

A Review of Basic Soil Constitutive Models for Geotechnical Application

Kok Sien Ti

*PhD student, Department of Civil Engineering,
University Putra Malaysia, Serdang, Selangor, Malaysia
e-mail: koksienti@yahoo.com*

Bujang B.K. Huat, Jamaloddin Noorzaei, Moh'd Saleh Jaafar

*Department of Civil Engineering,
University Putra Malaysia, Serdang, Selangor, Malaysia
e-mail: bujang@eng.edu.upm.my*

Gue See Sew

*CEO, G&P Geotechnics Sdn Bhd
Bandar Tasik Selatan, KL, Malaysia*

ABSTRACT

Solutions in soil constitutive modeling have been based upon Hooke's law of linear elasticity for describing soil behaviour under working loading condition and Coulomb's law of perfect plasticity for describing soil behaviour under collapse state because of its simplicity in applications. The combination and generalization of Hooke and Coulomb's law is formulated in a plasticity framework and is known as Mohr-Coulomb model. However, it is well known that soils are not linearly elastic and perfectly plastic for the entire range of loading. In fact, actual behaviour of soils is very complicated and it shows a great variety of behaviour when subjected to different conditions. Various constitutive models have been proposed by several researchers to describe various aspects of soil behaviour in details and also to apply such models in finite element modelling for geotechnical engineering applications. Recent published evolution of models used for soft soils and tunnels for the past 30 years was also presented. It must be emphasized here that no soil constitutive model available that can completely describe the complex behaviour of real soils under all conditions. This paper attempts to collaborate the efforts from various researchers and present the discussion on each models with advantages and limitations for the purpose of giving an overview comparison of various soil models for engineering applications.

KEYWORDS: soil constitutive models, finite element, elastic, plastic

INTRODUCTION

Soil is a complicated material that behaves non-linearly and often shows anisotropic and time dependant behaviour when subjected to stresses. Generally, soil behaves differently in primary loading, unloading and reloading. It exhibits non-linear behaviour well below failure condition with stress dependant stiffness. Soil undergoes plastic deformation and is inconsistent in dilatancy. Soil also experiences small strain stiffness at very low strains and upon stress reversal. These general behaviour was not possibly being accounted for in simple elastic-perfectly plastic Mohr-Coulomb model, although the model does offer advantages which makes it a favourable option as soil model.

Brinkgreve (2005) discussed in more detail the five basic aspects of soil behaviour. Briefly, the first aspect discussed on the influence of water on the behaviour of the soil from the effective stresses and pore pressures. Second aspect is the factor which influences the soil stiffness such as the stress level, stress path (loading and unloading), strain level, soil density, soil permeability, consolidation ratio and the directional-dependant stiffness (stiffness anisotropy) of the soil. The third aspect highlighted the irreversible deformation as a result of loading. Fourth aspect discussed on soil strength with its influencing factor includes loading speed of the tested specimen, age and soil density, undrained behaviour, consolidation ratio and directional-dependant shear strength (strength anisotropy). Other aspects of soil behaviour that should be considered also include factors such as compaction, dilatancy and memory of pre-consolidation stress.

In addition to soil behaviour, its failure in three-dimensional state of stress is extremely complicated. Numerous criteria have been devised to explain the condition for failure of a material under such a loading state. Among these three-, four-, and five-parameter model, Mohr-Coulomb model is a two-parameter model with criterion of shear failure and can also be a three-parameter model with criterion of shear failure with a small tension cut-off. Refer Fig. 1.

There exist a large variety of models which have been recommended in recent years to represent the stress-strain and failure behaviour of soils. All these models inhibits certain advantages and limitations which largely depend on their application. Alternatively, Chen (1985) provided three basic criteria for model evaluation. The first criteria is theoretical evaluation of the models with respect to the basic principles of continuum mechanics to ascertain their consistency with the theoretical requirements of continuity, stability and uniqueness.

Secondly, experimental evaluation of the models with respect to their suitability to fit experimental data from a variety available test and the ease of the determination of the material parameters from standard test data. The final criteria is numerical and computational evaluation of the models with respect to the facility which they can be implemented in computer calculations.

In general, the criterion for the soil model evaluation should always be a balance between the requirements from the continuum mechanics aspect, the requirements of realistic representation of soil behaviour from the laboratory testing aspect (also the convenience of parameters derivation), and the simplicity in computational application. Fig 2 shows the basic components for material models. It is a simple representation of a few basic types of soil constitutive models.

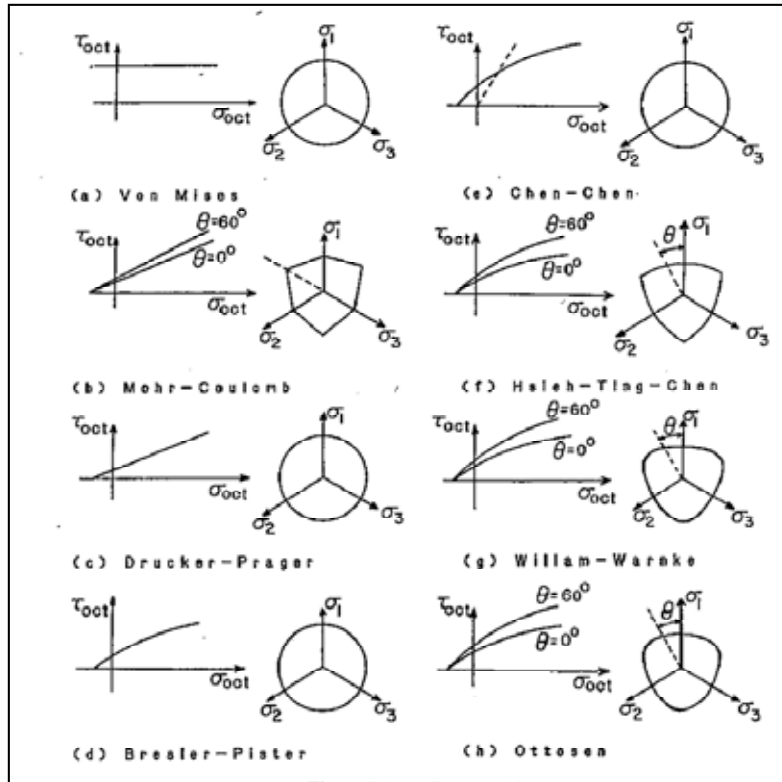


Figure 1: Failure models (Chen, 1985)

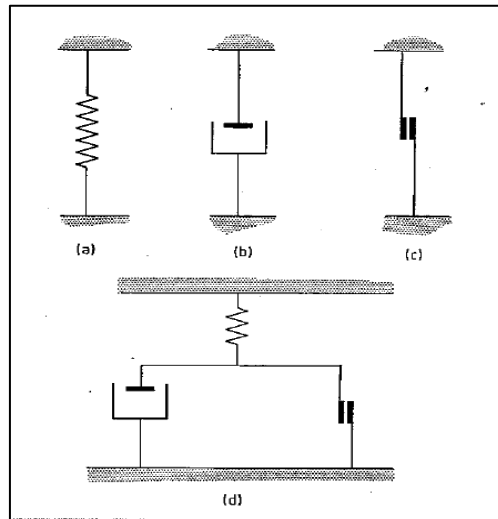


Figure 2: Basic components for material models. (a) Spring-reversible linear/nonlinear elasticity. (b) Dashpot-linear/nonlinear creep. (c) Slider-plastic resistance (strain dependant). (d) Possible elastic, viscoplastic assembly. (Zienkiewicz, 1985).

Few basic and practical soil constitutive models such as Hooke's law, Mohr-Coulomb, Drucker-Prager, Duncan-Chang or Hyperbolic (model), (Modified) Cam Clay, Plaxis Soft Soil

(Creep) and Plastic Hardening Soil Model was discussed and summarized by Brinkgreve (2005) according to the model's advantages and limitation. Application of each model was stated briefly in addition to selection of soil parameters from correlation and laboratory testing for application in finite element models. This paper aims to provide a more brief comparison between the soil models collaborated from various researchers in addition to a few more soil models which was not discussed by Brinkgreve (2005); e.g. Hyperelastic model, Hypoelastic model, Viscoelastic model, Viscoplastic model and the Hierarchical Single Surface model.

MOHR-COULOMB

Mohr-Coulomb model as shown in Fig 3 is an elastic-perfectly plastic model which is often used to model soil behaviour in general and serves as a first-order model. In general stress state, the model's stress-strain behaves linearly in the elastic range, with two defining parameters from Hooke's law (Young's modulus, E and Poisson's ratio, ν). There are two parameters which defines the failure criteria (the friction angle, ϕ and cohesion, c) and also a parameter to describe the flow rule (dilatancy angle, ψ which comes from the use of non-associated flow rule which is used to model a realistic irreversible change in volume due to shearing).

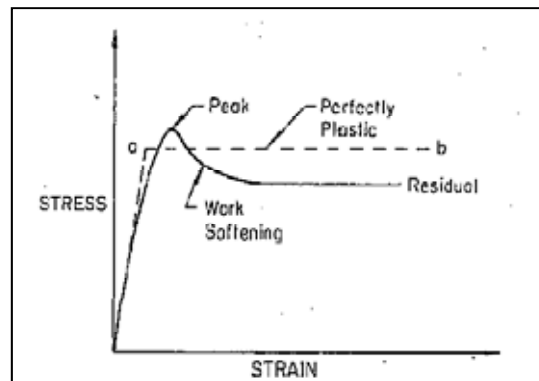


Figure 3: Elastic-perfectly plastic assumption of Mohr-Coulomb model.

In the conventional plastic theory, the flow rule is used as the evolution law for plastic strain rates. If the plastic potential function is the same as the yield function, the flow rule is called the associated flow rule and if it is different, it is called the non-associated flow rule. In soil mechanics, as associated flow rule has been used to model the behaviour in the region where negative dilatancy is significant, for example, the Cam clay model for normally consolidated clay. However, non-associated flow rule is frequently used to describe the behaviour of sands with both negative and positive dilatancy.

Mohr-Coulomb model is a simple and applicable to three-dimensional stress space model (Refer Fig 4) with only two strength parameters to describe the plastic behaviour. Regarding its strength behaviour, this model performs better. Researchers have indicated by means of true-triaxial tests that stress combinations causing failure in real soil samples agree quite well with the hexagonal shape of the failure contour (Goldscheider, 1984). This model is applicable to analyse the stability of dams, slopes, embankments and shallow foundations.

Although failure behaviour is generally well captured in drained conditions, the effective stress path that is followed in undrained materials may deviate significantly from observations. It is preferable to use undrained shear parameters in an undrained analysis, with friction angle set equal to zero. The stiffness (hence also deformation) behaviour before reaching the local shear is poorly modelled. For perfect plasticity, model does not include strain hardening or softening effect of the soil.

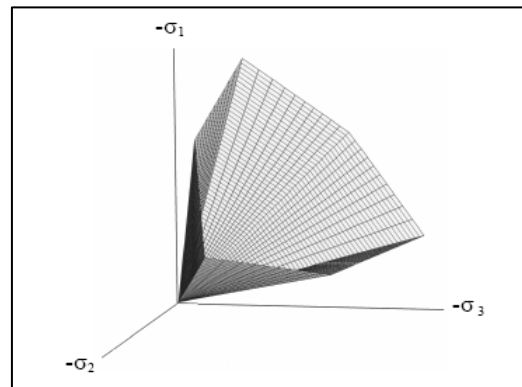


Figure 4: The Mohr-Coulomb yield surface in principal stress space ($c = 0$)

The simplification of Mohr-Coulomb model where the hexagonal shape of the failure cone was replaced by a simple cone was known as the Drucker-Prager model (Drucker & Prager, 1952). Generally, it shares the same advantages and limitations with the Mohr-Coulomb model but the latter model was preferred over this model.

(MODIFIED) CAM-CLAY

Long before the maximum stress has been reached, some irreversible straining has occurred as evidenced by the fact that reloading leaves a residual strain. Soil might be referred to as a strain hardening material since the onset of plastic yielding is not synonymous with the maximum stress. A few researchers have investigated the possibility of modeling soil as a strain hardening material, and this has been one of the major thrusts of the soil mechanics group at Cambridge University for the past thirty years (Roscoe, 1970). Roscoe et al (1963a) utilized the strain hardening theory of plasticity to formulate a complete stress-strain model for normally consolidated or lightly over-consolidated clay in triaxial test known as the Cam-clay model (Schofield and Wroth, 1968). Burland (1965) suggested a modified version of the Cam-clay model and this model was subsequently extended to a general three-dimensional stress state by Roscoe and Burland (1968).

The Modified Cam-clay is an elastic plastic strain hardening model where the non-linear behaviour is modelled by means of hardening plasticity. The model is based on Critical State theory and the basic assumption that there is a logarithmic relationship between the mean effective stress, p' and the void ratio, e . Virgin compression and recompression lines are linear in the e - $\ln p'$ space, which is most realistic for near-normally consolidated clays. (Refer Fig 5 below). Only linear elastic behaviour is modelled before yielding and may results in unreasonable values of v due to log-linear compression lines.

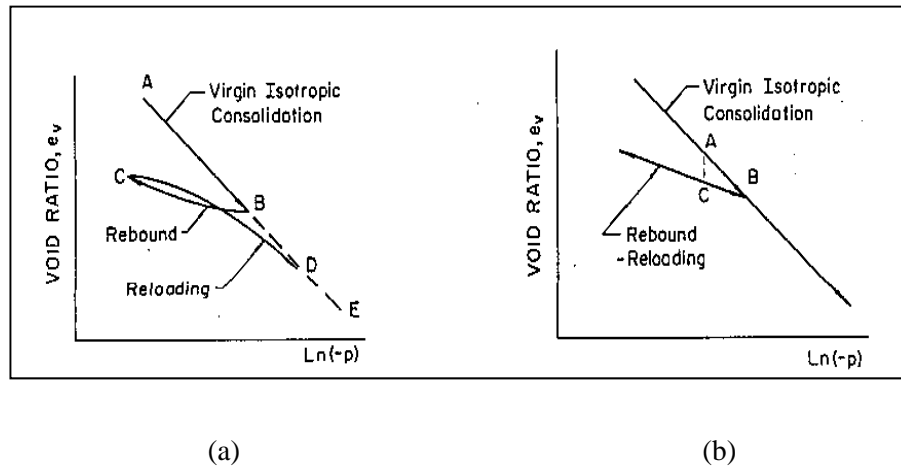


Figure 5: (a) Response of real soil to hydrostatic stress; (b) Response of idealized soil to hydrostatic stress.

This model is more suitable to describe deformation than failure especially for normally consolidated soft soils. The model also performs best in applications involving loading conditions such as embankment or foundation. It involves four parameters, i.e. the isotropic logarithmic compression index, λ , the swelling index, κ , Poisson's ratio for unloading and reloading, ν_{ur} , friction constant, M , pre-consolidation stress, p_c and the initial void ratio, e . Shear strength can only be modeled using the effective friction constant. In the case of primary undrained deviatoric loading of soft soils, the model predicts more realistic undrained shear strength compared to the Mohr-Coulomb model.

In addition to achieve better agreement between predicted and observed soil behaviour, a large number of modifications have been proposed to the standard Cam-clay models over the last two decades. Despite some successes in modifying the standard Cam-clay in the 1980s, Yu (1995, 1998) identified the limitations of this model. The yield surfaces adopted in many critical state models significantly overestimate failure stresses on the 'dry side'. These models assumed an associated flow rule and therefore were unable to predict an important feature of behaviour that is commonly observed in undrained tests on loose sand and normally consolidated undisturbed clays, and that is a peak in the deviatoric stress before the critical state is approached. The critical state had been much less successful for modeling granular materials due to its inability to predict observed softening and dilatancy of dense sands and the undrained response of very loose sands. The above limitations were confirmed by Gens and Potts (1988) where it is also noted that the materials modeled by critical state models appeared to be mostly limited to saturated clays and silts, and stiff overconsolidated clays did not appear to be generally modeled with critical state formulations.

DUNCAN-CHANG (HYPERBOLIC) MODEL

As known, soil behaves highly non-linear and it inhibits stress-dependant stiffness. Apart from the discussed elastic-plastic models, Duncan-Chang model is an incremental nonlinear stress-dependant model which is also known as the hyperbolic model (Duncan and Chang, 1970).

This model is based on stress-strain curve in drained triaxial compression test of both clay and sand which can be approximated by a hyperbolae with a high degree of accuracy (Kondner, 1936) as shown in Fig 6. It is also based on Ohde's (1939) idea that soil stiffness can be formulated as a stress-dependant parameter using a power law formulation. Its failure criteria is based on Mohr-Coulomb's two strength parameters. Most importantly, this model describes the three important characteristics of soil, namely non-linearity, stress-dependant and inelastic behaviour of cohesive and cohesionless soil.

At a given confining stress level, distinction is made between a (stress-dependant) primary loading stiffness, E_t and a (constant) unloading and reloading stiffness, E_{ur} . Loading is defined by the condition $d(\sigma_1/\sigma_3) > 0$. In this condition, plastic deformation occurs as long as the stress point is on the yield surface. For the plastic flow to continue, the state of stress must remain on the yield surface. Otherwise, the stress state must drop below the yield value; in this case, no further plastic deformation occurs and all incremental deformations are elastic. This by the condition $d(\sigma_1/\sigma_3) < 0$ is termed 'unloading'. The parameter E_{50} will be described in the following Hardening Soil model.

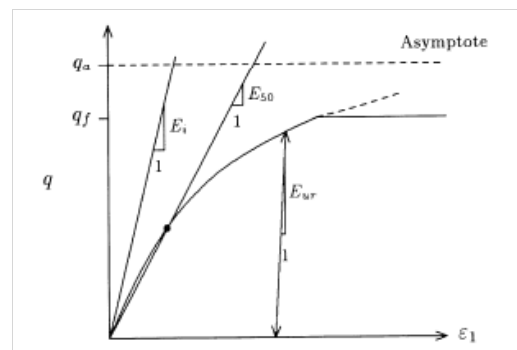


Figure 6: Hyperbolic stress-strain relation in primary loading for a standard drained triaxial test

Duncan-Chang model is widely used as its soil parameters can be easily obtained directly from standard triaxial test. It is a simple yet obvious enhancement to the Mohr-Coulomb model. In this respect, this model is preferred over the Mohr-Coulomb model. Failure itself is described by means of the Mohr-Coulomb failure criterion, but this is not properly formulated in the plasticity framework. As a result, dilatancy cannot be described. This model captures soil behaviour in a very tractable manner on the basis only two stiffness parameters and is very much appreciated for practical modeling. The major inconsistencies of this type of model is that, in contrast to the elasto-plastic type of model, a purely hypo-elastic model cannot consistently distinguished between loading and unloading. In addition, the model is not suitable for collapse load computations in fully plastic range. Potential numerical stability may occur when shear failure is approached. Therefore, ad hoc solution must be applied when the model is implemented in three-dimensional stress space. Nevertheless, Duncan-Chang model was quoted by Brinkgreve (2005) as the improved first order model for geotechnical engineering application in general.

PLAXIS HARDENING SOIL MODEL

The Hardening Soil model (Brinkgreve & Vermeer, 1997; Schanz, 1998) is a true second order model for soils in general (soft soils as well as harder types of soil), for any type of application (Brinkgreve, 2005). The model involves friction hardening to model the plastic shear strain in deviatoric loading, and cap hardening to model the plastic volumetric strain in primary compression. Distinction can be made between two main types of hardening, namely shear hardening and compression hardening. Shear hardening is used to model irreversible strains due to primary deviatoric loading. Compression hardening is used to model irreversible plastic strains due to primary compression in oedometer loading and isotropic loading. Both types of hardening are contained in the present model. Yield contour of the model in three-dimensional space is shown in Fig 7 below. Failure is defined by means of Mohr-Coulomb failure criterion. Because of the two types of hardening, the model is also accurate for problems involving a reduction of mean effective stress and at the same time mobilisation of shear strength. Such situations occur in excavation (retaining wall problems) and tunnel construction projects.

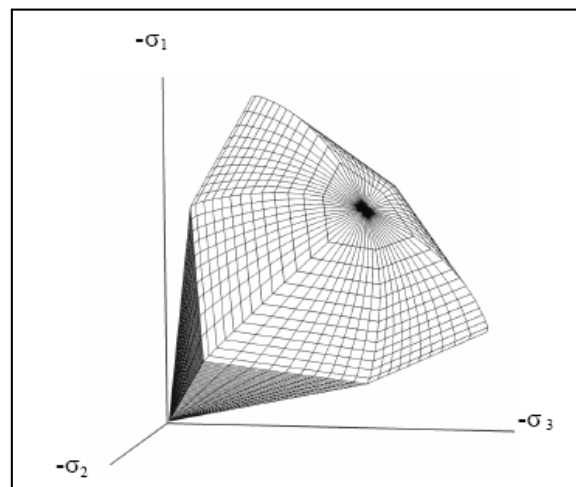


Figure 7: Total yield contour of Hardening-Soil model in principal stress space for cohesionless soil.

With respect to its stiffness behaviour the model involves a power law formulation for stress-dependant stiffness, similar as the one used in the Duncan-Chang model. In fact, the model shows correspondence with the Duncan-Chang model regarding its hyperbolic stress-strain response when simulating a standard drained triaxial test (Refer Fig 6). Since the Hardening Soil model is based on hardening plasticity rather than non-linear elasticity, it overcomes the limitations and inconsistencies of the Duncan-Chang model with respect to dilatancy and neutral loading. Besides that, this model also includes soil dilatancy and yield cap.

Some basic characteristics of the model are Stress dependent stiffness according to a power law (m), plastic straining due to primary deviatoric loading (E_{50}^{ref}), plastic straining due to primary compression (E_{oed}^{ref}), elastic unloading/reloading input parameters (E_{ur}^{ref} , v_{ur}) and failure criterion according to the Mohr-Coulomb model (c , ϕ and ψ).

In undrained loading, the model nicely shows a reduction of mean effective stress, as observed for soft soils, whereas it may also show the increase in mean effective stress for harder type of soils (dilative soils). This model can be used to accurately predict displacement and failure for general types of soils in various geotechnical applications. The model does not include anisotropic strength of stiffness, nor time-dependant behaviour (creep). Its capabilities for dynamic applications are limited.

HYPERELASTIC MODEL

Hyperelastic model or Green model for an elastic body is where the current state of stress depends only on the current state of deformation; that is, the stress is a function of current strain and not a function of history of strain. Materials which satisfy this fundamental requirement of an elastic body are called Cauchy elastic materials. A Cauchy elastic material is characterized as a material which does not depend on the history of the deformation process. The stress computed for a Cauchy elastic material using is independent of the deformation history, however the work done by the body and traction forces, may depend on the deformation history or load path. In other words, a Cauchy elastic material exhibits energy dissipation violating the energy dissipation properties of elastic materials.

For elastic bodies, there exists an unstressed state called the natural state and when external forces are applied, the body deforms and reaches a different energy state. When the external forces are removed, the body regains its natural state and there is no dissipation of energy during the deformation process. Materials for which the work which is performed is independent of the load path are said to be hyperelastic materials (or in the literature often called Green-elastic materials). Hence, hyperelastic materials are elastic materials, which are characterized by the existence of a strain energy function which is the potential for the stress or the complementary energy function which is the potential of the strain.

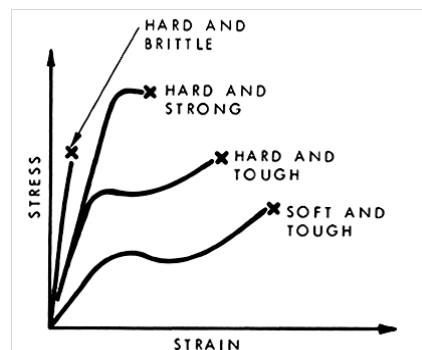


Figure 8: Tensile stress-strain curves for four types of polymeric material.

In order to deduce the strain energy functions, it is assumed, unless indicated, that the material is isotropic and constant volume (isotropic deformation). Also, hyperelastic materials are to be nearly or pure incompressible. The most common functions of deformation energy are the Mooney-Rivlin model, Neo-Hookean model, Ogden model (Ogden, 1972), Yeoh model (Yeoh, 1993), Arruda-Boyce (Arruda and Boyce, 1993), Gent model (Gent, 1996) and Blatz-Ko model (Blatz and Ko, 1962). In terms of mechanics of solid, the main application of the theory is to model the rubbery behaviour of a polymer material and polymeric foams that can be subjected to

large reversible shape change (Refer Fig 8). Generally, it is suitable for materials that respond elastically when subjected to very large strains.

This type of formulation can be quite accurate for concrete and rock in proportional loading. They satisfy the rigorous theoretical requirements of continuity, stability, uniqueness and energy consideration of continuum mechanics. However, this type of model fails to identify the inelastic behaviour of concrete and rock deformation, a shortcoming that becomes apparent when the materials experience unloading. The main objection to the hyperelastic model is that it often contains too many material parameters. For instance, a needed third-order isotropic model requires nine constants; while 14 constants are needed for a fifth-order isotropic hyperelastic model. A large number of tests is generally required to determine these constants, which limit the practical usefulness of the models (Chen, 1985).

HYPONELASTIC MODEL

Hyponelectricity is used to model materials that exhibit nonlinear, but reversible, stress strain behaviour even at small strains. Its most common application is in the so-called 'deformation theory of plasticity', which is a crude approximation of the behaviour loaded beyond the elastic limit. Similar to hyperelasticity, the strain in the material depends only on the stress applied to it, it does not depend on the rate of loading, or history of loading. The stress is a nonlinear function of strain, even when the strains are small, as shown in Fig 9 below.

Truesdell (1955, 1957) proposed a rate theory based on Cauchy formulations for such materials. From Truesdell's theory, incremental stress-strain laws can be developed. It can be shown that nonlinear elastic models can be considered as special forms of the hybrid hyperelastic model. Some of the formulations of piecewise linear elastic idealization have been proposed as hybrid hyperelastic models. Most of the plasticity models are in incremental forms similar to those of hyponelectric models. These models have to be modified suitably to represent the material behavior due to unloading. The Cauchy formulations are more general and complex than the Green models.

Newmark (1960) was the first to direct the attention of the researchers in soil mechanics toward the use of higher order models to represent soil behavior. Coon and Evans (1971) have proposed a hyponelectric model for the deformation behavior of cohesionless soils. Later, Corotis et al (1974) developed another hyponelectric model capable of describing the nonlinear stress-strain behavior of soils loaded along various axisymmetric paths.

Chen (1985) discussed that the path-independent behaviour implied in the previous secant type of stress-strain formulation can be improved by the hyponelectric formulation in which the incremental stress and strain tensors are linearly related through variable tangent material response moduli that are functions of the stress or strain state. In the simplest case of hyponelectric models, the incremental stress-strain relations are formulated directly as a simple extensions of the isotropic linear elastic model with the elastic constants replaced by variable tangential moduli which are taken to be functions of the stress and/or strain invariants.

Models of this type are attractive from both computational and practical viewpoints. They are well suited for implementation of finite element computer codes. The material parameters involved in the models can be easily determined from standard laboratory tests using well defined

procedures, and many of these parameters have broad data base. However, the application of this type of hypoelastic models should be confined to loading situations which do not basically differ from the experimental tests from which the material constants were determined or curve-fitted. Thus, the isotropic models should not be used in cases such as non-proportional loading loading paths or cyclic loadings.

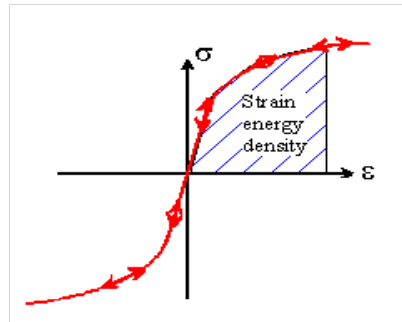


Figure 9: Nonlinear and reversible stress-strain behavior of hypoelastic model.

Chen (1985) also added the following two comments on problems associated with the hypoelastic modeling. The first problem is that, in the nonlinear range, the hypoelastic models exhibit stress induced anisotropy. This anisotropy implies that the principal axes of stress and strain are different, introducing coupling effect between normal stresses and shear strains. As a results, a total of 21 material moduli for general triaxial conditions have to be defined for every point of the material loading path. This is a difficult task for practical application.

The second problem is that under the uniaxial stress condition, the definition of loading and unloading is unclear. However, under multiaxial stress conditions, the hypoelastic formulation provides no clear criterion for loading or unloading. Thus, a loading in shear may be accompanied by an unloading in some of the normal stress components. Thus, a loading in shear may be accompanied by an unloading in some of the normal stress components. Therefore, assumptions are needed for defining loading-unloading criterion. Furthermore, the material tangent stiffness matrix for a hypoelastic model is generally unsymmetrical which results in a considerable increase in both storage and computational time. As a result of this, uniqueness of the solution of boundary value problems cannot be generally be assured.

VISCOPLASTICITY THEORY

The effect of time on the loading process is a salient feature of soils, especially in clayey soils. In general, there are two types of time dependant behaviour. One is due to the interaction of free pore water and the soil skeleton and is called consolidation of soils with low permeability. The other is brought by the inherent viscous characteristics of the soil skeleton. In this theory, Oka (1999) discussed on the constitutive modeling of time dependant behaviour due to the viscous nature of the materials, known as creep, relaxation, rate sensitivity and secondary compression and also common approaches in viscoplasticity theory. The viscous properties result from the microscopic structure of soils like clay. Clayey soils are composed of small clay particles with a high active, ion-water system between them, mainly trapped in the micro-pores.

Very interesting and elegant model of the microscopic nature of the viscous behaviour have been developed by Christensen & Wu (1965), Murayama & Shibata (1964), Ter-Stepanian (1975) and Mitchell (1976) using the rate process theory, and this approach was well reviewed by Sekiguchi (1985). It is well known that there are three approaches to the macroscopic modelling of time-dependant behaviour by using the empirical approach, the viscoelastic approach and the viscoplastic approach.

Empirical and Viscoelastic Approaches

Viscoelasticity consists of an elastic component and a viscous component where viscosity is a strain rate dependent on time. Generally, it has the following characteristics like hysteresis, stress relaxation and creep. Purely elastic materials do not dissipate energy (heat) when a load is applied, then removed. But a viscoelastic substance loses energy when a load is applied, then removed. Hysteresis is observed in the stress-strain curve, with the area of the loop being equal to the energy lost during the loading cycle. The two other main characteristics associated with viscoelastic materials are stress relaxation and creep. Stress relaxation refers to the behavior of stress reaching a peak and then decreasing or *relaxing* over time under a fixed level of strain

Creep is in some sense the inverse of stress relaxation, and refers to the general characteristic of viscoelastic materials to undergo increased deformation under a constant stress, until an asymptotic level of strain is reached. Any materials that exhibit hysteresis, creep or stress relaxation can be considered viscoelastic materials. In comparison, elastic materials do not exhibit energy dissipation or hysteresis as their loading and unloading curve is the same. Indeed, the fact that all energy due to deformation is stored is a characteristic of elastic materials. Furthermore, under fixed stress elastic materials will reach a fixed strain and stay at that level. Under fixed strain, elastic materials will reach a fixed stress and stay at that level with no relaxation as shown in Fig 10a below.

First, viscous or time-dependant behaviour was modeled by an empirical relation based on experimental results observed in a creep test and a relaxation test. The explicit relation between strain and the logarithm of time during which creep occurs is commonly used. Garlanger (1972) proposed a compression model by including a secondary compression. This relation depends explicitly on time, Therefore, it is not an objective constitutive relation but a solution of the adequate constitutive relation. The constitutive relation that contains time explicitly should only be used to describe the inherently time dependant phenomena such as the attenuation of radioactivity. The constitutive relation should generally be described by differential equations. The explicit introduction of time violates the principle of objectivity in continuum mechanics (Eringen, 1962). Consequently, this type of empirical relation is strictly limited to the specific boundary and loading conditions (Singh & Mitchell, 1968). In addition, this type of relation is often one-dimensional and is not applicable to general loading conditions.

The most popular method of modeling time-dependant behaviour of materials is based on viscoelastic approach. This approach has been widely applied to many materials including metals, polymers, soils, concrete, rocks, etc. In the linear viscoelastic model, the Maxwell model, the Voigt model and the three-parameter model with a Voigt element and an elastic spring, called the linear spring-Voigt model, are representative.

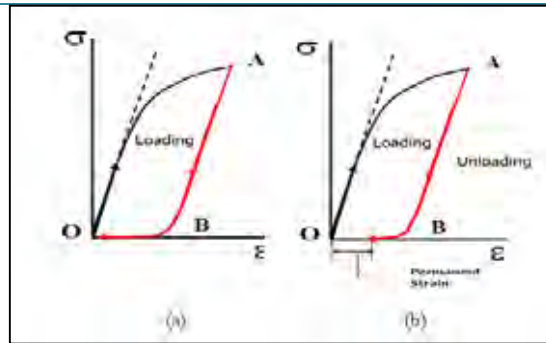


Figure 10: Stress-strain diagram for (a) viscoelastic model and (b) Viscoplastic model.

Viscoplastic Approach

Viscoplasticity theory is relatively simple extension of viscoelastic model where permanent strain is observed (Refer Fig. 10b) in this model. Various elasto-viscoplastic models have been proposed to describe the rheological behaviour of clay. Murayama & Shibata (1964) proposed a rheology model based on the rate process theory, the leading study in this field. Adachi & Okano (1974) proposed an elasto-viscoplastic constitutive model that extends the critical state energy (Roscoe et al, 1963).

For this model, Perzyna's elasto-viscoplastic theory (1963) was introduced to described the rate sensitive behaviour of normally consolidated clay. This theory was reconsidered and generalized by Oka (1981) and Adachi & Oka (1982), in which it is assumed that normally consolidated clay never reaches the static equilibrium state even at the end of primary consolidation, and viscoplastic strain is taken as a hardening parameter. Perzyna's theory is called the overstress model. The key assumption of this model is that viscous effects becomes pronounced only after the material yields, and that the viscous effects are not essential in the elastic domain.

Sekiguchi (1977) proposed an elasto-viscoplastic model for normally consolidated clay based on a non-stationary flow surface. A viscoplastic potential was introduced so that this model could describe universally rate sensitive behaviour of clay, such as creep rupture. The Sekiguchi model is classified as a non-stationary flow surface model because of the response functional includes state variables and time.

In addition for this type of model, it is not commonly applied to critical state models and is believed by some researchers to give extremely poor results for other than very simple perfectly plastic soil models.

APPLICATION IN EMBANKMENTS AND TUNNELS

A recent compilation by Mestat *et al.* (2004) deals with a bibliographic database dedicated to the comparison between FEM results and in situ measurements for geotechnical structures. To date, it comprises a total of 416 case studies, among which 133 deal with embankments on soft soils and 135 with underground works. The majority of the embankments studied have been built on soft soils and in some instances on very soft soils. The constitutive laws employed to describe

the behaviour of such soils are the following; linear and non-linear elasticity, elastoplasticity without strain hardening; elastoplasticity with strain hardening and elastoplasticity. (Fig 11).

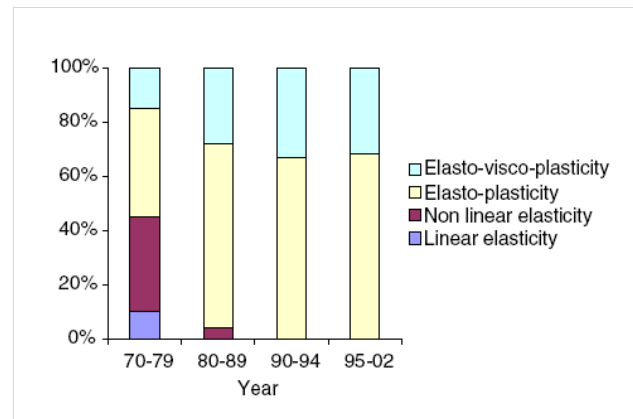


Figure 11: Evolution of the type of constitutive model used in numerical models for soft soils over the past 30 years (Mestat, 2004).

Over the past 10 years or so, the trend has favored use of elastoplastic models coupled with empirically based creep laws. When the embankment is described by finite elements, the most widespread constitutive law is isotropic linear elasticity (55%), followed by perfect elastoplasticity (36%) and non-linear elasticity (9%). The choice of such simple models stems from limited information available on both the materials and their mode of implementation.

As for tunnels, the constitutive models used for describing soil behaviour (whether excavated or not) encompass the following types of laws: linear and non-linear elasticity (these tend to comprise the oldest reference); elastoplasticity without strain hardening; elastoplasticity with strain hardening and elasto-viscoplasticity (Refer Fig 12). Generally speaking, the most widespread constitutive law is the Mohr-Coulomb perfect elastoplasticity model with isotropic linear elasticity. Among the elastoplastic laws with strain hardening, the Cam-clay models remains the most widely used.

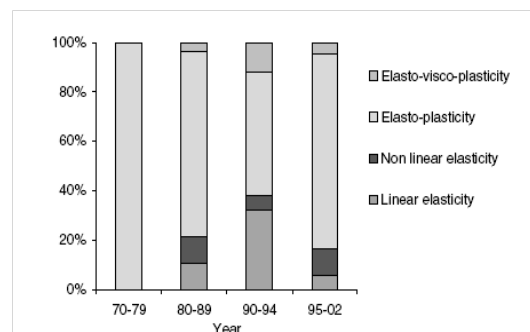


Figure 12: Evolution of the type of constitutive model used in numerical models of tunnels over the past 30 years.

SUMMARY

Various models like Mohr-Coulomb, Duncan-Chang, Plaxis Hardening Soil, hyperelastic, hypoelastic, viscoelastic and viscoplastic model have been discussed earlier on. With advanced computing technology, the incorporation of these models in finite element modeling can be easily done. Although computers is able to produce accurate numerical results does not mean that the results based on a specific problem will also exhibits this level of accuracy. In modeling, it is important to choose a soil constitutive model which is the 'right' answer to the problem being considered. Even with 'right' models, it should be emphasized that there are approximations within this level of accuracy. For example, approximation in the finite element method, approximation in assumptions about the constitutive soil response of the soil and the detailed description of the numerical model and its boundary conditions.

Geotechnical engineers often faces challenges to choose the most appropriate soil model applicable in their numerical modeling. Therefore, there should be in depth understanding on the concepts, advantages, limitation and also output of each model for each problem being modelled. Engineers should also make use of constitutive model which provides a reasonable fit to data obtained from range of laboratory test. It is important to conduct various computation-measurement comparisons along with additional full-scale experiments to ascertain the degree of realism in the models in order to adjust and refine them to each type of different modelling application.

ACKNOWLEDGEMENT

Authors wishes to record their appreciation to the Ministry of Higher Education Malaysia for funding this research under the Fundamental Research Grant Scheme.

REFERENCES

1. Adachi, T and Oka, F. (1982) "Constitutive equations for normally consolidated clay based on elasto-viscoplasticity", *Soils & Foundations*, Vol. 22, No. 5, pp 57-70.
2. Adachi, T and Okano, M. (1974) "A constitutive equation for normally consolidated clay", *Soils & Foundations*, Vol. 14, No. 4, pp 55-73.
3. Arruda, EM and Boyce MC. (1993) "A three-dimensional constitutive model for the large stretch behaviour of rubber elastic material", *Journal of the Mechanics and Physics of Solids*, Vol. 41, No. 2, pp 389-412.
4. Blatz, PJ and Ko WL. (1962) "Application of finite elastic theory to the deformation of rubbery materials", *Transaction of the Society of Rheology*, Vol VI, pp 223-52.
5. Brinkgreve, RBJ. (2005) "Selection of soil models and parameters for geotechnical engineering application". In: Yamamuro, JA and Kaliakin, VN, editors. Geotechnical Special Publication No.128, ACSE, pg 69-98.

6. Brinkgreve, RBJ and Vermeer, PA.(1997) "Plaxis finite element code for soil and rock analysis-Version 7", Balkema, Rotterdam.
7. Burland, JB.(1965) "The yielding and dilation of clay;correspondence", *Geotechnique*, Vol. 15, No. 2, pp 211-219.
8. Chen, CF. (1985) "Mechanics of Geomaterials", In: Z. Bazant; editor. John Wiley & Sons Ltd. pg 65-86.
9. Christensen, RW and Wu, PL. (1965) "Analysis of clay deformation as the rate process", *J. Soil Mech. Found. Div.* 91 (SM5), ASCE, pp 125-157.
10. Coon, MD and Evans, RJ. (1971) "Recoverable Deformation of Cohesionless Soils", *Journal of the Soil Mechanics and Foundations Division*. ASCE, Vol. 97, No SM 2.
11. Corotis, RB, Farzin, MH. and Krizeck, RJ (1974) "Nonlinear Stress- Strain Formulations for Soil", *Journal of the Geotechnical Engineering Division*. ASCE. Vol. 100, No. GT9.
12. Drucker, DC and Prager, W. (1952) "Soil mechanics and plastic analysis or limit design", *Quart. Applied Math.* Vol. 10, No. 2, pp 157-165.
13. Duncan, JM and Chang, CY. (1970) "Nonlinear analysis of stress and strain in soils", ASCE *J. of the Soil Mech. And Found. Div.* Vol. 96, pp 1629-1653.
14. Eringen, C. (1962) "Non-linear theory of continous media", McGraw Hill.
15. Garlanger, J.E. (1972) "The consolidation of soils exhibiting creep under constant effective stress", *Geotechnique*, Vol. 22, No.1, pp 71-78.
16. Gens, A., Potts, D.M. (1988) "Critical state models in computational geomechanics", *Engineering Computations*. Vol. 5, pp 178 – 197.
17. Goldscheider, M. (1984) "True triaxial tests on dense sands". In: G. Gudehus, F.Darve, I. Vardoulakis; editors. Constitutive relations for soils. Balkema, Rotterdam.
18. Gent, A.N. (1996) "A new constitutive relation for rubber", *Rubber Chem. Technol.* Vol 69, pp 59-61.
19. Kondner, R.L. (1963) "Hyperbolic stress-strain response: cohesive soils", *J of the Soil Mech. And Found. Eng.* ASCE ,Balkema, Rotterdam.
20. Mestat, PH, Bourgeois, E and Riou, Y. (2004) "Numerical modelling of embankments and underground works", *Computers and Geotechnics*, Vol. 31, pp 227-236.
21. Mitchell, JK. (1976) "Fundamentals of Soils Behaviour", John Wiley & Sons.
22. Murayama, S and Shibata, T. (1964) "Flow and stress relaxation of clays". IUTAM symp. On Rheology and Soil Mechanics, Grenoble, pp 99-129.

23. Newmark, N. M. (1960) "Failure Hypothesis for Soils," Proceedings Society Of Civil Engineers, Boulder, Colorado, June.
24. Ogden, RW. (1972) "Large deformation isotropic elasticity on the correlation of theory and experiment for incompressible rubberlike solids", Proc. R Soc. Lond. A
25. Ohde, J. (1939) "Zur theorie der druckverteilung im Baugrund." Der Bauingenieur, Vol 20, pp 451-459.
26. Oka, F. (1999) "Continuum theory of granular materials" An Introduction Mechanics of Granular Materials. Oda, M & Iwashita, K, editors. Balkema, Netherlands, pp 107-113.
27. Perzyna, P.(1963) "The constitutive equations for work hardening and rate sensitive plastic materials" Proc. Vibrational Problems, Vol 4, No. 3, Warsaw, pp 281-290.
28. Roscoe, KH. (1970) "The influence of strains in soil mechanics" Tenth Rankine lecture. *Geotechnique*, Vol. 20, No. 2,pp 129-170.
29. Roscoe, KH and Burland, JB. (1968) "On the generalized stress-strain behaviour of wet clay." In: J. Heyman and FA Leckie, editors. Engineering Plasticity. Cambridge University Press, London, pp. 535-609.
30. Roscoe, KH, Schofield,AN and Thrairajah, A. (1963) "Yielding of clays in states wetter than critical", *Geotechnique*, Vol.13, No.3, pp 211-240.
31. Schanz, T. (1998) "Zur modellierung des mechanischen verhaltens von reibungsmaterialien. Habilitation." Stuttgart University.
32. Schofield, A.N and Wroth, CP. (1968) "Critical State Soil Mechanics", McGraw-Hill, New York.
33. Yeoh, OH. (1993) "Some forms of the strain energy function for rubber", *Rubber Chem Technol.* Vol 66, pp 754-71.
34. Sekiguchi, H. (1977) "Rheological characteristic of clays", Proc. 9th Int. Conf. On SMFE, 1, Tokyo, pp 289-292.
35. Sekiguchi, H. (1985). "Macroscopic approaches-static-intrinsically time-dependant in Constitutive Laws of Soils", Pro. Discussion Session IA:In: 11th Conf. On SMFE, pp 66-98
36. Singh, A and Mitchell, JK. (1968) "General stress-strain-time function for soils", *J. Soil Mech. Found. Div.*94 (SM 1), ASCE, pp 21-46.
37. Ter-Stepanian. (1975) "Creep of clays during shear and its rheological model", *Geotechnique*. Vol 25, No. 2.
38. Truesdell, C. (1955a) "The simplest rate theory of pure elasticity," *Comm Pure Appl Math*: 8. pp 123-132.

39. Truesdell, C. (1955b) "Hypoelasticity", *J. Rational Mech. Anal.* 4, pp 83-133
40. Yu, H.S. (1995) "A unified critical state model for clay and sand", Civil Engineering Research Report No. 112.08.1995. University of Newcastle, NSW 2308, Australia.
41. Yu, H.S. (1998) "CASM: A unified state parameter model for clay and sand", *International Journal of Numerical and Analytical Methods in Geomechanics*, 22, pp 621–653
42. Zienkiewicz, OC. (1985) "Mechanics of Geomaterials". In: Z. Bazant; editor. John Wiley & Sons Ltd.

