Deformation of Geocell with Different Tensile and Compressive Modulus

Qu, Cheng-zhong

Northeast Dianli University, Jilin 132012, China

e-mail: quchengzhong@mail.nedu.edu.cn

ABSTRACT

The analytical solution for the deflection of geocell with different tension and compression modulus is presented in this paper. The factors affecting the solution of geocell's deflection are analyzed. The analysis indicates that geocell's deflection is affected strongly by width, height, tension modulus, compression modulus of geocell, and Poisson ratio and elastic modulus of foundation. Of these factors, foundation's elastic modulus is the great one. And meanwhile, the influence of the ratio of geocell's tension to compression modulus to geocell's deflection is researched. From the analysis, we find that the geocell's deflection greatly changes with the change of the ratio. It indicates that, in the latter design of geocell, the designers should take the influence of the different modulus values in tension and compression into account.

KEYWORDS: tension modulus, compression modulus, geocell, deflection

PREFACE

As a new geotextile material, geocell has been widely used in many fields including public roads, railways, constructions and shore protection. The initial work performed at the U.S. Army Engineer Waterways Experimental Station led to the development of available geocell systems. Two types of geocell systems are referred to. The first type consists of strips of polymer sheets welded together to form a mattress interconnected cells. Another type of geocell referred to consisting of strips of geogrids connected to form three dimensional cells. From then on, a number of researchers were devoting to the theoretical research and application of geocell.

Dash (2003) et al. found an improvement in the load bearing capacity of the buried foundation mattresses with an increase in the mattress thickness, up to a geocell height of twice the width of the footing, beyond which the improvement is only marginal duo to the local failure of the geocell wall taking place [1].

Richardson (2004、2005) evaluated the function of geocell, and discussed its application to the no-paved stones transport roads and the good results [2]-[4].

Latha et al. (2006) investigated geocell's advantages of constructing embankment on the soft ground and come to the conclusion that geocell could increase the bearing capacity of foundation and reduce the foundation settlement effectively [5].

Boyle et al (2007) expounded the effects and results achieved by applying geocell to the restoration and the reinforcement works of the eroded slopes and roadbeds [6].

Thallak et al. (2007) proved that foundation settlement can be reduced a lot through a geocell reinforced soft clay ground test [7].

Through the analysis to nine lateral restricted tests, Chen et al. (2008) checked the invalidating mechanism on the condition of limiting the structure deformation and the structure overload [8].

Almost all researches performed the model studies on the basis of a hypothesis that geocell works with the single elastic modulus whether it is under tensile or compressive state. But in fact, the material elastic modulus of geocell is comparatively different when geocell is stretched or compressed. Thus, a certain error will occur when single elastic modulus is considered to determine the deflection of geocell subjected to external load.

For solving the problem, a geocell with different tension and compression elastic modulus is analyzed in this paper and the analytical solution of geocell's deflection is gotten so as to make the calculating results close to practice..

THE ASSUMPTION OF MATERIAL WITH DIFFERENT MODULUS IN TENSION AND COMPRESSION

Some researchers had studied the theory of material with different tension and compression elastic modulus. The basic concept, fundamental assumptions, the analytical solutions under uniaxial compression, and the finite element numerical calculation methods had been obtained [9]-[11].

The fundamental assumptions of material with different tension and compression elastic modulus include:

The nonlinear relationship between stress and strain in $\sigma > 0$ and $\sigma < 0$ sections is denoted with corresponding linear relationship, as shown in Fig. 1, where σ = the stress, ε = the strain, E_t = the tension elastic modulus, and E_c = the compression modulus of material.

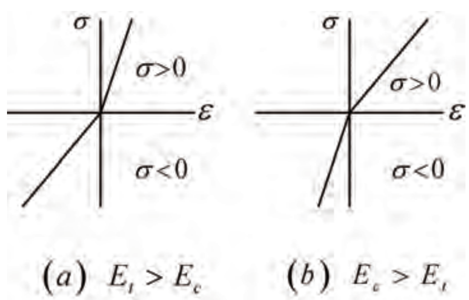


Figure 1: Stress-strain of material with different tensile and compressive modulus

That is to say that the tension and compression relationship of material with different modulus can be expressed with a bilinear model.

The research object is supposed to be continuous, homogeneous and isotropic. It also conforms to elastic small deformation assumption and the general rules of the continuous medium.

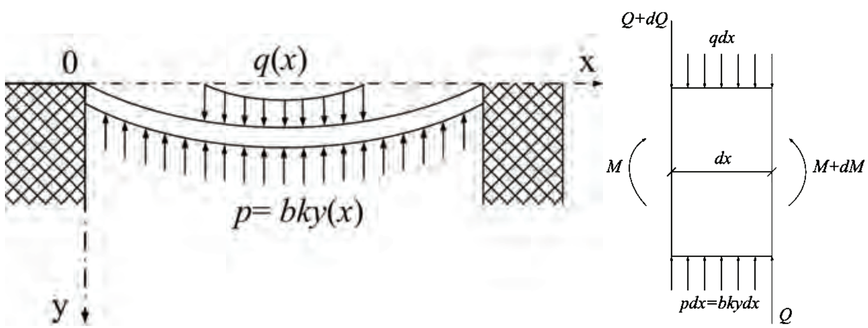
Based on this, the difference between the classical elastic theory and the difference modulus elastic theory rely on the physical equation for the relationship between stress and strain.

It is assumed that the elastic modulus is E_t and Poisson ratio is γ_t at all directions for $\sigma > 0$, and E_c and γ_c respectively for $\sigma < 0$.

GEOCELL MODEL WITH DIFFERENT MODULUS

A geocell with width of *b* is supposed as the research object, which loading distribution, deformation is shown in Fig. 2a, where $p = bky$ represents the ground reaction per unit length on the basis of Winkler's foundation, *k*= the foundation modulus, and *y*= the deflection of geocell, *q*= the load density.

The force diagram of material element of geocell is shown in Fig. 2b, where *M*= the bending moment, *Q*= the shear force.



(*a*) (*b*)

Figure 2: Load distribution, deflection, and force diagram of material element of geocell

Two independent equilibrium equations can be obtained from Fig. 2b:

$$\sum Y = Q - (Q + dQ) + qdx - pdx = 0 \tag{1}$$

$$\sum M = M - (M + dM) + Qdx + \frac{1}{2}[q - p]dx^2 = 0 \tag{2}$$

Equation (1) can be rewritten as:

$$\frac{dQ}{dx} = bky - q \tag{3}$$

According to mechanics of materials:

$$Q = dM/dx \quad (4)$$

Eq. 3 is transformed to:

$$\frac{dQ}{dx}=\frac{d^2M}{dx^2}=bky-q \tag{5}$$

From mechanics of materials, we know that:

$$\frac{d^2y}{dx^2}=-\frac{M}{EI} \tag{6}$$

where E= the Young's modulus, I= the inertial moment of cross section of geocell.

The equivalent elastic modulus E_0 of material with different modulus is [12][13]:

$$E_0=\frac{4E_tE_c}{\left(\sqrt{E_t}+\sqrt{E_c}\right)^2} \tag{7}$$

where E_t = the tension modulus of geocell and E_c the compression modulus of geocell.

Replacing E with E_0, the equation (6) will be transformed into:

$$\frac{d^2y}{dx^2}=-\frac{M}{E_0I} \tag{8}$$

The simultaneous equations (5) and (8), the differential equation of geocell is obtained:

$$\frac{d^4y}{dx^4}+\frac{bk}{\frac{4E_tE_c}{\left(\sqrt{E_t}+\sqrt{E_c}\right)^2}I}y=\frac{q}{\frac{4E_tE_c}{\left(\sqrt{E_t}+\sqrt{E_c}\right)^2}I} \tag{9}$$

Supposing:

$$4\beta^4=\frac{bk}{\frac{4E_tE_c}{\left(\sqrt{E_t}+\sqrt{E_c}\right)^2}I} \tag{10}$$

Then:

$$\beta = \sqrt[4]{\frac{bk}{\frac{16E_tE_c}{\left(\sqrt{E_t}+\sqrt{E_c}\right)^2}I}} \tag{11}$$

Substituting equation (11) into equation (9) obtains:

$$\frac{d^4y}{dx^4}+4\beta^4y=\frac{q}{\frac{4E_tE_c}{\left(\sqrt{E_t}+\sqrt{E_c}\right)^2}I} \tag{12}$$

Eq. 12 is the basic differential equation of the geocell with different modulus.

THE ANALYTICAL SOLUTION OF DEFLECTION OF GEOCELL WITH DIFFERENT MODULUS

Supposing q =0, Eq. 12 can be rewritten as:

$$\frac{d^4y}{dx^4}+4\beta^4y=0 \tag{13}$$

The deflection y of geocell can be solved from Eq. 13, and then the solutions of rotation angle $\theta=\frac{dy}{dx}$, moment $M=-E_0I\frac{d\theta}{dx}=-E_0I\frac{d^2y}{dx^2}$ and shear force $Q=\frac{dM}{dx}=-E_0I\frac{d^3y}{dx^3}$ can be achieved by differentiation to the deflection equation of geocell:

$$\begin{aligned}
&y=e^{\beta x}(A\cos\beta x+B\sin\beta x)+e^{-\beta x}(C\cos\beta x+D\sin\beta x)\\
&\theta=\beta\left\{e^{\beta x}\left[(A+B)\cos\beta x+(B-A)\sin\beta x\right]\text{-}\ e^{-\beta x}\left[(C-D)\cos\beta x+(C+D)\sin\beta x\right]\right\}\\
&\mathrm{M}=-2\mathrm{E}_0\mathrm{I}\beta^2\left\{e^{\beta x}(B\cos\beta x-A\sin\beta x)\text{-}e^{-\beta x}(D\cos\beta x-C\sin\beta x)\right\}\\
&\mathrm{Q}=-2\mathrm{E}_0\mathrm{I}\beta^3\left\{e^{\beta x}\left[(B-A)\cos\beta x-(A+B)\sin\beta x\right]+e^{-\beta x}\left[(C+D)\cos\beta x-(C-D)\sin\beta x\right]\right\}
\end{aligned} \tag{14}$$

As shown in Fig. 2a, supposing when x = 0, $y(0) = y_0$, $\theta(0)=\theta_0$, $M(0)=M_0$, and $Q(0)=Q_0$, substituting these values for the ones in Eq. 14, we can obtain that:

$$\left.\begin{aligned} &A+C=y_0 \\ &\beta(A+B-C+D)=\theta_0 \\ &-2E_0I\beta^2(B-D)=M_0 \\ &-2E_0I\beta^3(A+B+C+D)=Q_0 \end{aligned}\right\} \tag{15}$$

Solving Eq. 15, the coefficients of A, B, C, and D can be gotten:

$$\left.\begin{aligned} &A=\frac{y_0}{2}+\frac{\theta_0}{4\beta}+\frac{Q_0}{8E_0I\beta^3} \\ &B=\frac{\theta_0}{4\beta}-\frac{M_0}{4E_0I\beta^2}-\frac{Q_0}{8E_0I\beta^3} \\ &C=\frac{y_0}{2}-\frac{\theta_0}{4\beta}-\frac{Q_0}{8E_0I\beta^3} \\ &D=\frac{\theta_0}{4\beta}+\frac{M_0}{4E_0I\beta^2}-\frac{Q_0}{8E_0I\beta^3} \end{aligned}\right\} \tag{16}$$

Substituting A, B, C, and D in Eq. 16 for Eq. 14 obtains:

$$y=y_0\phi_1(\beta x)+\frac{\theta_0}{\beta}\phi_2(\beta x)-\frac{M_0}{E_0I\beta^2}\phi_3(\beta x)-\frac{Q_0}{E_0I\beta^3}\phi_4(\beta x) \tag{17}$$

Where $\phi_1(\beta x)$, $\phi_2(\beta x)$, $\phi_3(\beta x)$, and $\phi_4(\beta x)$ are the Krylov's function

$\phi_1(\beta x)=ch\beta x\cos\beta x$, $\phi_2(\beta x)=\frac{1}{2}(ch\beta x\sin\beta x+sh\beta x\cos\beta x)$, $\phi_3(\beta x)=sh\beta x\sin\beta x$,

$\phi_4(\beta x)=\frac{1}{4}(ch\beta x\sin\beta x-sh\beta x\cos\beta x)$ and $ch\beta x$ and $sh\beta x$ are the hyperbolic functions:

$$ch\beta x=\frac{e^{\beta x}+e^{-\beta x}}{2}\ ,\quad sh\beta x=\frac{e^{\beta x}-e^{-\beta x}}{2}$$

The geocell model shown in Fig. 3, where *B*=half of the loading rang, and *L*=half of the length of geocell.

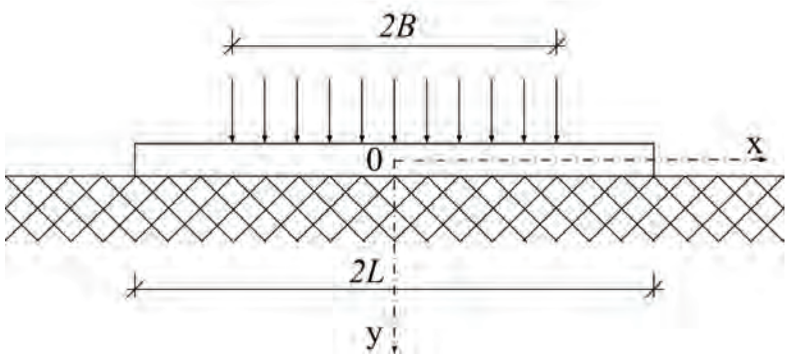


Figure 3: Geocell Model With Different Modulus

In section $0 \le x \le B$ of Fig. 3, the deflection of geocell, because of q≠0, is changed as:

$$y = y_0\phi_1(\beta x) + \frac{\theta_0}{\beta}\phi_2(\beta x) - \frac{M_0}{E_0 I \beta^2}\phi_3(\beta x) - \frac{Q_0}{E_0 I \beta^3}\phi_4(\beta x) + \frac{q}{k}[1 - \phi_1(\beta x)] \quad (18)$$

Based on the boundary conditions, when $x = 0$, $\theta_0 = 0$ and $Q_0 = 0$, thus equation (18) can be concluded:

$$y = y_0\phi_1(\beta x) - \frac{M_0}{E_0 I \beta^2}\phi_3(\beta x) + \frac{q}{k}[1 - \phi_1(\beta x)] \quad (19)$$

In section $x > B$, the deflection of geocell is:

$$y = y_0\phi_1(\beta x) - \frac{M_0}{E_0 I \beta^2}\phi_3(\beta x) + \frac{q}{k}\{\phi_1[\beta(x - B)] - \phi_1(\beta x)\} \quad (20)$$

According to the boundary conditions, when $x = L$, $Q_L = 0$, and $M_L = 0$, the following results can be obtained from Eq. 20 and $M = -E_0 I \frac{d^2 y}{dx^2}$ and $Q = -E_0 I \frac{d^3 y}{dx^3}$:

$$\frac{d^3 y}{dx^3} = -4\beta^3 y_0 \phi_2(\beta L) + \frac{4\beta M_0}{E_0 I}\phi_4(\beta L) - \frac{4q\beta^3}{k}\phi_2[\beta(L - B)] + \frac{4q\beta^3}{k}\phi_2[\beta L] = 0 \quad (21)$$

$$\frac{d^2 y}{dx^2} = -4\beta^2 y_0 \phi_3(\beta L) - \frac{M_0}{E_0 I}\phi_1(\beta L) - \frac{4q\beta^2}{k}\phi_3[\beta(L - B)] + \frac{4q\beta^2}{k}\phi_3[\beta L] = 0 \quad (22)$$

Solving Eqs. 21 and 22 can obtain y_0, namely the deflection of geocell at origin coordinate (x=0):

$$y_0 = \frac{q}{k}\{1 - C\} \tag{23}$$

Where $C = \dfrac{\phi_1(\beta L)\phi_2\left[\beta(L-B)\right] + 4\phi_3\left[\beta(L-B)\right]\phi_4(\beta L)}{\phi_1(\beta L)\phi_2(\beta L) + 4\phi_3(\beta L)\phi_4(\beta L)}$.

y_0 in equation (23) is the analytical solutions of geocell's deflection at origin coordinate (x=0).

THE FACTORS INFLUENCING THE DEFLECTION OF GEOCELL WITH DIFFERENT MODULUS

The relative factors in Eq. 23 are assigned as foundation modulus $k = \dfrac{0.65E_s}{(1-v^2)}\sqrt[12]{\dfrac{E_sB^4}{E_0I}}$ from Biot (1937) [14], foundation's elastic modulus E_s=2-10×10^6 N/m^2, foundation's Poisson ratio v=0.2-0.5, geocell's tension modulus E_t=3-50×10^6 N/m^2, geocell's compression modulus E_c=30-100×10^6 N/m^2, geocell's width b=0.3-1.0m, geocell's height h=0.1-0.5m, geocell's length l=1.2m, width of load on geocell B=0.2m, load density q=50×10^4 N/m。

The influence of each factor in Eq. 23 to the deflection of geocell is analyzed as follows.

Fig. 4 through Fig. 9 are plotted to express the change of y_0 (as ordinate) with the change of b, h, v, E_s, E_t, and E_c (respectively as abscissas).

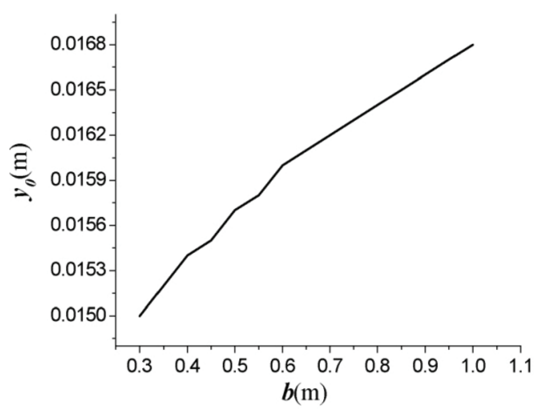


Figure 4: y_0 - b curve

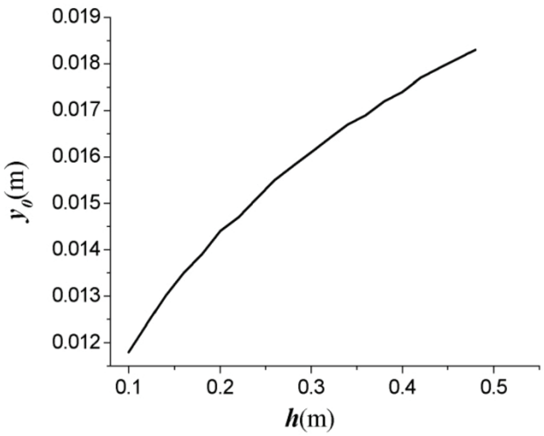


Figure 5: y_0 - h curve

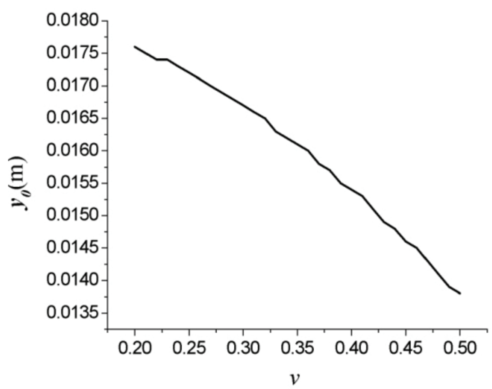


Figure 6: y_0-v curve

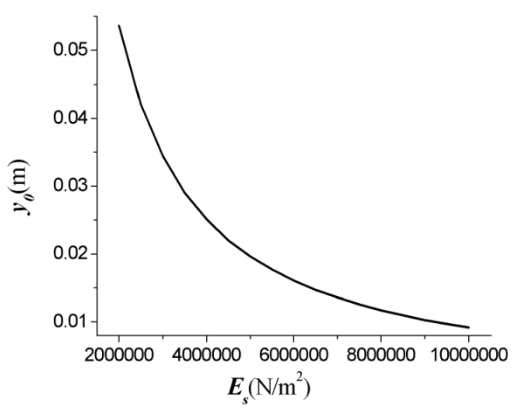


Figure 7: y_0 - E_s curve

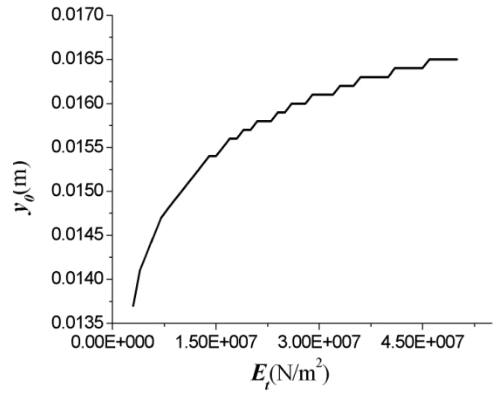


Figure 8: y_0 - E_t curve

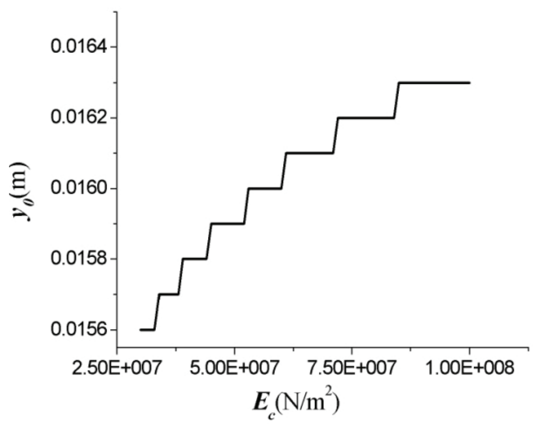


Figure 9: y_0 **-** $\boldsymbol{E_c}$ **curve**

From Figure 4 through Fig. 9, it can be concluded that all the factors above have influence to y_0, the most significant one is E_s.

In addition, in order to verify the influence of different modulus to y_0, Fig. 10 is plotted with y_0 as ordinate and E_s / E_c as abscissas.

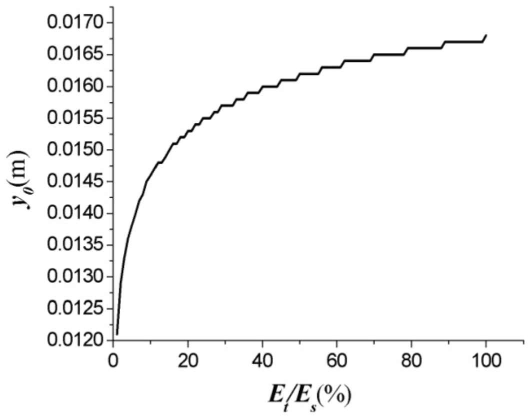


Figure 10: $y_0 - E_s/E_c$ curve

Fig. 10 indicates that the ratio of tension to compression plays a very important role in the calculating result for y_0.

When the ratio ranges from 1% to 100% (100% represents the condition of equal modulus), the calculating results of y_0 range from 0.0121m to 0.0168m, which is to say that y_0 increases 38.8%.

This should be paid enough attention in design of geocell.

CONCLUSIONS

The different modulus theory is introduced into the calculation of geocell’s deflection in this paper, and the analytical solution of geocell’s deflection is obtained.

Through the analysis to a number of factors influencing the deflection of geocell, the following conclusions can be drawn:

Geocell’s width, height, foundation’s Poisson ratio, foundation’s elastic modulus, geocell’s tension and compression modulus have influence to geocell’s deflection. The foundation’s elastic modulus is the most important influence to the deflection of geocell.

The ratio of geocell’s tension to compression modulus has great influence to the deflection of geocell.

Different modulus should be considered as a key factor in the future design of geocell.

REFERENCES

1. Sujit Kumar Dash, S. Sireesh, T.G. Sitharam (2003). Model Studies on Circular Footing Supported on Geocell Reinforced Sand Underlain by Soft Clay. Geotextiles and Geomembranes, 2003, 21(4): 197-219.

2. Gregory N. Richardson (2004). Geocells: a 25-year Perspective Part 1: Roadway Applications. Geotechnical Fabrics Report, 2004, 22(6): 14-19.

3. Gegory N. Richardson (2004). Geocells, a 25-year Perspective Part 2: Channel Erosion Control and Retaining Walls. Geotechnical Fabrics Report, 2004, 22(8): 22-27.

4. Greg Richardson (2005). Geocells for Channel and Retaining Wall Erosion Control. Fabric Architecture, 2005, 17(1): 30-35.

5. G. Madhavi Latha, K. Rajagopal, N. R. Krishnaswamy (2006). Experimental and Theoretical Investigations on Geocell-Supported Embankments. International Journal of Geomechanics, 2006, 6(1): 30-35.

6. Stan Boyle, Kathy Robertson (2007). Geocell, Grid, and Reinforced-soil in Restoring Eroded Steep Slopes. Geosynthetics, 2007, 25(2): 20-24.

7. Sitharam G. Thallak, Sireesh Saride, Sujit Kumar Dash (2007). Performance of Surface Footing on Geocell-reinforced Soft Clay Beds. Geotechnical and Geological Engineering, 2007, 25(5): 509-524.

8. R.H. Chen, Y.M. Chiu (2008). Model Tests of Geocell Retaining Structures. Geotextiles and Geomembranes, 2008, 26(1): 56-70.

9. Medri G. A nonlinear elastic model for isotropic materials with different behavior in tension and compression. Transactions of the ASME, 1982, 26(104): 26-28.

10. Srinivasan R.S, Ramachandra L. S. Large deflection analysis of bimodulus annular and circular plates using finite elements. Computers & Structures, 1989, 31(5): 681—691.

11. Srinivasan R.S, Ramachandra L S. Axisymmetric buckling and post-bucking of bimodulus annular plates. Eng Struct, 1989, 11(7): 195-198.

12. YAO Wen-juan, YE Zhi-ming. Analytical Solution of Bending-Compression Column Using Different Tension-Compression Modulus. Applied Mathematics and Mechanics, 2004, 25(9): 901-909.

13. YAO Wen-juan, YE Zhi-ming. Analytical Solution for Bending Beam Subject to Lateral Force With Different Modulus. Applied Mathematics and Mechanics, 2004, 25(10): 1014-1022.

14. Biot, M..A. Bending of an Infinite Beam on an Elastic Foundation. Journal of Applied Physics, 1937, 12(2): 155~164.