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Extension of Mononobe-Okabe Theory to Evaluate Seismic Active Earth Pressure Supporting c-ϕ Backfill

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ABSTRACT

An effort is made to evaluate the formulation of seismic active earth pressure behind a non-vertical retaining wall supporting c-Φ backfill. The formulation is prepared to get a single critical wedge surface for simultaneous action of weight, surcharge, cohesion and adhesion. The success of any retaining structure involves the awareness of seismic active earth pressure. The effect of various parameters viz. internal friction (ϕ), angle of wall friction (δ), wall inclination angle(α),backfill inclination angle (i), cohesion (c), adhesion (c_a), seismic accelerations (k_h, k_v), surcharge loading (q), unit weight (γ), height (H) are also taken into account to provide the variation of seismic active earth pressure coefficient.

KEYWORDS: seismic active earth pressure, pseudo-static, retaining wall, *c-Φ* backfill, single wedge.

INTRODUCTION

Many researchers have investigated the mechanism of earth pressure acting on the back of retaining wall. The pioneering work in determining the static earth pressure was done by Coulomb (1773). Mononobe-Okabe extended the theory including earthquake loads introducing horizontal and vertical inertia forces for retaining wall having cohesionless backfill. Next onward significant development has been made in this field. The Coulomb's theory is further extended to evaluate seismic active earth pressure considering c-ϕ backfill by Prakash and Saran (1966), Saran and Prakash (1968), Saran and Gupta (2003), Ghosh and Saran (2007), Ghosh et al. (2008). In all these methods three separate wedge surfaces generated for the individual action of unit weight, surcharge and cohesion considering unit adhesion is equal to unit cohesion. This assumption is not at all true. Das and Puri (1996) improved the analysis by considering different value of cohesion and adhesion. Thereafter, Shukla et al. (2009) was the first person who give the idea to extend this Mononobe-Okabe concept for c-Φ backfill in such a way to get single critical wedge surface. Ghosh and Saran (2010) and Sharma and Ghosh (2010) have given a solution for seismic active earth pressure in such a way that they are getting single critical wedge angle for the simultaneous action of weight, surcharge and cohesion. Therefore this paper aimed to give a satisfactory formulation to evaluate seismic active earth pressure including the influence of both

adhesion and cohesion for a non-vertical retaining wall. The influence of cohesion, adhesion, wall inclination and backfill inclination angle is given graphically with detailed parametric study.

METHOD OF ANALYSIS

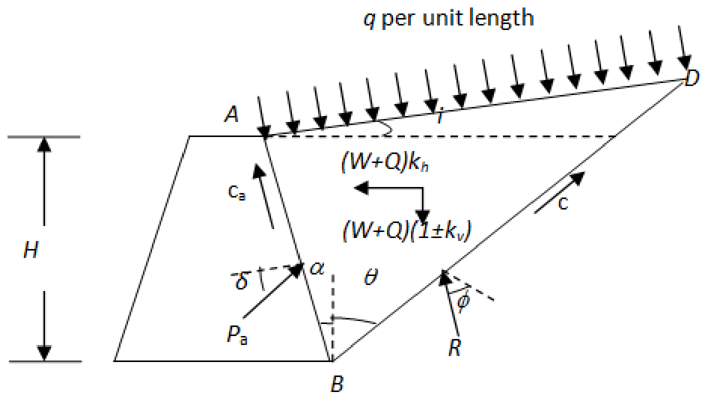

Figure 1: Forces acting on retaining wall – soil wedge system during active state of equilibrium

A schematic diagram of seismic active earth pressure is shown in Fig.1. Here a rigid retaining wall of height H supporting c-ϕ backfill of unit weight γ, unit cohesion c, unit adhesion c_a, angle of wall friction δ, angle of soil friction ϕ, retaining wall inclination angle α, backfill inclination angle *i* is shown. On the top of the backfill a surcharge load of intensity q per unit length is acting. At any stage of earthquake (having seismic acceleration coefficients k_h and k_v) during active state of equilibrium, if the planer wedge surface BD generates an angle θ with the vertical, then the forces acting on the wedge system as shown in Fig.1, P_a and R being the force on the retaining wall and reaction offered by the retained earth on the sliding wedge ABD at the face BD respectively.

The other forces are total cohesion C = (cHSecα sina)/sind, total adhesion $C_a = c_a H \, Sec\alpha$, weight of wedge, $W = (\gamma H^2 Sec^2\alpha \; Sina \; Sinb)/2Sind$, surcharge load, Q = (qHSecα Sinb)/Sind, horizontal inertia force = (W+Q) K_h and vertical inertia force = (W+Q)K_v.

where,

a = (90-α-i), b=(α+θ), d=(90-i-θ).

Applying the equilibrium conditions, ∑H = 0 and ∑V = 0 we get respectively,

$$P_a \cos(\alpha+\delta) - R\cos(\phi+\theta) + \frac{cHSec\alpha Sina Sin\theta}{Sind} - c_a H \tan\alpha = (W+Q)k_h \quad (1)$$

$$P_a \sin(\alpha+\delta) + R\sin(\phi+\theta) + \frac{cHSec\alpha Sina Cos\theta}{Sind} + c_a H = (W+Q)(1 \pm k_v) \quad (2)$$

On simplification of Eqn 1 and 2 substituting the values of C, Q, W etc. we obtain,

$$P_a \sin(\alpha+\phi+\delta+\theta) = [\gamma + \frac{2q}{H}](1 \pm k_v)\frac{H^2}{2}\frac{SinbSec\alpha Cos(\phi+\theta-\psi)}{SindCos\psi} - \frac{cHSec\alpha Sina Cos\phi}{Sind} - \frac{c_a HCos(\alpha+\phi+\theta)}{Cos\alpha} \quad (3)$$

Replacing [(γ+2q)/H] by γ_e, and substituting

$$\frac{2c}{\gamma_e H(1 \pm k_v)} = n_c \quad , \quad \frac{2c_a}{\gamma_e H(1 \pm k_v)} = m_c$$

Eq. 3 can be written as

$$P_a = \gamma_e \frac{H^2}{2}(1 \pm k_v)\frac{\left[\frac{SinbSec\alpha Cos(\phi+\theta-\psi)}{SindCos\psi} - n_c \frac{Sec\alpha SinaCos\phi}{Sind} - m_c \frac{Cos(\alpha+\phi+\theta)}{Cos\alpha}\right]}{\sin(\alpha+\phi+\delta+\theta)} \tag{4}$$

which can also be written as

$$P_a = \gamma_e \frac{H^2}{2}(1 \pm k_v)k_a \tag{5}$$

where

$$k_a = \frac{[\sin(\alpha+\theta)\cos(\phi+\theta-\psi) - n_c \sin a \cos\phi \cos\psi - m_c \cos(i+\theta)\cos\psi \cos(\alpha+\phi+\theta)]}{\sin(\alpha+\phi+\delta+\theta)\cos(i+\theta)\cos\psi\cos\alpha} \tag{6}$$

In Eqn 7, all the terms are constant except θ. On optimizing this coefficient for seismic active earth pressure we get the value of θ which is represented here as θ_c and given by

$$\theta_c = \cos^{-1}\sqrt{\frac{(r+s)s + t^2 + t\sqrt{s^2+t^2-r^2}}{2(s^2+t^2)}} \tag{7}$$

where

$$r = -\sin(\psi+\delta+i) - m_c \cos\psi \cos\delta \tag{8}$$

$$s = 2n_c \sin a \cos\phi \cos\alpha \cos\psi \cos(\alpha+\phi+\delta+i) + m_c \cos\psi[\cos(\alpha+\phi+\delta-i)\cos(\alpha+\phi+i) + \sin(\alpha+\phi+\delta+i)\sin(\phi+\alpha-i)] + \sin(\phi-\psi-i)\cos(2\alpha+\phi+\delta) + \sin(\phi+\delta)\cos(\phi-\psi+i) \tag{9}$$

$$t = \sin(\phi-\psi-i)\sin(2\alpha+\phi+\delta) + 2n_c \sin a \cos\phi \cos\psi \sin(\alpha+\phi+\delta+i) + m_c \cos\psi[\cos(\alpha+\phi+\delta-i)\sin(\alpha+\phi+i) - \sin(\alpha+\phi-i)\cos(\alpha+\phi+\delta+i)] + \sin(\phi+\delta)\sin(\phi-\psi+i) \tag{10}$$

Putting this value of θ_c in Eqn 6, the coefficient of seismic active earth pressure kae and substituting this value of k_{ae} in Eqn 5 the magnitude of total seismic active force (P_{ae}) on the back of a retaining wall supporting c-Φ backfill is obtained. In case of cohesionless soil to avoid the phenomenon of shear fluidization the value of should satisfy the following Eqn.

$$\phi > \tan^{-1}\frac{K_h}{1 \pm K_v} \tag{11}$$

PARAMETRIC STUDY

Parametric study is done to shed light on the effects of different soil and wall parameters on the variation of seismic active earth pressure coefficient k_{ae}.

Effect of angle of internal friction of soil (ϕ)

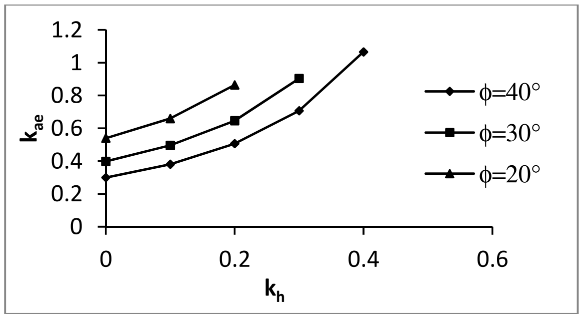

Figure 2: Variation of seismic active earth pressure coefficient (k_a) with k_h for $k_v=k_h/2$, $\delta=\Phi/2$, γ = 18 kN/m^3, c=10 kN/m^2, c_a = 8 kN/m^2, q = 15 kN/m, H=10 m, α=20°, i=10°

Fig. 2 shows the variation of seismic active earth pressure coefficient (k_{ae}) with k_h for $k_v=k_h/2$, $\delta= \Phi/2$, γ =18 kN/m^3, c =10 kN/m^2, c_a = 8 kN/m^2, q = 15 kN/m, H=10 m, α= 20°, i= 10° for different value of Φ. From the plot it is apparent that increase in Φ decreases the magnitude of seismic active earth pressure coefficient k_{ae} but increase in seismic acceleration coefficient k_h increases the magnitude of seismic active earth pressure coefficient k_{ae}. For example at k_h = 0.2, at Φ =20°, 30°, 40° the value of k_{ae} are 0.38, 0.496 and 0.659 respectively.

Effect of angle of wall friction (δ)

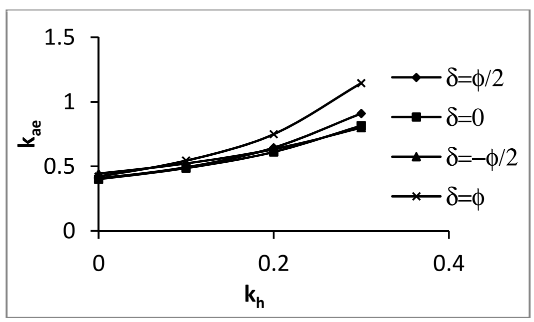

Figure 3: Variation of seismic active earth pressure coefficient (k_{ae}) with k_h for $k_v=k_h/2$, ϕ=30°, γ = 18 kN/m^3, c=10 kN/m^2, c_a = 8 kN/m^2, q = 15 kN/m, H=10 m, α=20°, i= 10°

Fig. 3 shows the variation of seismic active earth pressure coefficient (k_{ae}) with k_h for $k_v=k_h/2$, Φ = 30°, γ = 18 kN/m^3, c=10 kN/m^2, c_a = 8 kN/m^2, q = 15 kN/m, H=10 m, α=20°, *i*= 10° for different value of δ. From the plot, it is clear that at k_h=0.1, the value of k_{ae} for δ=0 and ϕ/2 is more or less same and for δ= -ϕ/2 and ϕ the values are 0.523 and 0.544 respectively.

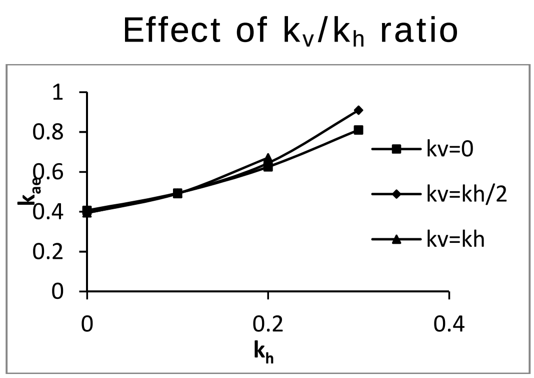

Figure 4: Variation of seismic active earth pressure coefficient (k_{ae}) with k_h for ϕ=30°, δ=Φ/2, γ = 18 kN/m^3, c=10 kN/m^2, c_a = 8 kN/m^2, q = 15 kN/m, H=10 m, α=20°, i=10°

Fig.4 shows the variation of seismic active earth pressure coefficient (k_{ae}) with kh for δ=Φ/2, Φ = 30°, γ = 18 kN/m^3, c=10 kN/m^2, c_a = 8 kN/m^2, q = 15 kN/m, H=10 m, α=20°, i= 10° for different ratio of k_v/k_h. From the plot, it is seen that after k_h=0.1, increase in $_{kv}/k_h$ ratio increases the magnitude of seismic active earth pressure coefficient k_{ae} and this increase becomes more for higher value of k_h. At k_h=0.1, the value of k_{ae} is approximately equal for k_v/k_h= 0, ½, 1 and the value is 0.49.

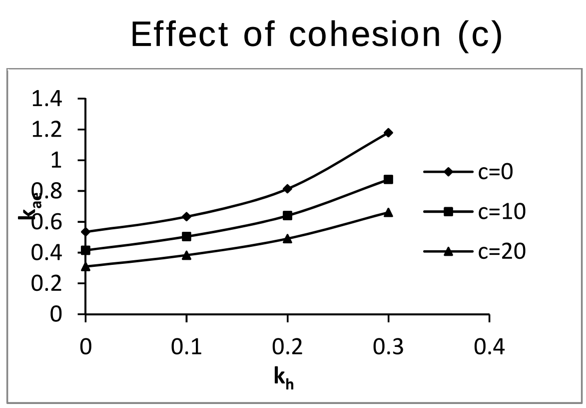

Figure 5: Variation of seismic active earth pressure coefficient (k_{ae}) with k_h for kv=k_h/2, ϕ=30°, δ=Φ/2, γ = 18 kN/m^3, c_a = 0 kN/m^2, q = 15 kN/m, H=10 m, α=20°, i= 10°

Fig. 5 shows the variation of seismic active earth pressure coefficient (k_{ae}) with k_h for Φ =30°, k_v=k_h/2, δ=Φ/2, γ = 18 kN/m^3, c_a = 0, q =15 kN/m, H=10 m, α=20°, *i*=10° for different values of unit cohesion. From the plot, it is seen that cohesion decreases the magnitude of seismic active earth pressure coefficient k_{ae}. For example, at k_h = 0.2, for c= 0, 10 kN/m^2 and 20 kN/m^2, the value of seismic active earth pressure coefficient k_{ae} are 0.814, 0.640 and 0.491. But previously Saran and Gupta (2003), Ghosh and Saran (2008) demonstrate that cohesion c affects the magnitude of total active force P_{ae} but it does not influence seismic active earth pressure coefficient k_{ae}. In contrast, Ghosh and Sharma (2010) presented the effect of c on k_{ae} but they have considered both the magnitude of unit cohesion and unit adhesion are identical.

Effect of adhesion (c_a)

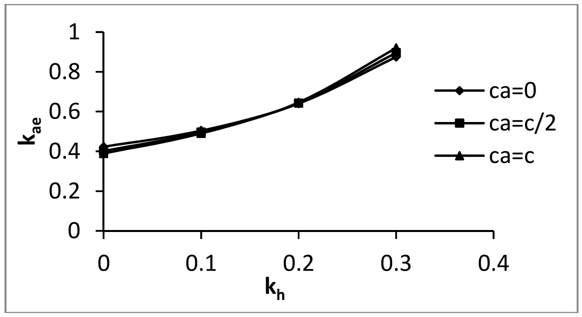

Figure 6: Variation of seismic active earth pressure coefficient (k_{ae}) with k_h for $k_v=k_h/2$, $\phi= 30°$, $\delta=\Phi/2$, $\gamma = 18$ kN/m^3, c=10 kN/m^2, q = 15 kN/m, H=10 m, $\alpha=20°$, i= 10°

Fig. 6 shows the variation of seismic active earth pressure coefficient (k_{ae}) with kh for Φ =30°, $k_v=k_h/2$, $\delta=\Phi/2$, γ = 18 kN/m^3, c = 10 kN/m^2, q = 15 k/m, H=10 m, α=20°, i= 10° for different ratio of c_a/c. There is very nominal decrease in seismic active earth pressure coefficient k_{ae} due to increase c_a/c ratio up to k_h=0.2. But after that, it shows totally opposite character where k_{ae} increases with the increases in the value of ca/c ratio.

Effect of surcharge (q)

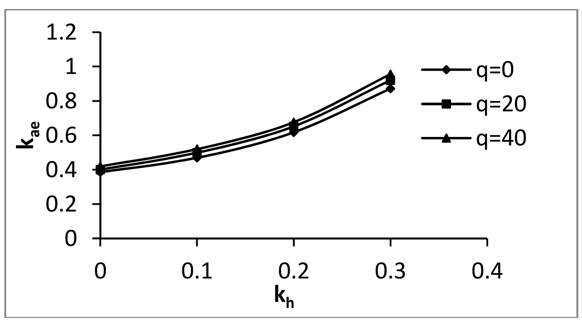

Figure 7: Variation of seismic active earth pressure coefficient (k_{ae}) with k_h for $k_v=k_h/2$, $\phi=30°$, $\delta=\Phi/2$, $\gamma = 18$ kN/m^3, c=10kN/m^2, c_a = 8 kN/m^2, H=10 m, $\alpha=20°$, i= 10°

Fig. 7 shows the variation of seismic active earth pressure coefficient (kae) with kh for Φ =30°, $k_v=k_h/2$, $\delta=\Phi/2$, γ = 18 kN/m^3, c = 10 kN/m^2, c_a = 8 kN/m^2, H=10 m, α=20°, i= 10° for different value of surcharge loading. From the plot, it is seen that surcharge loading q affects significantly the magnitude of k_{ae}. This fact is also established in Ghosh and Saran (2010) and Ghosh and Sharma (2010). k_{ae} increases with the increase in the value of surcharge loading.

Effect of unit weight of backfill material (γ)

Fig.8 shows the variation of seismic active earth pressure coefficient (k_{ae}) with kh for Φ =30°,

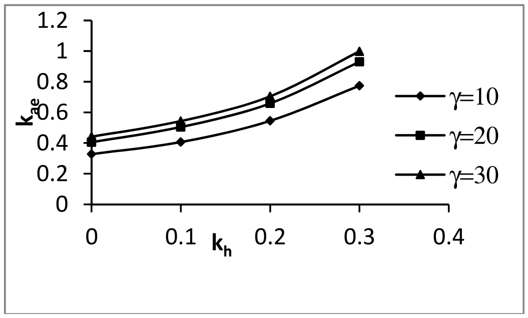

Figure 8: Variation of seismic active earth pressure coefficient (k_{ae}) with k_h for k_v=k_h/2, ϕ=30°, δ=Φ/2, c=10 kN/m^2, c_a = 8 kN/m^2, q=15 kN/m, H=10 m, α=20°, i= 10°

Effect of height of retaining wall (H)

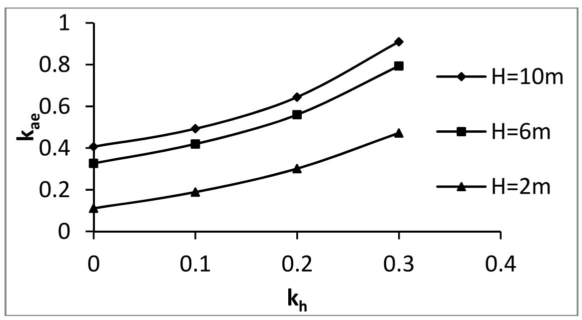

Figure 9: Variation of seismic active earth pressure coefficient (k_{ae}) with k_h for k_v=k_h/2, ϕ=30°, δ=Φ/2, c=10 kN/m^2, c_a = 8 kN/m^2, q=15 kN/m, H=10 m, α=20°, i= 10°

Fig.9 shows the variation of seismic active earth pressure coefficient (kae) with kh for Φ =30°, k_v = k_h/2, δ = Φ/2, q = 15 kN/m, c = 10 kN/m^2, c_a = 8 kN/m^2, γ=18 kN/m^3, α=20°, i= 10° for different height of retaining wall. From the plot, it is seen that height of the retaining wall affects significantly the magnitude of k_{ae}. For smaller height of retaining wall effect of kh is small. At kh= 0.2, for H= 2 m, 6 m 1nd 10 m the value of k_{ae} are 0.301, 0.559 and 0.644 respectively.

Effect of wall inclination angle (α)

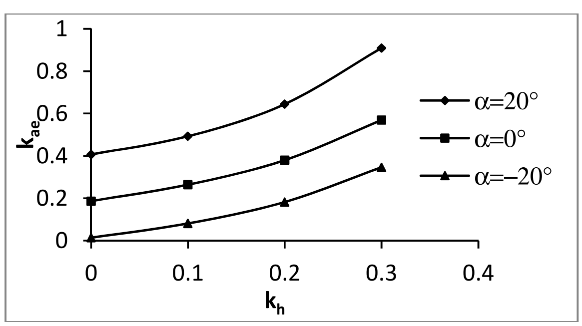

Figure 10: Variation of seismic active earth pressure coefficient (k_{ae}) with k_h for kv=k_h/2, ϕ=30°, δ=Φ/2, γ = 18 kN/m^3, c=10 kN/m^2, c_a = 8 kN/m^2, q=15 kN/m, H=10 m, i= 10°

Fig.10 shows the variation of seismic active earth pressure coefficient (k_{ae}) with k_h for for Φ =30°, k_v=k_h/2, δ=Φ/2, q= 15 kN/m, c = 10 kN/m^2, c_a = 8 kN/m^2, γ=18 kN/m^3, H=10 m, i= 10° for different wall inclination angle (α). From the plot, it is seen that the effect of wall inclination angle is very a major factor for the determination of seismic active earth pressure coefficient (Kae). For example at K_h=0.3, for α = -20°, 0° and 20° the values of k_{ae} are 0.346, 0.569 and 0.909 respectively.

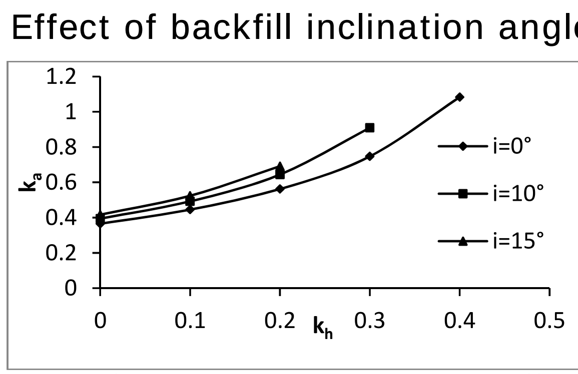

Figure 11: Variation of seismic active earth pressure coefficient (k_{ae}) with k_h for k_v=k_h/2, f=30°, δ=Φ/2, γ = 18 kN/m^3, c=10 kN/m^2, c_a = 8 kN/m^2, q=15 kN/m, α=20°, H=10 m

Fig.11 shows the variation of seismic active earth pressure coefficient (k_{ae}) with k_h for for Φ =30°, k_v=k_h/2, δ=Φ/2, q= 15 kN/m, c = 10 kN/m^2, c_a = 8 kN/m^2, γ=18 kN/m^3, H=10 m, α= 20° for different wall inclination angle (α). At k_h= 0.2, for i= 0°, 10°, 15° the values of k_{ae} are 0.563, 0.644 and 0.692 respectively. k_{ae} increases with the increase in the value of backfill inclination angle.

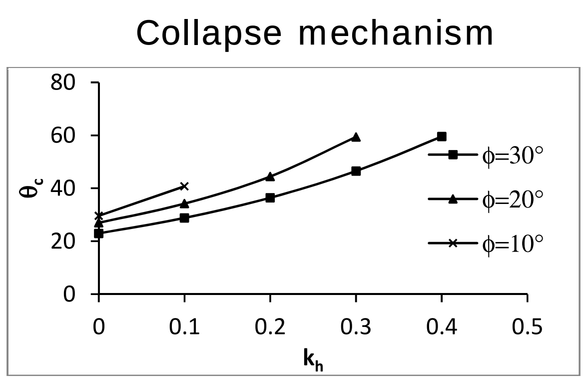

Figure 12: Variation of seismic critical wedge angle (θ_c) with k_h for k_v=k_h/2, δ=Φ/2, γ = 18 kN/m^3, c=10 kN/m^2, c_a = 8 kN/m^2, q = 15 kN/m, H=10 m, α=20°, i= 10°

In the case of evaluating seismic active earth pressure the critical wedge angle is the maximum angle made by the failure surface. The critical wedge angle θc with vertical is given by the Eqn.7. The wedge angle is presented graphically in Fig.12 for different value of ϕ viz 10°, 20°, 30°. θc decreases with the increases in the value of ϕ which means lesser participation of soil mass in vibration due to increase in ϕ.

COMPARISON OF RESULTS

Table 1 shows the comparison of results as obtained from present study with Sharma and Ghosh'2010.

Table 1: Comparison of the results obtained from present study with Sharma and Ghosh'2010 [Φ = 30°, δ = 2Φ/3, γ = 18kN/m^3, q = 15 kN/m, H = 10 m, i= 10, α=30°]

Value of cohesion and adhesion	K_h=0, K_v=0		K_h=0.1, K_v=0.05	
	Sharma and Ghosh(2010)	**Present study**	**Sharma and Ghosh(2010)**	**Present study**
$c = c_a$ =10 kN/m^2	0.623	0.570	0.741	0.715
$c = c_a$ =20 kN/m^2	0.493	0.446	0.602	0.575

CONCLUSION

The present study illustrates an analytical formulation for the coefficient of total active force on the back of a retaining wall supporting a c-Φ backfill considering simultaneous action weight, surcharge, adhesion and cohesion to get a single critical wedge angle. The significance of cohesion, adhesion, wall inclination, backfill inclination over the seismic active earth pressure coefficient is obtainable in this report. Depending on the developed formulation, a detailed parametric study is done for the variation of various soil and wall parameters. From the basis of parametric study, it is seen seismic earth pressure coefficient shows an inverse relation with the angle of soil internal friction, cohesion, ca/c ratio. On the other hand, it increases with the increase in the value of k_v/k_h ratio, surcharge loading, unit weight of backfill material, backfill inclination angle. The effect of K_{ae} is greater for high retaining wall. The value of k_{ae} increases due to change of wall inclination from negative to positive.

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