possible.

The matrix of using functions of highways to site types is formed as shown in Fig. 6.

Highway Class \ Site Type	A	B	C	D	E
Group I	①				
Group II	②	③	④		
Group III			⑤	⑥	
Group IV				⑦	⑧

**Figure 6:** Using functions - site types matrix

Eight conditions are gotten from table 1, namely condition ①, condition ②, condition ③, condition ④, condition ⑤, condition ⑥, condition ⑦ and condition ⑧. According to the analysis above, foundation modulus should be gradually increasing from condition ① to condition ⑧.

## IMPROVEMENT TO FOUNDATION MODULUS

Eq. 1 can be rewritten as:

$$k = 0.65 \frac{E_s}{(1-\nu^2)} \left( \frac{E_s b^4}{EI} \right)^{1/12} \quad (3)$$

and Eq. 2 as:

$$k = 0.95 \left( \frac{1}{1-\nu^2} \right)^{0.108} \frac{E_s}{(1-\nu^2)} \left( \frac{E_s b^4}{EI} \right)^{0.108} \quad (4)$$

Generally the foundation's Poisson's ratio ranges from 0.15 to 0.4, so  $\left( \frac{1}{1-\nu^2} \right)^{0.108}$  in Eq. 4 ranges from 1.0025 to 1.019. That is to say,  $0.95 \left( \frac{1}{1-\nu^2} \right)^{0.108}$  in Eq. 4 is 1.9 percent higher than 0.95 at most.

Therefore,  $\left( \frac{1}{1-\nu^2} \right)^{0.108}$  in Eq. 4 can be ignored and Eq. 4 can be transformed to:

$$k = 0.95 \frac{E_s}{(1-\nu^2)} \left( \frac{E_s b^4}{EI} \right)^{0.108} \quad (5)$$

So Eq. (1) and Eq. (5) can be combined as:

$$k = a \frac{E_s}{(1-\nu^2)} \left( \frac{E_s b^4}{EI} \right)^{\gamma} \quad (6)$$

From Eq. 6, the calculating result of  $k$  is controlled by coefficient  $a$ , and the change trend by coefficient  $r$ .

The calculation of  $k$  for different conditions can be obtained through adjusting the constants  $a$  and  $r$  in Eq. 6. The values of  $a$  and  $r$  corresponding to condition ① are  $a=0.65$  and  $\gamma=1/12$ , and  $a=0.95$  and  $\gamma=0.108$  to condition ⑧. For other conditions,  $a$  can be calculated through linear interpolation between 0.65 and 0.95, and  $r$  between 1/12 and 0.108.

The calculated  $a$  and  $r$  values and the equations used to calculate  $k$  corresponding to condition ① through condition ⑧ are shown in Table 1.

**Table 1:** The calculating formulae for  $k$  under different conditions

Condition	$a$	$\gamma$	Foundation modulus formula
①	0.65	1/12	$k = 0.65 \frac{E_s}{(1 - \nu^2)} \left( \frac{E_s B^4}{EI} \right)^{\frac{1}{12}}$
②	0.69	0.087	$k = 0.69 \frac{E_s}{(1 - \nu^2)} \left( \frac{E_s B^4}{EI} \right)^{0.0868}$
③	0.74	0.090	$k = 0.74 \frac{E_s}{(1 - \nu^2)} \left( \frac{E_s B^4}{EI} \right)^{0.0903}$
④	0.78	0.094	$k = 0.78 \frac{E_s}{(1 - \nu^2)} \left( \frac{E_s B^4}{EI} \right)^{0.0938}$
⑤	0.82	0.097	$k = 0.82 \frac{E_s}{(1 - \nu^2)} \left( \frac{E_s B^4}{EI} \right)^{0.0973}$
⑥	0.87	0.101	$k = 0.87 \frac{E_s}{(1 - \nu^2)} \left( \frac{E_s B^4}{EI} \right)^{0.1008}$
⑦	0.91	0.104	$k = 0.91 \frac{E_s}{(1 - \nu^2)} \left( \frac{E_s B^4}{EI} \right)^{0.1043}$
⑧	0.95	0.108	$k = 0.95 \frac{E_s}{(1 - \nu^2)} \left( \frac{E_s B^4}{EI} \right)^{0.108}$

The following Fig. 7 through Fig. 11 are used to compare the calculated results from the new formulae in Table 2, where con.1 through con.8 are used to express condition ①, through condition ⑧.

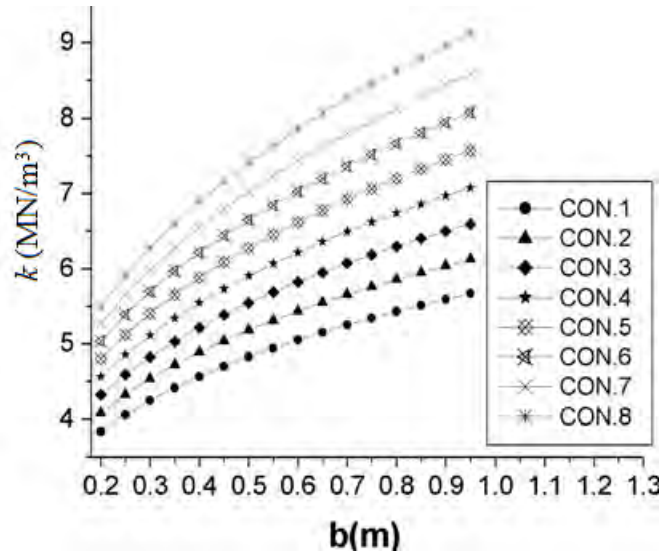


Figure 7:  $k - b$  curves under different conditions

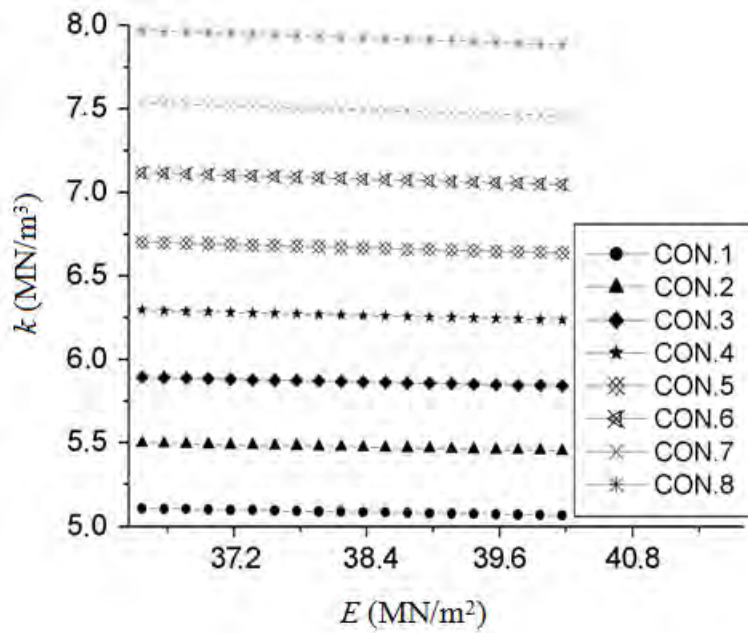


Figure 8:  $k - E$  curves under different conditions

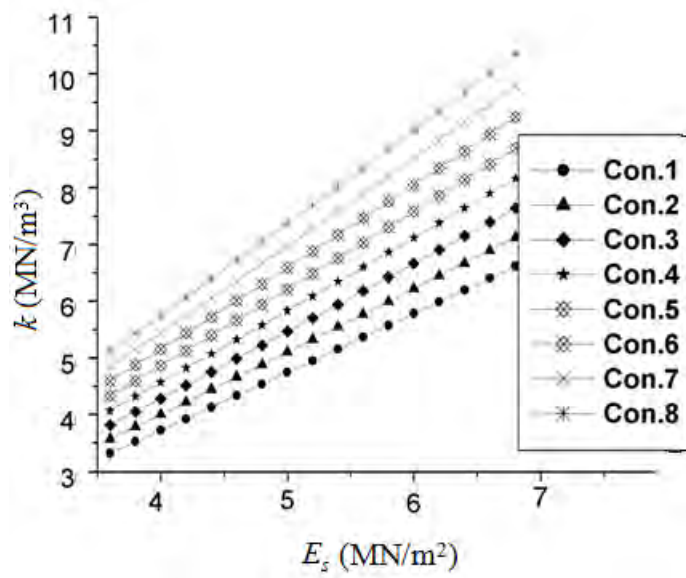


Figure 9:  $k - E_s$  curves under different conditions

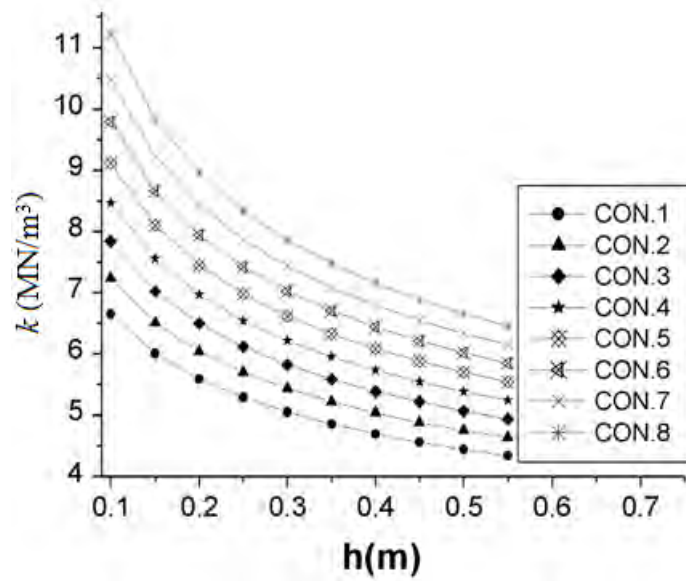
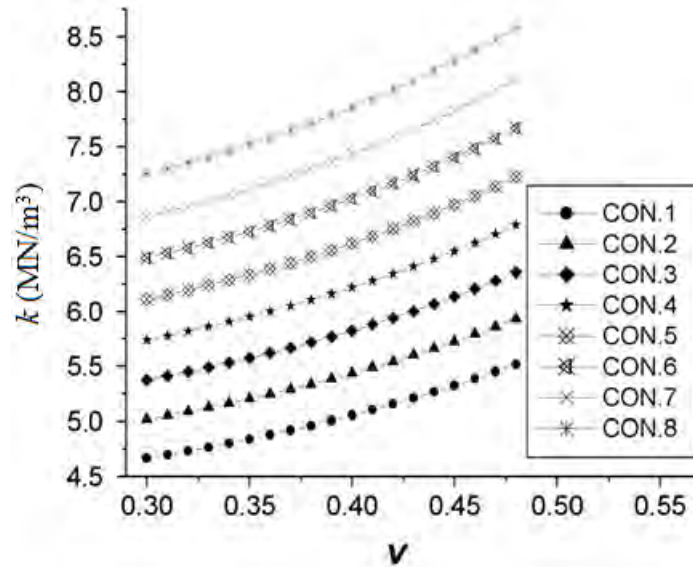


Figure 10:  $k - h$  curves under different conditions



**Figure 11:**  $k - \nu$  curves under different conditions

Thus foundation modulus for different conditions are established.

## CONCLUSION

The following conclusions are reached as a result of this study.

The results of calculating  $k$  from Biot and Vesic methods are not only greatly different from each other numerically, but also on the change trend of  $k$  with  $b$ ,  $h$  and  $\nu$ .

The matrix of using functions of highways to site types is formed and eight conditions are gotten from the matrix.

A uniform formula for foundation modulus is established through the analysis to Biot's and Vesic's formulae.

The new calculating formula for foundation modulus under different conditions are gotten.

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