Grey Prediction Research of Slope Deformation

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ABSTRACT

After introducing the basic theories related to grey system such as grey information, gray generation series, grey correlation analysis, the author unfolds a discussion on data transform method of gray mathematics sequence and general grey prediction mode in deformation forecast. Thus, Metabolism forecast model is suggested to be adopted in deformation observation of the forecast analysis considering the influence of general grey prediction mode on the process as well as the result when operating on the relationship between the new and the original information. Taking one of the slope deformation observation data as an example, the author carries some exclusive studies on general grey forecasting, new information grey forecasting and metabolism grey forecasting. Compared with the results predicted by general grey forecasting and new information grey forecasting, metabolism grey forecasting enjoys the minimum residual mistakes of slope deformation, being the top conformable to the actual measured data and the weakest relative errors-maker. It is thus clear that metabolism grey forecasting is the best choice for slope deformation forecast. Moreover, problems like heavy data, high computing cost and low efficiency led by prolonged periodic time in observing can also be avoided with metabolism grey forecasting.

KEYWORDS: Grey system, grey model, data transform, deformation forecast, grey forecasting model

INTRODUCTION

The stability of a slope is subject to earthquakes, geotechnical characteristics, topography, human activities, atmosphere, rainfall, and so on. And these influencing factors do not act alone and abide by no rules in different period [1]. As for the influencing factors of slope deformation, some of them are hidden in the veil, yet there are still some being already known. Say, all these factors share the obvious gray characteristics. Therefore, grey theory is putted forward for analysis and research in slope deformation forecast.
GREY SYSTEM THEORY AND METHOD

Grey information

Grey system theory has been put forward and developed by Chinese professor Deng Julong. It mainly concerns with the “few samples and inadequate information” unstable system, half of which are unknown and half known. After forming and exploiting “those known” information, people can extract the valuable ones from it and thus realize the correct description as well as effective control of the system’s operation on the rule. The clarity of information is marked by lightness of color: black represents the information unknown, white information completely clear, and grey information partial clear. In this way, “the information is not adequate” is the basic implication of grey. The nature of grey theory is being gray, with inadequate information which is half known. Gray represents the information lying between black and white, which denotes the grey system made up by some uncertain information, some clear information, part being not completely clear and part being completely clear [2].

Grey generation sequence

(1) Smooth ratio and level ratio

Definition 1:

\[
\rho(k) = \frac{x(k)}{\sum_{i=1}^{k-1} x(i)}, \quad k = 2, 3, \ldots, n
\]  

This is regarded as smooth ratio of sequence \(X\).

Smooth ratio reflects the smoothness of data; ratio \(\rho(k)\) is used to see whether data changes are stable. It is clear that the more stable data are, the smaller smooth ratio is.

Definition 2: if sequence \(X\) meets the following requirements; \(X\) is a quasi smooth sequence

\[
\frac{\rho(k + 1)}{\rho(k)} < 1, k = 2, 3, \ldots, n - 1
\]  

\[
\rho(k) \in [0, 0.5], k = 3, 4, \ldots, n
\]  

Definition 3: sequence \(X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))\),

\[
\sigma(k) = \frac{x(k)}{x(k-1)}, \quad k = 2, 3, \ldots, n
\]  

This is the level ratio of sequence \(X\).
(2) Accumulation generation operator

The process from gray to white is called accumulation generation of grey information, which reveals some characteristics of the discrete data.

Definition 4: The original data sequence is set to be \( X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \), \( D \) is sequence operator, \( X^{(0)}D = (x^{(0)}(1)d, x^{(0)}(2)d, \ldots, x^{(0)}(n)d) \), in which

\[
x^{(0)}(k)d = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \ldots, n
\]  

(5)

\( D \) is accumulate operator of \( X^{(0)} \), noted as \( 1 \rightarrow AGO \).

(3) Cumulative reduce generation operator

Definition 5: The original data sequence is \( X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \), \( D \) is sequence operator \( X^{(0)}D = (x^{(0)}(1)d, x^{(0)}(2)d, \ldots, x^{(0)}(n)d) \), in which

\[
x^{(0)}(k)d = x^{(0)}(k) - x^{(0)}(k-1)
\]  

(6)

\( D \) is \( X^{(0)} \) A Cumulative reduce operator, noted as \( IAGO \).

(4) Grey index law of the accumulation generation

Generally speaking, new data sequences generated by accumulation of nonnegative smooth sequence tend to be increasing, and show the characteristics of index increasing. For example, monitoring data sequence

\[ X^{(0)} = (0.02, 0.03, 0.06, 0.04, 0.05) \]

After accumulation, it generates sequence

\[ X^{(1)} = (0.02, 0.05, 0.11, 0.15, 0.20) \]

Its original data and accumulation generation date are shown here in figure 1 and 2.
Definition 6: Sequence $X = (x(1), x(2), \ldots, x(n))$, if

1. $\forall k, \sigma(k) \in (0, 1]$, then, sequence $X$ has negative grey index rule.
2. $\forall k, \sigma(k) \in (1, b]$, then, sequence $X$ has Positive grey index rule.
3. $\forall k, \sigma(k) \in [a, b], b - a = \delta$, then, sequence $X$ has grey index rule whose absolute gray is $\delta$.
4. When $\delta < 0.5$, say, it shows a quasi index law.

Theorem 1: if $x^{(0)}$ is a positive quasi smooth sequence, then, accumulating $x^{(0)}$ once will generate sequence $x^{(1)}$ which shows a quasi index law.

Since $\sigma^{(1)}(k) = \frac{x^{(1)}(k)}{x^{(1)}(k-1)} = \frac{x^{(0)}(k) + x^{(1)}(k-1)}{x^{(1)}(k-1)} = 1 + \rho(k)$, according to the definition of quasi smooth sequence, there exists $\rho(k) < 0.5$, then

$$\sigma^{(1)}(k) \in (1, 1.5), \quad \delta < 0.5$$

So $x^{(1)}$ possesses the quasi index law [3-5].

**Grey correlation degree**

In many grey relation analyses, Deng correlation is the most widely applied.

Definition 7 $X_0 = (x_0(1), x_0(2), \ldots, x_0(n))$ is System feature sequence, and

$X_1 = (x_1(1), x_1(2), \ldots, x_1(n))$

...
\[ X_i = (x_i(1), x_i(2), \ldots, x_i(n)) \]

\[ \ldots \]

\[ X_m = (x_m(1), x_m(2), \ldots, x_m(n)) \]

As for \( \xi \in (0,1) \),

\[
 r(x_0(k), x_i(k)) = \frac{\min \min_{k} |x_0(k) - x_i(k)| + \xi \max_{i} \max_{k} |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \xi \max_{i} \max_{k} |x_0(k) - x_i(k)|} 
\]

\[
 r(X_0, X_i) = \frac{1}{n} \sum_{k=1}^{n} r(x_0(k), x_i(k)) 
\]

Then \( r(X_0, X_i) \) suffice the four axioms of grey correlation, namely, standardization, even symmetry, integrity and approximation. In this formula, \( \xi \) is a distinguishing coefficient, \( r(X_0, X_i) \) shows the degree of grey correlation between \( X_0 \) and \( X_i \).

**GREY MODEL**

The basic ideas of grey prediction model goes as follows: first, carrying on the accumulation generation of the original data sequence and making the implied law shown in data sequence; then, evenly dispersing these data through homogeneous differential equation and establishing discrete grey model with parameters; after that, estimating the parameters value by Least square method; and decreasing gradually the newly-gotten forecasting series, thus getting the original approximate sequence; lastly, accurately testing the results [6-8].

**Data transform**

Before data analysis, data should be checked to see the feasibility of setting up the grey model. If the data meet quasi smooth requirement (\( \rho(t) < 0.5 \)) and index law (\( \delta < 0.5 \)), then it meets the basic conditions of grey modeling. Otherwise, data should be processed by data transform, and the new transformed data meet modeling conditions.

Original data is \( X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \), transform as follow:

**Exponential conversion:**

\[
 y = m^x, (0 < m < 1) \quad (7)
\]

**Log conversion:**

\[
 y = \ln x \quad (8)
\]
Root conversion:

\[ y = \sqrt[q]{x}, \quad (q = 2 \text{ or } 3) \quad (9) \]

Translation conversion:

\[ y = x + a \quad (a \text{ is translation constant}) \quad (10) \]

The sequence after conversion is \( Y^{(0)} = (y^{(0)}(1), y^{(0)}(2), \ldots, y^{(0)}(n)) \). The transformed data, on the one hand, can improve the smoothness of original data; on the other hand, can deduce the error between predicted data with the measured data, and thus the accuracy of prediction can be improved \(^9\).

**General grey prediction model**

Grey derivative of data sequence is defined on the basis of the general differential equation idea. Thus, it’s convenience to establish the approximate differential equation model with discrete data sequence \([6-10]\).

Differential equation:

\[
\frac{dX}{dt} + aX = b
\]

Here, \( \frac{dX}{dt} \) is the first derivative of \( X \), \( X \) is the background value of \( \frac{dX}{dt} \).

Discretization of this equation makes differential turn into difference:

\[
z^{(i)}(k) = 0.5x^{(i)}(k-1) + 0.5x^{(i)}(k) \quad (12)
\]

The gray differential equation:

\[
X^{(0)}(K) + aZ^{(i)}(K) = b \quad (13)
\]

When \( X^{(0)} \) is a non-negative sequence,

\[
X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \quad (14)
\]

In which, \( x^{(0)}(k) \geq 0, k = 1, 2, \ldots, n \), \( X^{(1)} \) is a first order accumulating sequence of \( X^{(0)} \),

\[
X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)) \quad (15)
\]

\( Z^{(i)} \) is the sequence generated by the mean values around \( X^{(i)} \),

\[
Z^{(i)} = (z^{(i)}(1), z^{(i)}(2), \ldots, z^{(i)}(n)) \quad (16)
\]

If \( \hat{a} = [a, b]^{T} \) is a parameter sequence, besides
So the smallest squares estimate parameters sequence of model \( x^{(0)}(k) + a z^{(1)}(k) = b \) meets the requirement:

\[
\hat{\alpha} = (B^T B)^{-1} B^T Y
\]  

(18)

\( a, b \) can be solved by this formula, then, they are put into \( GM(1,1) \) of model

\[
x^{(0)}(k) + a z^{(1)}(k) = b
\]  

(19)

Time response sequence of model \( x^{(0)}(k) + a z^{(1)}(k) = b \) :

\[
x^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a} , \quad k = 1, 2, \ldots, n
\]  

(20)

Original value

\[
x^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = (1 - e^a)(x^{(0)}(1) - \frac{b}{a})e^{-ak} , \quad k = 1, 2, \ldots, n
\]  

(21)

When \( t \leq n \), \( \hat{x}^{(0)}(t) \) are Model fitted values;

When \( t > n \), \( \hat{x}^{(0)}(t) \) are Model prediction values;

Here, \( a \) is a developing coefficient, showing the developing situation among sequences; \( b \) is a gray role, showing the relationship of data variation.

**Metabolism forecast model**

Original data: \( X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \)

Based on \( X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \), \( GM(1,1) \) model is All data \( GM(1,1) \).

Based on \( X^{(0)} = (x^{(0)}(k_0), x^{(0)}(k_0+1), \ldots, x^{(0)}(n)) \), \( GM(1,1) \) model is Part data \( GM(1,1) \).
\( x^{(0)}(n+1) \) is new information, \( x^{(0)}(n+1) \) is regarded as Last Data, \( X^{(0)} = (x^{(0)}(k_0), x^{(0)}(k_0+1), \ldots, x^{(0)}(n), x^{(0)}(n+1)) \), this model generated from this is regarded as new information GM(1,1).

As time passes by, power of original data in Signal processing gradually reduces. When the new information is being continuously added, old information should be removed accordingly. In this manner, the data can be updated timely and the characteristic of signal can be clearly shown. Thus, it’s reasonable to erase the old information so that a new model—Metabolism forecast model can be formed.

Original data:

\[
X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))
\]  

(22)

\( x^{(0)}(n+1) \) is new information, \( x^{(0)}(n+1) \) is regarded as last data

\[
X^{(0)} = (x^{(0)}(k_0+1), \ldots, x^{(0)}(n), x^{(0)}(n+1))
\]  

(23)

Compare with general grey model which causes superfluous data, Metabolism keeps date consistent. Moreover, the older the information is, the less its influence imposes on data prediction. In this way, there’s no need to expand the computer memory and the forecasting data gets more accurate owing to the reduced model computation\[8].

**Model test**

Methods to appraise common grey model GM(1,1) are as follows:

(1) Residual inspection

Original error data \( x^{(0)}(k) = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \), \( \hat{x}^{(0)}(k) \) is simulation data got through grey model.

Residual error:

\[
e(k) = x^{(0)}(k) - \hat{x}^{(0)}(k)
\]  

(24)

Relative error:

\[
\varepsilon(k) = \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}
\]  

(25)

(2) Variance inspection

Original error data \( x^{(0)}(k) = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \), \( \hat{x}^{(0)}(k) \) is simulation data, residual error sequence is \( e(k) \), then
\(x^{(0)}(k)\) mean value,

\[
\overline{x}^{(0)}(k) = \frac{1}{n} \sum_{k=1}^{n} x^{(0)}(k)
\]  

(26)

e(k) mean value,

\[
\overline{e}(k) = \frac{1}{n} \sum_{k=1}^{n} e(k)
\]  

(27)

\(x^{(0)}(k)\) variance,

\[
s_1^2 = \frac{1}{n} \sum_{k=1}^{n} [x^{(0)}(k) - \overline{x}^{(0)}(k)]^2
\]  

(28)

e(k) variance,

\[
s_2^2 = \frac{1}{n} \sum_{k=1}^{n} [e(k) - \overline{e}(k)]^2
\]  

(29)

Thus, variance ratio:

\[
C = \frac{s_2}{s_1}
\]  

(30)

Original error inspection is a common method which tests errors between model value and measured value. Variance inspection is a method to test statistical distribution characteristics of original error.

**DEFORMATION FORECAST BASED ON GREY MODEL**

No. 16 monitoring sites of a slope monitoring is taken as an example. First, the noise of the deforming data of a deforming slope is to be removed by wavelet packet; then, accumulation generation and forecast analyses of the de-noising data are to be operated. During the prediction of deformation monitoring data process, the length of sequence is subjected to the sequence forms and particular needs. If the sequence is too long, pre-data are weak in correlation with predicting results, which is likely to provoke errors. Normal sequence length is among 4 and 10, in this paper the length is chosen to be 8.

We need to solve the deformation of each phase and explain the feasibility of the gray model by using appropriate conditions because monitoring data are deformation accumulating. 33-40 monitoring data are shown in Table 1.

**Table 1:** form 33 to 40 period date (unit: mm)

<table>
<thead>
<tr>
<th>periods</th>
<th>Deformation value</th>
<th>Accumulate data</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>1.2085</td>
<td>24.9826</td>
</tr>
</tbody>
</table>
From table 1 we can see that both Smooth Ratio $\rho(t)$ and absolute grey degree $\delta$ < 0.5 are less than 0.5. They satisfy the exponential law. Using matlab software [10-13], displacement data of 16 monitoring points is forecasted by models of General GM (1,1), new information GM (1,1), and renewal GM(1,1). Tables 2 and 3 are respectively the predicted values and predicted results residuals. The relation curves between monitoring value and predictive value is shown in Figure 3.

**Table 2:** Predicted value of Different methods (unit: mm)

<table>
<thead>
<tr>
<th>periods</th>
<th>Actual monitoring data</th>
<th>GM predicted value</th>
<th>New information GM predicted value</th>
<th>Metabolism GM predicted value</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>39.5</td>
<td>40.4177</td>
<td>40.1693</td>
<td>39.2303</td>
</tr>
<tr>
<td>42</td>
<td>42.6</td>
<td>43.0686</td>
<td>42.7415</td>
<td>41.6311</td>
</tr>
<tr>
<td>43</td>
<td>44.9</td>
<td>45.8933</td>
<td>45.4784</td>
<td>44.1789</td>
</tr>
<tr>
<td>44</td>
<td>47.6</td>
<td>48.9032</td>
<td>48.3906</td>
<td>46.8826</td>
</tr>
<tr>
<td>45</td>
<td>50.3</td>
<td>52.1106</td>
<td>51.4893</td>
<td>49.7517</td>
</tr>
<tr>
<td>46</td>
<td>53.1</td>
<td>55.5283</td>
<td>54.7863</td>
<td>52.7965</td>
</tr>
<tr>
<td>47</td>
<td>55.2</td>
<td>58.1702</td>
<td>58.2945</td>
<td>56.0276</td>
</tr>
<tr>
<td>48</td>
<td>58.3</td>
<td>60.051</td>
<td>60.0273</td>
<td>59.4564</td>
</tr>
</tbody>
</table>

**Table 3:** Residual and absolute error (unit: mm)

<table>
<thead>
<tr>
<th>periods</th>
<th>GM predicted value</th>
<th>New information GM predicted value</th>
<th>Metabolism GM predicted value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Residual</td>
<td>absolute error</td>
<td>Residual</td>
</tr>
<tr>
<td>41</td>
<td>0.9177</td>
<td>0.0232</td>
<td>0.6693</td>
</tr>
<tr>
<td>42</td>
<td>0.4686</td>
<td>0.0110</td>
<td>0.1415</td>
</tr>
<tr>
<td>43</td>
<td>0.9933</td>
<td>0.0221</td>
<td>0.5784</td>
</tr>
<tr>
<td>44</td>
<td>1.3032</td>
<td>0.0273</td>
<td>0.7906</td>
</tr>
<tr>
<td>45</td>
<td>1.8106</td>
<td>0.0359</td>
<td>1.1893</td>
</tr>
<tr>
<td>46</td>
<td>2.4283</td>
<td>0.0457</td>
<td>1.6863</td>
</tr>
<tr>
<td>47</td>
<td>2.9702</td>
<td>0.0538</td>
<td>3.0945</td>
</tr>
<tr>
<td>48</td>
<td>1.751</td>
<td>0.0300</td>
<td>1.7273</td>
</tr>
</tbody>
</table>
ANALYSIS AND DISCUSSION

As it is shown in table 3, the residual maximum absolute error of new supersedes the old. GM (1,1) is 1.1564, and its maximum value is 0.0227, and both of them are smaller than GM (1,1) and GM (1,1), which explains that the metabolism model of GM (1,1) error is more stable and its prediction is more accurate. At the mean time, Figure 2 shows that GM (1,1) and GM (1,1) have been straying from the tested results and result in great deviation; the prediction results of the metabolism GM (1,1) are more approximate to the measured data, so it fits better. Therefore, comparing the gray prediction model GM (1,1) with the new GM (1,1) prediction model, residual of the metabolism GM (1,1) is the least in the prediction of slope deformation and fits the best to the measured data and makes the fewest relative errors. That’s to say, the metabolism model of GM (1,1) is the most accurate way adopted in slope deformation prediction.

REFERENCES


