The Effect of Failure Criterion on Slope Stability Analysis

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ABSTRACT

The stability of a slope is analyzed by using the pseudo-static approach. The influence of the introduction of principal effective normal intermediate stress (σ'_2) in the formulation of sliding block stability by using Drucker-Prager criterion is explored. The results show that this approach indicates the greatest stability of the studied slope.
To facilitate the task, a visual program of calculation has been created, which is interactive with a database software and utilizes the concept of object-oriented programming.

KEYWORDS: slope; Mohr-Coulomb; Pseudo-static; Drucker-Prager; intermediate effective principal stress

INTRODUCTION

The General Limit Equilibrium Method (GLEM) is often used to evaluate slope stability. In fact, since Fellenius (1936) has quantified slope stability with linear safety factor, the GLEM method has known much refinement to approach the in situ real soil behavior (Bishop, 1955; Janbu, 1957; Morgenstern and Price, 1965; Spencer, 1967; Newmark, 1965, Sarma 1979 and Zhu, 2005; and others).

The basic concept of this method (GLEM) consists of evaluating the stability of a sliding block (usually divided into slices) in terms of a safety factor, which is defined as the ratio of the sum of the stabilizing and destabilizing forces in the inclined ground. In other words, a slope becomes unstable when the shear stresses acting on potential failure plane exceed the shearing resistance of soil. In addition, in most methods the problem is analyzed in two dimensions, thus only two stresses σ'_3 and σ'_1 are used to evaluate the shear stress at a slip surface in the Mohr-Coulomb criterion.

In this work, in order to approach the in situ real conditions and because the confined intermediate normal effective stress (σ'_2) is an inherent property of soil, one proposes, in this paper, to replace the Mohr-Coulomb model by Drucker-Prager criterion, and to reformulate the slope's stability to estimate the safety factor via a pseudo-static analysis. Also, t (shear stress in slip surface) is substituted by (q') (deviator stress, which is also a shear stress)

(Figure 5) in the same surface. The results show that this approach gives most stability of the studied slope.

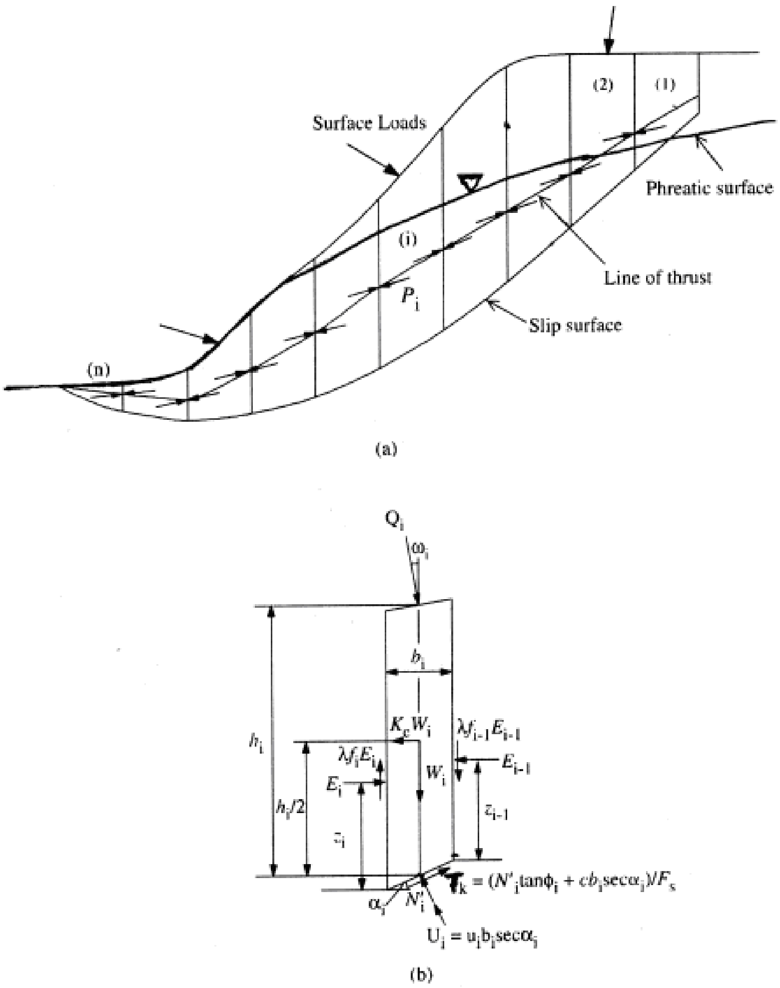

Figure 1: (a) Sliding body. (b) Typical slice. P_i: Inter slice force

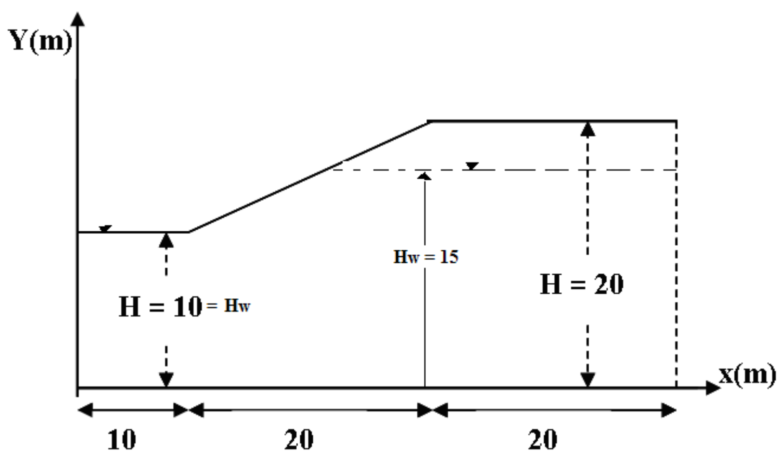

Figure 2: Soil profile with one layer.

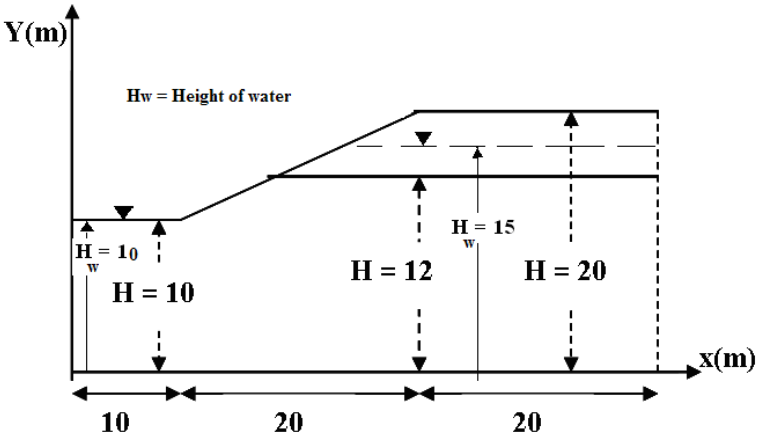

Figure 3: Soil profile with two layers.

To make our task easy, a visual computer program (interactive with access software database and using a concept of programming oriented object -P.O.O-) is developed.

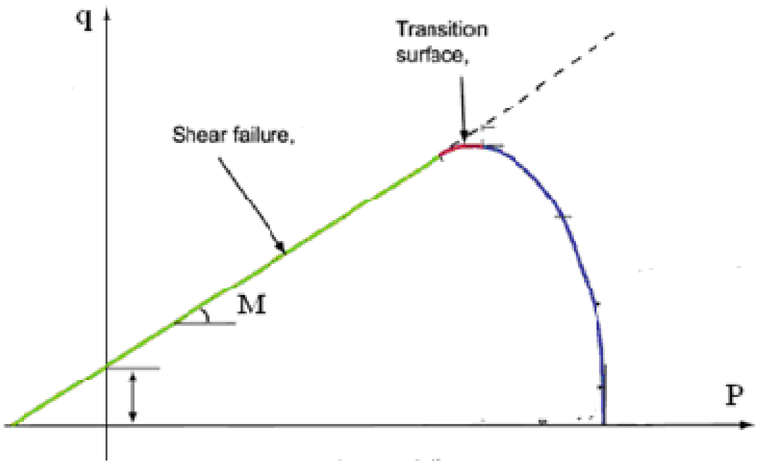

Figure 4: Drucker-Prager Model in p - q plane

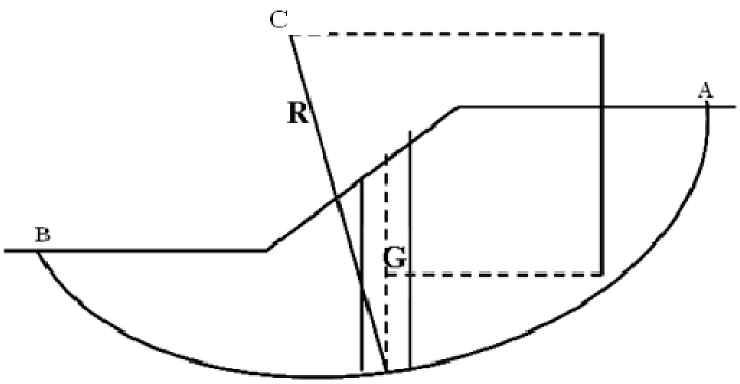

Figure 5: Slope failure mechanism.

DRUCKER-PRAGER CRITERION

In general, the soil behavior in slope stability is established by the Mohr-Coulomb criterion and governed by the following equation:

$$\tau_f = c' + \sigma' \tan\phi' \tag{1}$$

With: τ_f is the shear stress in the soil on the failure plane (or, shear strength), σ' is the effective normal stress on that plane, c' is the effective cohesion, and ϕ' is the effective friction angle of soil.

In terms of principal effectives stress, we can rewrite eq. (1) as follow:

$$\sigma_3' = \sigma_1'\left[\frac{1 - Sin(\varphi')}{1 + Sin(\varphi')}\right] - 2C'\left[\frac{Cos(\varphi')}{1 + Sin(\varphi')}\right] \tag{2}$$

Where, σ'_3 and σ'_1 are respectively the minor and the major principal effective stresses.

It appears that Eq. 2 neglects the confining stress (σ'_2) which is an inherent property of the soil. In order to study its influence on the slope stability, one proposes to rewrite Eq. 2 based on the following mathematical elementary manipulations: [17]

$$\frac{q'}{P' + [C' / Tan(\varphi')]} = M \tag{3}$$

p' is given by the following equation:

$$P' = \frac{\sigma_1' + 2\sigma_3'}{3} \tag{4}$$

And q', M are given as follows:

$$q' = \sigma_1' - \sigma_3' \tag{5}$$

$$M = \frac{6Sin(\varphi')}{3 - Sin(\varphi')} \tag{6}$$

Eq. (3) is equivalent to eq. (2) and has the advantage that the parameters p' (Eq. 4) and q' (Eq. 5) are invariant terms. For simplification purpose, one takes $\sigma'_2 = \sigma'_3$, then Eq. 3 becomes:

$$q' = \left(C' M / Tan(\varphi')\right) + MP' \tag{7}$$

The Mohr-Coulomb criterion (Eq. 1) is rewritten in terms of triaxial stresses variables (p', q') to obtain the relation (7) [17]. Thus, (7) is the modified Mohr-Coulomb criterion in the space (p', q') and takes into account the confined effective stress (σ'_2). The failure of slope, in this case, is not under the shear stress τ (Classical Mohr-Coulomb criterion (Eq. 1), but under the deviator stress (q') for the same slip surface (Figure 4) (Modified Mohr-Coulomb criterion (Eq. 7).

Eq. 7 represents a Drucker-Prager criterion. Thus, this last one is given as:

$$\sqrt{J_2} = A + BI_1 \; ; \quad A = \frac{6C' \, Cos(\varphi')}{(3 - Sin(\varphi'))\sqrt{3}} \quad ; \quad B = \frac{2Sin(\varphi')}{(3 - Sin(\varphi'))\sqrt{3}}$$

$$J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2] \quad ; \quad I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

Where I_1 is the first invariant of the Cauchy stress and J_2 is the second invariant of the deviatoric part of the Cauchy stress.

By taking $\sigma'_2 = \sigma'_3$ (to simplify our problem) in the relations of Drucker-Prager model above and making some elementary manipulations, one obtains Eq. 7. Hence, by using Eq. 7, the failure of slope is not estimated in the space (τ, σ'); i.e. by using a classical Mohr-Coulomb criterion, but in (p', q') space (Figure 5) by using Drucker-Prager criterion which includes (σ'_2). The two states of stress spaces (τ, σ') and (q', p') are not the same in the same slip surface.

In order to perform a pseudo static analysis of the slope stability by using Eq. 7, the method developed by Zhu *et al.* (2005) [16] is used and Mohr-Coulomb criterion is replaced by Drucker-Prager criterion (by using Eq. 7) for the same slip surface.

EQUILIBRIUM EQUATION AT SLIP SURFACE

By combining equations (2) and (4) and (7) and (8), one obtains respectively equations (8) and (9):

$$P' = (1 + 2K_O)\sigma'_1 / 3 - 4C'\sqrt{K_O} / 3 \tag{8}$$

$$q' = M(1 + 2K_O)\sigma'_1 / 3 - 4MC'\sqrt{K_O} / 3 + MC' / Tan(\varphi') \tag{9}$$

The area at the foot of each sliding corner slice, is given by

$$\eta = b / Cos(\alpha) \tag{10}$$

Where, α is a slope angle, K_O is an active coefficient of earth; N' is a normal (to slip surface) effective force at the bottom of slice, b the width of slice (Fig.1-b).

From equations (9) and (10), one writes the shear force equation at the bottom of the slice (Fig.1-b) in the following compact form:

$$T = \xi_1 N' + \xi_2 \tag{11}$$

In Eq. 11, $N' = \sigma'_1 \, \eta = \sigma'_1 \, bSec(\alpha)$ *where* $\sigma'_1 = \sigma'_v$ (is the effective in situ major vertical normal stress), and,

$$\xi_1 = M(1 + 2K_O) / 3 \tag{12}$$

$$\xi_2 = -4M\,\eta\,C'\sqrt{K_O}/3 + M\,\eta\,C'/Tan(\varphi') \tag{13}$$

Where, T = q'η is the shear force corresponding to the deviator stress q'. Really q' is the shear stress (Figure 5) at failure plan obtained because we include a tree principals stresses $\sigma_1', \sigma_2', \sigma_3'$ (with $\sigma_2' = \sigma_3'$ to simplify our problem in this study) (see Eq.7 of Drucker-Prager). The difference between $(\tau \text{ and } q')$ is that q' is a shear stress (see Figure 5) obtained by including the effect of σ_2' and (τ) has not submitted to the effect of σ_2'. In others terms, because (1) can be rewritten as (7) [17], and (7) is failure criterion, and (q') (see Figure 5) is a shear stress, then, (q'η) is the shear force at failure plan; i.e. at slip surface of slope. In addition, at the given extremely small point of soil, one considers that the effect of σ_2' and σ_3' is the same or homogenous; i.e. $\sigma_2' = \sigma_3'$ to simplify our problem.

SAFETY FACTORS FS

By considering the force equilibrium of the ith slice, and by resolving along the perpendicular to the slip surface (Fig.1 – b), one finds:

$$\begin{aligned} N_i' = {} & [W_i + \lambda f_{i-1}E_{i-1} - \lambda f_i E_i + Q_i Cos(\omega_i)]Cos(\alpha_i) \\ & + [-K_c W_i + E_i - E_{i-1} + Q_i Sin(\omega_i)]\,Sin(\alpha_i) - u_i\eta_i \end{aligned} \tag{14}$$

Where, *K*c is the horizontal seismic coefficient; λ is a scaling factor; W_i is weight at slice i; α_i is the slope of slice i; external force *Qi*, making angle ω*i* with the vertical (positive as indicated in Fig. 1- b); u_i is the average water pressure.

And along the parallel to the slip surface (Fig.1 –b) one finds:

$$\begin{aligned} (\xi_1 N_i' + \xi_2)/Fs = {} & (W_i + \lambda f_{i-1}E_{i-1} - \lambda f_i E_i + Q_i Cos(\omega_i))Sin(\alpha_i) \\ & - (-K_c W_i + E_i - E_{i-1} + Q_i Sin(\omega_i))\,Cos(\alpha_i) \end{aligned} \tag{15}$$

Normal interslice forces *Ei* and *Ei*-1 acting on the left and right boundaries of the slice at vertical distances z_i and z_{i-1} from the bottom, respectively; f_i is the interslice function.

Then, by substituting Eq. (14) into Eq. (15) one obtains,

$$\begin{aligned} & E_i[\xi_{1i}(Sin(\alpha_i) - \lambda f_i Cos(\alpha_i)) + Fs(\lambda f_i Sin(\alpha_i) + Cos(\alpha_i))] = \\ & E_{i-1}[\xi_{1i}(Sin(\alpha_i) - \lambda f_{i-1}Cos(\alpha_i)) + Fs(\lambda f_{i-1}Sin(\alpha_i) + Cos(\alpha_i))] + \\ & Fs[(W_i Sin(\alpha_i) + K_c W_i Cos(\alpha_i) - Q_i Sin(\omega_i - \alpha_i)] - [W_i Cos(\alpha_i) \\ & - KcWiSin(\alpha i) + Q_i Cos(\omega_i - \alpha_i) - u_i\eta_i]\xi_{1i} - \xi_{2i} \end{aligned} \tag{16}$$

Equation (16) is rearranged in the following compact form:

$$E_i\phi_i = \psi_{i-1}E_{i-1}\phi_{i-1} + FsT_i - R_i \tag{17}$$

With

$$\phi_i = \xi_{1i}(Sin(\alpha_i) - \lambda f_i Cos(\alpha_i)) + Fs(\lambda f_i Sin(\alpha_i) + Cos(\alpha_i)) \tag{18}$$

$$\phi_{i-1} = \xi_{1(i-1)}(Sin(\alpha_{(i-1)}) - \lambda f_{(i-1)}Cos(\alpha_{(i-1)})) + Fs(\lambda f_{(i-1)}Sin(\alpha_{(i-1)}) + Cos(\alpha_{(i-1)})) \quad (19)$$

$$T_i = W_i Sin(\alpha_i) - Q_i Sin(\omega_i - \alpha_i) + K_c \, W_i Cos(\alpha_i) \quad (20)$$

$$R_i = [W_i Cos(\alpha_i) - K_c W_i Sin(\alpha_i) + Q_i Cos(\omega_i - \alpha_i) - u_i \eta_i]\xi_{1i} + \xi_{2i} \quad (21)$$

$$\psi_{(i-1)} = [\xi_{1i}(Sin(\alpha_i) - \lambda f_{(i-1)}Cos(\alpha_i)) + Fs(\lambda f_{(i-1)}Sin(\alpha_i) + Cos(\alpha_i))] / \phi_{(i-1)} \quad (22)$$

With the limit conditions $E_0=0$ and $E_n=0$ (E_0 and E_n are the inter-slice horizontals forces at the upper and the lower ends, respectively) from Eq (17), the force equilibrium equation is derived in the form of an expression for the factor of safety FS:

$$Fs = \frac{\sum_{i=1}^{n-1} R_i (\prod_{j=i}^{n-1} \psi_j) + R_n}{\sum_{i=1}^{n-1} T_i (\prod_{j=i}^{n-1} \psi_j) + T_n} \quad (23)$$

With R_i is the sum of the shear resistances contributed by all the forces acting on the slices except the normal shear inter-slice forces, and T_i is the sum of the component of these forces tending to cause instability.

SCALING FACTOR λ

Now, consider the moment equilibrium of the i^{th} slice. The moments of all the forces acting on the slice about the center of the base (Fig.1 – b) lead to:

$$E_i(z_i - \frac{b_i}{2} Tan(\alpha_i)) = E_{(i-1)}(z_{(i-1)} + \frac{b_i}{2} Tan(\alpha_i)) - \lambda \frac{b_i}{2}(f_i E_i + f_{(i-1)} E_{(i-1)}) + KcW_i \frac{h_i}{2} - Q_i h_i Sin(\omega_i) \quad (24)$$

If we assume that (Fig.1 – b)

$$M_i = E_i \, z_i \text{ and } M_{(i-1)} = E_{(i-1)} \, z_{(i-1)}$$

Equation (24) takes the following form:

$$M_i = M_{(i-1)} - \lambda \frac{b_i}{2}\left(f_i E_i + f_{(i-1)} E_{(i-1)}\right) + K_c W_i \frac{h_i}{2} + \frac{b_i}{2}\left(E_i + E_{(i-1)}\right) Tan(\alpha_i) - Q_i h_i Sin(\omega_i) \quad (25)$$

As $M_0 = 0$ and $M_n = 0$, the moment equilibrium equation is derived in the form of an explicit expression for the scaling factor λ :

$$\lambda = \frac{\sum_{i=1}^{n}\left[b_i\left(E_i + E_{(i-1)}\right)Tan(\alpha_i) + A_1 + A_2\right]}{\sum_{i=1}^{n}\left[b_i\left(f_i E_i + f_{(i-1)} E_{(i-1)}\right)\right]} \quad (26)$$

With $A_1 = K_c W_i h_i$ and $A_2 = 2 Q_i h_i Sin(\omega_i)$

In all the above equations, the index (i) means number of slice.

APPLICATIONS

In order to study the influence of the confined effective intermediate normal stress, two examples are treated. The first one is a 1 layer soil (Figure 2) and the second one is a 2 layers soil (figure. 3). The characteristics of each soil are given in Table 1. For both two examples, one chooses the same probable failed circle (radius = 20.6m, x_c = 16m and y_c =22m, C is center of circle) (found the critical slip surface is not the objective of this paper) and a constant number of slices equal to (280). The tolerance taken for two successive iterations for FS (safety factor) and for λ is set to 0.0001 (ΔFs = Δλ = 0.0001). The two extremes points of a failed circle (Figure 4) are A (x_A=34.8m, y_A= 20 m) and B (x_B = 2 m, y_B =10 m).

Table 1: Characteristics of soil

	Figure 3 Two layers		Figure 2 one layer
N° Layer	1	2	1
Height (m)	8	12	20
c′ (kN/m^2)	28.74	23	23
γ_{sat}(kN/m^3)	18.85	19	19
ϕ′ (Deg)	20	18	18

DISCUSSIONS

The computation of the pseudo-static analysis is done for the both examples. The results obtained by the 1-layer and 2-layers are compared with the other methods (Zhu, Bishop, Janbu and Fellenius) and showed respectively in Tables 2 and 3. It appears that the present method which takes into account the confined intermediate normal stress effect lead to a most important safety factor, which is underestimated by the other methods (i.e. Zhu, Bishop, Janbu, and Fellenius). This means that the soil present a better resistance by introducing (σ'_2) (effective confined stress) in our formulation to establish the safety factor (FS) expression.

This result shows that the confined stress, which is a natural property of soil, has a real effect in the soil stability when one considers the pseudo static method.

Table 2: for one layer example (Figure 2)
(280 slices were used in this example)

$f_i = f_{i-1}$= 1.85 (see Fig.1-b); Kc = 0.02; ΔFs = Δλ = 0.0001				
Methods	Water		No Water	
	Fs	λ	Fs	λ
Present method	0.920	0.1856	1.552	0.1941
Zhu method	0.633	0.1855	1.068	0.1940
Bishop*	0.701	-	1.116	-
Janbu*	0.588	-	1.004	-
Fellenius*	0.666	-	1.062	-

*The safety factors expressions were taken in Mallkawi & al [8]

Table 3: for two layers example (Figure 3)
(280 slices were used in this example)

fi = fi-1= 1.85 (see Fig.1-b); Kc = 0.02; ΔFs = Δλ = 0.0001				
Methods	Water		No Water	
	Fs	λ	Fs	λ
Present method	0.941	0.1734	1.583	0.1867
Zhu method	0.651	0.1702	1.095	0.1845
Bishop*	0.722	-	1.145	-
Janbu*	0.610	-	1.042	-
Fellenius*	0.690	-	1.092	-

*The safety factors expressions were taken from Mallkawi & al [8]

Also, because σ'_2 exists in the soil for each slice, then, the shear stress taken in our calculation must take the effect of all stresses existing in the soil. That why, it is very important to choose a failure criterion which take all stresses in three dimensional stresses space. In fact, if we use the Mohr-Coulomb criterion, then the effect of σ'_2 on the comportment of slip surface is not possible. The shear stress (τ) in the slip surface is not the result of application of all the tree principals stresses. But, if we use the Drucker-Prager criterion, the deviator stress q' (which is also a shear stress) is more appropriate to replace the shear stress (τ). The q', in this case, encompasses all the tree principals effective normal stresses. Under this approach, the results will be more realistic and better. We say better, because σ'_2 is an effective confined stress. Then the soil has a better strength.

In others terms, because (1) can be rewritten as (7), and (7) is failure criterion, and (q') (Figure 5) is a shear stress, then, ($q'\eta$) is the shear force at failure plan; i.e. at slip surface of slope.

Computations were done by considering and neglecting the water presence. The results show in the two cases that the safety factor is greater when one neglects the water presence. This means that the presence of water in soil decreases the strength of the sliding corner.

CONCLUSION

In their efforts to approach the in situ real conditions in the behavior of soils, scientists have been inclined to employ powerful formulations to satisfy this imperative. These formulations have taken a great importance, especially in their stochastic form. However, the idea to include, at first, the maximum parameters that inherently exist in the soil and have an influence on the studied phenomenon in our formulation is, also, a non negligible manner to describe the in situ reality of soil. To satisfy this aim, we focused our efforts to reformulate the slope stability by including the intermediate principal effective normal stress (σ'_2). The introduction of this stress is, in fact, correct because, it exists really in the soil and may have a direct effect on the stability of sliding slope under external actions. Hence, the formulations, which don't consider this intermediate stress, overshadow the real behavior of soil as it is shown in the results of pseudo static method established in this study.

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