

# Slope Slip Band Based on Circular Slip Surface

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## ABSTRACT

For selection of control variables for a slope's circular slip surface, center coordinates and radius of circular slip surface were considered. Their search ranges are too large, so it is not easy to find the minimum of factor of safety. Proposed control variables are end-points abscissa and arc height of circular slip surface. Their search ranges are small, so it can rapidly find minimum of factor of safety. Many researchers focus on find slip surface with the smallest factor of safety. Different methods can indicate different slip surfaces, though usually they are almost the same. Obtaining a single slip surface is not very important issue in practice. Because of this uncertainty, a slip band composed of many slip surfaces may be more effective. Numerical computation shows that, for homogeneous soil slope, slip bands are almost the same based on Fellenius and Bishop methods, and width of slip band usually is less than 1 m.

**KEYWORDS:** control variable; slip band; circular slip surface; slice method.

## INTRODUCTION

The method of slices is widely used by engineers and researchers, and this is a traditional and well established method. Although the method of slices does not consider the stress-strain relation of soil, it can provide an estimate for the factor of safety of a slope without the knowledge of the initial conditions, which is one of the reasons why the slice method is favored by many engineers (Cheng, 2007). Fellenius method and Bishop method are two such methods. They both assume that the slip surface with the minimum of factor of safety to be a circular surface (*Editor's note: the actual slip surface is roughly a cylinder; the circle is its cross section in the plane of analysis in which the sliding motion occurs*). The calculated factor of safety by Fellenius method for a given slip surface is often slightly less than that by Bishop method. Some research show that Fellenius and Bishop methods, although they give different factor of safety values, they can give almost the same slip surface for homogeneous soil slope. For heterogeneous multi-soil layer slope, if the profile is not very complex, the slip surface can be assumed to be a circular one (Ning, 2008). Usually center coordinates and radius of circular slip surface are selected as control variables for circular slip surface. Because their search ranges are too large, it is difficult to find minimum of factor of safety. So new control variables are proposed in this paper.

Some research suggested that use Fellenius method as initial computation, and then use “rigorous” method (such as “rigorous” Janbu method and Morgenstern-Price method) for further computation. Fellenius method considers circular failure surface, factor of safety is calculated without an iteration, making their method one of the most commonly used in engineering practice. Bishop method uses a circular failure mechanism with very thin slices and need iterative calculation (Bardet, 1989). So Fellenius method completes more rapidly on a computer than Bishop method. Also, iteration sometimes can fail to converge (Cheng, 2008), and, often, “rigorous” method uses iteration, so they cannot avoid failure to converge in some case. For slip surface, many papers listed single slip surface of their search. For the same slope, different researchers can search different slip surface, different method can have different factor of safety for the same slip surface (Chen, 2006; Cheng, 2007; Li, 2006; Que, 2008; Wan, 2006; Zhao, 2006; Zolfaghari, 2005). Different methods are based on different assumptions. The assumptions can lead error for factor of safety. It is not important that different slip surface with different similar factors of safety. We can think these slip surfaces are acceptable. In this paper, we proposed slip band (many slip surfaces with similar factors of safety) instead of single slip surface. It is more effective as shown below.

## FELLENIUS AND BISHOP METHODS BASED ON CIRCULAR SLIP SURFACE

The forces applied on the  $i^{\text{th}}$  slice are defined in figure 1.  $E_i$  and  $E_{i+1}$  are horizontal inter-slice force.  $X_i$  and  $X_{i+1}$  are vertical inter-slice force.  $W_i$  is the weights of  $i^{\text{th}}$  slice.  $N_i$  and  $T_i$  are the resultant of the normal and tangential forces acting on the  $i^{\text{th}}$  slice base of length  $l_i$  and inclination  $\alpha_i$  with respect to the horizontal.  $\overline{AB}$  is the circular slip surface with radius  $R$ .

Fellenius method assumed composition of all inter-slice forces is equilibrium. The factor of safety for the method can be defined for the moment:

$$F_s = \frac{\sum [c'_i l_i + (W_i \cos \alpha_i - u_i l_i) \tan \phi'_i]}{\sum W_i \sin \alpha_i} \quad (1)$$

where  $c'_i$  is the effective cohesion,  $\phi'_i$  is the effective friction angle,  $u_i$  is the average pore water pressure acting on the  $i^{\text{th}}$  slice.

Bishop method assumed  $X_i$  and  $X_{i+1}$  are zero. The factor of safety for the method can be defined for the moment:

$$F_s = \frac{\sum \frac{1}{m_{\alpha i}} [c'_i b_i + (W_i - u_i b_i) \tan \phi'_i]}{\sum W_i \sin \alpha_i} \quad (2)$$

$$m_{\alpha i} = \cos \alpha_i + \frac{\tan \phi'_i \sin \alpha_i}{F_s} \quad (3)$$

According equations (2) and (3), the factor of safety for Bishop method was calculated with iteration. “Rigorous” methods, such as “rigorous” Janbu method and Morgenstern-Price method, also used iteration based on random slip surface. Although these methods were thought as “rigorous”, but they computer slow than Fellenius method without an iteration, and failure to convergence was found in some case.

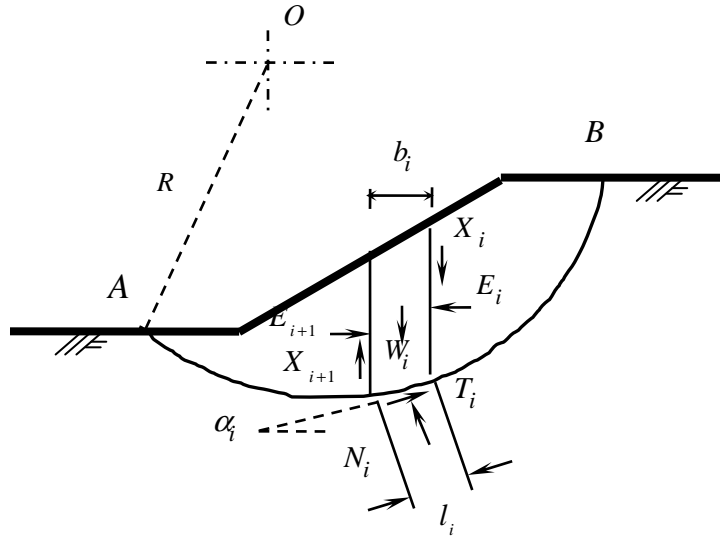


Figure 1: Forces acting on an individual slice

## CONTROL VARIABLES SELECTION OF CIRCULAR SLIP SURFACE

Many researchers proposed selected  $(x_o, y_o, R)$  as control variables (see Figure 2), where  $(x_o, y_o)$  is coordinates for center  $O$  of circular slip surface,  $R$  is radius of circular slip surface. Search ranges of  $(x_o, y_o, R)$  are too large. It is difficult to search minimum of factor of safety.

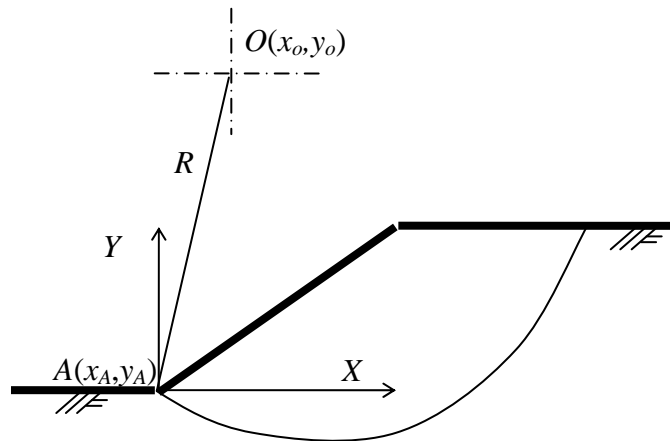


Figure 2: Control variables  $(x_o, y_o, R)$

New control variables  $(x_A, x_B, H_r)$  was proposed (see Figure 3). Points  $A$  and  $B$  are endpoints of slip surface.  $(x_A, y_A)$  and  $(x_B, y_B)$  are coordinates of points  $A$  and  $B$ . If profile of slope is known,  $y_A$  and  $y_B$  can be derived by  $x_A$  and  $x_B$ .  $H_r$  is arc height for slip surface.  $L$  is length of segment  $AB$ . Then it is obvious that maximum of  $H_r$  is  $L/2$ . If  $A$  is at right side of toe of slope, minimum of  $H_r$  is 0. If  $A$  is at left side of toe of slope, it is obvious that toe of slope is over circular slip surface.  $(x_C, y_C)$  are coordinates of toe of slope. Then minimum  $H_{r\min}$  of  $H_r$  can be written as follow:

$$a_1 = 2(x_C - x_B) \quad (4)$$

$$b_1 = 2(y_C - y_B) \quad (5)$$

$$a_2 = 2(x_C - x_A) \quad (6)$$

$$b_2 = 2(y_C - y_A) \quad (7)$$

$$c_1 = x_B^2 + y_B^2 - x_C^2 - y_C^2 \quad (8)$$

$$c_2 = x_A^2 + y_A^2 - x_C^2 - y_C^2 \quad (9)$$

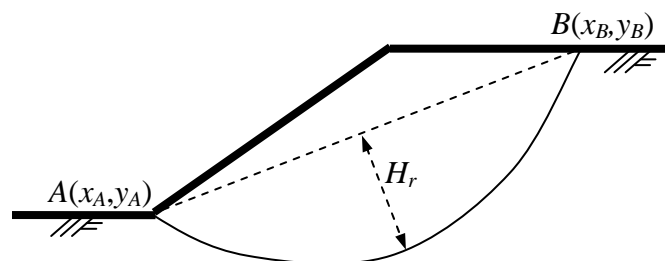
$$y_o = \frac{a_1 c_2 - a_2 c_1}{a_2 b_1 - a_1 b_2} \quad (10)$$

$$x_o = -\frac{c_1 + b_1 y_o}{a_1} \quad (11)$$

$$r = \sqrt{(x_C - x_o)^2 + (y_C - y_o)^2} \quad (12)$$

$$H_{r\min} = r - \sqrt{r^2 - \frac{L^2}{4}} \quad (13)$$

New control variables  $(x_A, x_B, H_r)$  are more effective than control variables  $(x_o, y_o, R)$ . Search ranges of new control variables are smaller. So it is easy to search the minimum factor of safety.



**Figure 3:** Control variables ( $x_A$ ,  $x_B$ ,  $H_r$ )

## SLIP BAND NUMERICAL COMPUTATION FOR CIRCULAR SLIP SURFACE

A homogeneous soil slope with a slope height equal to 6 m and slope angle equal to  $45^\circ$  (Cheng, 2007) is considered. The slip surface is evaluated by the modified genetic algorithm techniques as proposed by Zhou (Zhou, 2005), and total number of slices is 50. For the case 1 in which the friction angle is 5 degree, the cohesive strength of the soil is 2 kPa, and the unit weight of the soil is  $20 \text{ kN/m}^3$ . We found 10 slip surface with factor of safety between 0.2440 and 0.2457 based on Fellenius method, and found 10 slip surfaces with factor of safety between 0.2554 and 0.2572 based on Bishop method. 10 slip surfaces with similar factor of safety formed slip band see Figure 4(a), (b). In Figure 4 and 5, the grid element size is  $1\text{m} \times 1\text{m}$ . For the case 2 in which the friction angle is 35 degree, the cohesive strength of the soil is 2 kPa, and the unit weight of the soil is  $20 \text{ kN/m}^3$ . We found 10 slip surfaces with factor of safety between 1.0407 and 1.0499 based on Fellenius method, and found 10 slip surfaces with factor of safety between 1.1708 and 1.1792 based on Bishop method. 10 slip surfaces with similar factor of safety formed slip band see Figure 4(c), (d). For the case 3 in which the friction angle is 35 degree, the cohesive strength of the soil is 20 kPa, and the unit weight of the soil is  $20 \text{ kN/m}^3$ . We found 10 slip surfaces with factor of safety between 2.2958 and 2.2814 based on Fellenius method, and found 10 slip surfaces with factor of safety between 2.2257 and 2.2689 based on Bishop method. 10 slip surfaces with similar factor of safety formed slip band see Figure 4(e), (f).

Another homogeneous soil slope with a slope height equal to 8.5 m and slope horizontal length equal to 17 m [5] is considered, where the friction angle is 20 degree, the cohesive strength of the soil is 14.7 kPa, and the unit weight of the soil is  $18.62 \text{ kN/m}^3$ . We found 10 slip surfaces with factor of safety between 1.6418 and 1.6425 based on Fellenius method, and found 10 slip surfaces with factor of safety between 1.7394 and 1.7401 based on Bishop method. 10 slip surfaces with similar factor of safety formed slip band see Figure 5(a), (b).



**Figure 4:** Slip band using Fellenius method (FM) and Bishop method (BM) in three cases for a homogeneous soil slope

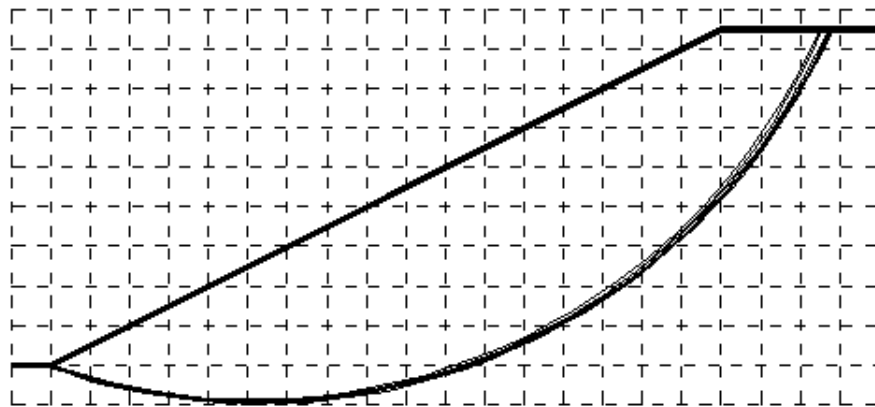
From Figure 4 and 5, width of slip band usually is less than 1 m. Because of uncertain factor, it seems that slip band is more reasonable and effective than slip surface. Fellenius method and

Bishop method can obtain almost same slip band for homogeneous soil slope. From Figure 4, the right end of the failure surface moves closer to the crest of the slope as the friction angle of the soil is increased (which is a well know result). Also, the right end of the failure surface moves closer to the crest of the slope as the cohesion of the soil is decreased.

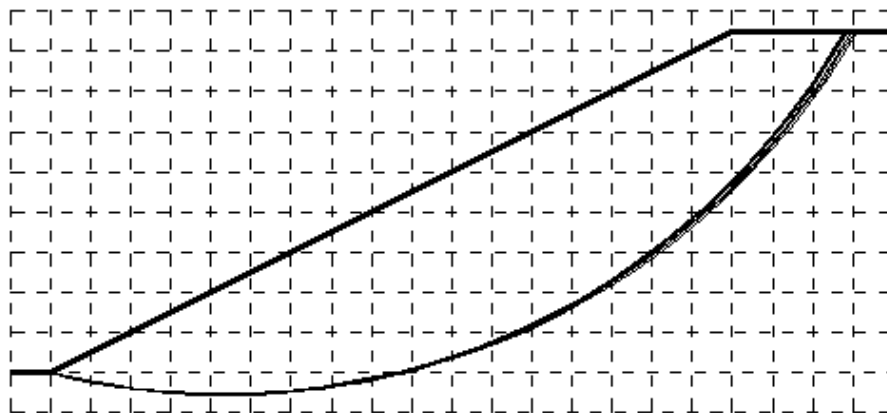
## DISCUSSION AND CONCLUSION

For selection of control variables for circular slip surface, the present study shows that control variables ( $x_o$ ,  $y_o$ ,  $R$ ) as general selection, search ranges are too large, so it is not easy to find minimum of factor of safety. But control variables ( $x_A$ ,  $x_B$ ,  $H_r$ ) as proposed selection, search ranges are small, so it can rapidly find minimum of factor of safety.

Many researchers focus on find slip surface with the smallest factor of safety. Different methods can derive different slip surfaces, which usually they almost are the same. Single slip surface is not very important. Slip band composed of many slip surfaces is more effective and reasonable. Numerical computation shows that, for homogeneous soil slope, slip bands are almost the same based on Fellenius and Bishop methods, and width of slip band usually is less than 1 m.



(a) Slip band using FM



(b) Slip band using BM

**Figure 5:** Slip band using Fellenius method (FM) and Bishop method (BM) for another homogeneous soil slope

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## REFERENCES

1. Bardet, J. P. and M. M. Kapuskar (1989) "A simplex analysis of slope stability" *Computers and Geotechnics*, Vol. 8, pp.329-348.
2. Chen Y. M., X. J. Wei and Y. C. Li (2006) "Locating non-circular critical slip surfaces by particle swarm optimization algorithm" *Chinese Journal of Rock Mechanics and Engineering*, Vol. 25, No. 7, pp. 1443-1449.
3. Cheng, Y. M., T. Lansivaara and W. B. Wei (2007) "Two-dimensional slope stability analysis by limit equilibrium and strength reduction methods" *Computers and Geotechnics*, Vol. 34 ,pp.137-150.
4. Cheng, Y. M., T. Lansivaara and J. Siu (2008) "Impact of convergence on slope stability analysis and design" *Computers and Geotechnics*, Vol. 35, No. 1, pp. 105-113.
5. Li L., S. C. Chi and G. Lin (2006) "Chaos harmony search method and its application to local factor of safety method for soil slopes" *Chinese Journal of Rock Mechanics and Engineering*, Vol. 25, No. S1, pp. 2763-2769.
6. Ning, S. J., L. H. Yao and Y. N. Zhao (2008) "Application of improved decimal strings genetic algorithm to searching for the most critical slip surface of soil slope" *Journal of Engineering Geology*, Vol. 16, No.01, pp.109-115.
7. Que J. S., J. P. Chen, Q. Wang, et al.(2008) "Genetic algorithm applied to entire stability analysis of soil slope" *Chinese Journal of Rock and Soil Mechanics*, Vol. 29, No.2, pp. 415-419.
8. Wan W., P. Cao, T. Feng (2006) "Application of accelerating hybrid genetic algorithm to searching for the most dangerous slip surface of slope" *Chinese Journal of Rock Mechanics and Engineering*, Vol. 25, No. S1, pp. 2270-2276.
9. Zhao H. B.(2006) "A new global optimization method of non-circular slip surface for slope stability analysis" *Chinese Journal of Rock Mechanics and Engineering*, Vol. 25, No. S2, pp. 3725-3729.
10. Zhou Y. P., S. J. Li, Y. X. Liu (2005) "Searching critical failure surface in slope stability analysis with genetic algorithm" *Chinese Journal of Rock Mechanics and Engineering*, Vol. 24, No. S1, pp. 5226-5320.
11. Zolfaghari, Ali R., Andrew C. Heath and Paul F. McCombie (2005) "Simple genetic algorithm search for critical non-circular failure surface in slope stability analysis" *Computers and Geotechnics*, Vol. 32, No. 3, pp. 139-152.

