Application of Principal Component Analysis in the Microstructure of Frozen Soil

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ABSTRACT

The strength of ice and frozen soil interaction is depended on the water content and the temperature of the environment, the principal component analysis is used to analyze the effects of temperature and pressure on the strength of frozen soil through triaxial test simulation and its microstructure parameters. Studies display that the message superposition is clear and severe among micro structural parameters. The first, the second and the third principal components, whose accumulative contribution is 95%, can exhibit almost all messages expressed by 11 original microstructure parameters. The proposed method can be employed to establish the principal components for any kind of soil even it is illustrated by the given clay samples. At a constant temperature, the shear strength of sample increases with the confining pressure; In constant confining pressure, shear strength of frozen soil increases with decreasing temperature. From the analysis of the microstructure of principal components of soil, the decrease of temperature and the increase of confining pressure has little effect on the number of soil particles, but the effect on the other parameters of soil particles.

KEYWORDS： The temperature; The confining pressure; The strength of frozen soil; The triaxial test of frozen soil; The microscopic parameters; Principal component analysis

INTRODUCTION

When the temperature is below zero degrees Celsius, the soil pore water will freeze and form part of frozen soil[1-2]. There is different from conventional soil due to the frozen of soil ice[3-4]. The conventional soil is usually considered a three-phase system, and the permafrost is often referred to as the four-phase system duo to ice. Even at low temperatures, the presence of liquid water in frozen soil is common[5-6]. At constant temperature, due to changes in stress conditions unfrozen water and ice have corresponding changes. Therefore, the nature of conventional soil mechanics properties of frozen soil depends on not only the corresponding properties of soil, but also the soil temperature and confining pressure [7-9]. The effect of water content on strength of frozen soil is very important, but the study results of the relationship between water content and strength of frozen soil is often inconsistent. Effect of water content on the strength of frozen soil depends on the soil temperature, the soil particles, the ice forms of connectivity, and the shear rate etc. [10-11]. For a given soil, the strength of frozen soil is determined by corresponding to temperature, moisture content, shear rate, ice-soil interaction between particles and the interaction of these factors. The complex reasons causing the relationship between temperature, ice content, loading rate and other factors and the strength of frozen soil, mainly lies in the interaction among various factors rather than factors [12].

PCA (Principal, Component Analysis) is a method of multivariate statistical analysis that chooses a less number of important variables form multiple variables through a linear transformation. Study on the correlation analysis and principal component analysis of statistics into soil microstructure parameters can not only eliminate "messages" phenomenon of various microstructure parameters reflecting soil internal mechanism, but also greatly reduce the dimension and microstructure parameters. Based on difference of the cumulative contribution rate of the requirements, the most information can be expressed by using principal components with different number of approximation carrying all the microstructure parameters. Reducing principal component index dimension can be used to establish the constitutive equations, and the strength criterion of soil mechanics, to reflect the inherent relationship between soil micro structure and macro mechanics phenomenon; and to study the microstructure reengineering process using the soil of relationship building [13].

TRIAXIAL TEST OF FROZEN SOIL

The experiments were carried out in 9 specimen (the height of sample is 124.80mm, the diameter is 61.80mm. the mass is 766kg, and the moisture content is 17.02%). Nine samples are respectively in different constant confining pressure (300kPa, 400kPa, 500kPa, respectively) and different temperature (-5 °C-10 °C, C, -15 °C, respectively), being applied to axial pressure until shear failure; then based on the limit stress circle envelope, the shear strength parameters are obtained[14].

Lanzhou silty clay soil is used for the sample. According to the predetermined dry density and water content, the samples are divided into 5 layers in compaction, equal in number, each layer of the contact surface being roughed. The prepared samples are measured for its diameter and height. Then the moisture content is measured. The average diameter shall be calculated as follows:

$$d_0 = \frac{d_1 + d_2 + d_3}{3} \tag{1}$$

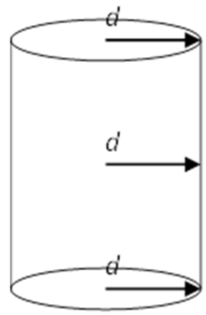

Figure 1: Diameter of the specimen

In formula (1), d_1, d_2, d_3 are respectively the diameter of the specimen, in part, as following the Figure 1.

(a) strain control triaxial shear instrument type of frozen soil (b) scanning electron microscope

Figure 2: Instrument and equipment

Instrument and equipment is strain control triaxial shear instrument type of frozen soil, seen from the Figure 2 (a), that composed of the ambient pressure system, the pressure system, the pore water pressure measurement system, and the host. Ancillary equipment includes the compaction device, the saturator, the cutting device, the divider, the cutting disc, the membrane tube and the mold.

Firstly, a constant process more than 12h to the sample in the required temperature, while the triaxial loading facility for testing machine needs be cooled by the corresponding temperature conditions for a certain length of time. Then, the strain control triaxial shear instrument type of frozen soil is started up, and the samples are loaded after, gradually being increased confining pressure and kept 1-2 hours after the start of shear. When the shear is the peak or deformation reached 25mm loading will be stopped. In the experimental process, confining pressure and loading rate is for 0.033MPa/s, and the shear rate is for 1.25mm/min.

OBSERVATION OF SEM MICROSTRUCTURE AND PARAMETER EXTRACTION

(a) -5°C 300kPa (b) -10°C 400kPa (c) -15°C 500kPa

Figure 3: The observed images under different conditions (parts)

Abbreviation of SEM is Scanning electron microscope, shown in Figure 2 (b). In the process of sample preparation, the soil samples must be remained the original state without disturbance. The specimens are made slow drying by dry method until the shrinkage limit is less than 10%[15]. After broke by hand, samples are chosen the different direction site of destroy shear zone and the flat section of the site is under the magnifier. In order to remove the particles on the sample surface, a layer of glue is coated lightly and tore until the glue dries. The particles are loosed away by rubber ball, and the fresh section can obtained. A thin layer of platinum is sprayed in the surface of samples to increase the conductivity of the samples. Therefore, the sample in the incident electron beam irradiation surface is not easy to accumulate charge to affect image quality. Three specimen of electron microscope in the sample, and each electron microscope specimen is got a total of 4 SEM photos respectively at the end of the transverse, longitudinal, transverse, in the end under the voltage of 30KV, spot 4, and high pressure. The scanning results are shown in Figure 3.

The image analysis software of Leica QWin is the core part of Leica image workstation, quantitative image analysis. And microscopy applications are suited ideally, met the modern needs of digital image. Image software is easy to solve and speed up the completion of the conventional and research on image analysis. Leica QWin is effective and powerful software for general image processing and analysis, and provides the mega pixel accuracy compared with the quantitative microscopy optimization. Leica QWin module is arranged and extended so that the needs of different applications from simple manual measurement and automatic measurement can be met, and various parameters can be achieved. Leica QWin is based on soil SEM quantitative image analysis system in platform of MATLAB, and 4 functional modules to the image reading, image processing, parameter extraction and data storage as a quantitative analysis of the core of the system. The extraction of microstructure parameters of soil are realized by the coordination between each module[16]. The extraction results are shown in Table 1.

Table 1: Original data of soil microstructure characteristic parameters

Serial number of soil sample	*a*	*b*	*c*	*d*	*e*	*f*	*g*
1	1156.95	1161.18	1270.19	3618.38	2549	0.91	4.01
2	1159.44	1004.73	1183.75	3341.51	2392	0.85	4.05
3	1150.74	791.07	872.65	2533.77	2140	0.91	3.90
4	1152.05	1113.04	1196.47	3562.17	3627	0.93	3.92
5	1157.40	1018.84	1298.32	3576.54	2139	0.78	4.01
6	1165.05	1436.90	1596.01	4535.97	2941	0.90	4.15
7	1165.39	1025.30	1182.35	3344.21	2853	0.87	4.16
8	1145.80	1199.97	1473.39	4107.45	2919	0.81	3.82
9	1145.78	749.73	882.06	2541.83	2258	0.85	3.82

Test results are analyzed in the pore ratio of 80%. Of course, the soil will become close together, and the void ratio is changed as the pressure increases. The contrast results may have inaccuracies in the same pore, but the difference is ignored in test.

EVALUATION METHODS OF MICROSCOPIC PARAMETERS

Principal component analysis is a method that many original indexes that have certain relevance indicators form a group of new independent comprehensive index to replace the original index. The more Var (F_1) larger, the more information contained in the F_1. Therefore, F_1 should be the maximum variance in all the linear combinations, and is the first principal component. If the first principal component is not sufficient to represent the original index information, and then the second linear combinations of F_2 is chosen. The information Existing in F_1 no longer appear in F_2, namely Cov (F_1, F_2) = 0. F_2 is called the second principal component. The third, the fourth, ... , *P*, can be constructed out of. Then the main components are ordered, and the reasonable and appropriate interpretation combined with professional knowledge of principal component of the selected is provided.

TEST RESULTS AND ANALYSIS

Results of triaxial test of frozen soil

A shear strength formula is

$$\tau_f = c + \sigma \tan\varphi \tag{2}$$

T_f is for the shear strength (kPa), c is for the cohesion (or cohesion), kPa, σ is for the total stress (kPa) and φ is the angle of internal friction (kPa).

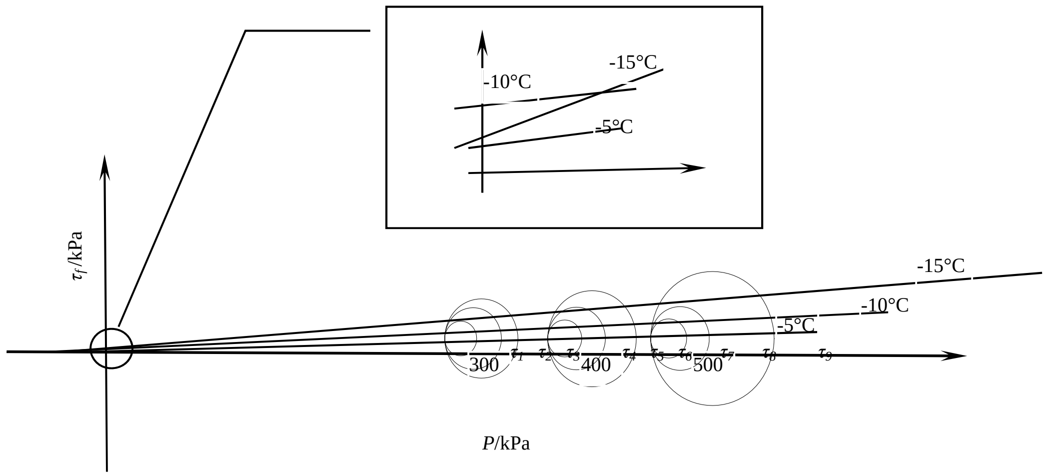

Figure 4: Mohr's circle under different confining pressure and temperature conditions

Table 2: Test damage value of sample in different temperature and confining pressure

T/°C \ τ_f/kPa \ P/kPa	300	400	500
−5	τ_1 (331)	τ_2 (433)	τ_3 (535)
−10	τ_4 (355)	τ_5 (455)	τ_6 (557)
−15	τ_7 (376)	τ_8 (486)	τ_9 (610)

Table 3: Shear strength indexes of different confining pressure and temperature

T/°C	c/kPa	φ/°
−5	10.34	0.85
−10	23.76	0.57
−15	11.52	7.00

With temperature decreasing, *c* increased first and then decreased the angle of internal friction decreases as the decrease of temperature firstly and then increases.

Study of microscopic parameters

The variable microscopic parameters of *p* (p=7) are respectively repressed by $\boldsymbol{X_1}$, $\boldsymbol{X_2}$… $\boldsymbol{X_7}$. 7 dimensional random variable vector is composed of $\boldsymbol{X}=(\boldsymbol{X_1}, \boldsymbol{X_2}, \ldots\ldots, \boldsymbol{X_7})^{T}$.

By

$$X_i = \frac{x_{ij} - \overline{x}_j}{S_j} \tag{3}$$

among them, $s_j^2 = \frac{\sum_{i=1}^{n}(x_{ij} - \overline{x}_j)}{n-1}$, $\overline{x}_j = \frac{1}{n}\sum_{i=1}^{n} x_{ij}$, a standard array obtained.

$$S = (s_{ij})_{7\times 7} = \frac{1}{n-1}\sum_{k=1}^{n}(x_k - \overline{x})(x_k - \overline{x})^T , \quad (4)$$

$$R = (r_{ij})_{7\times 7} = \left(\frac{s_{ij}}{\sqrt{s_{ii}s_{jj}}} \right), \quad (5)$$

among them $\overline{x} = (\overline{x}_1, \overline{x}_2, ..., \overline{x}_7)^T$, $s_{ij} = \frac{1}{n-1}\sum_{k=1}^{n}(x_{ki} - \overline{x}_i)(x_{kj} - \overline{x}_j)$, $i, j = 1, 2, ..., 7.$

The solution of characteristic equation of $|\lambda I\text{-}R|=0$ is used Jacobi to calculate eigenvalues, which arranged in order of size as $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3 \ldots\ldots \geqslant \lambda_p \geqslant 0$. Eigenvalues is the variance of different components, and reflects the size of each principal component influence. The contribution rate of Z_i is $W_i = \lambda_j \Big/ \sum_{j=1}^{p}\lambda_j$, and the accumulative contribution rate is $P_i = \sum_{j=1}^{m}\lambda_j \Big/ \sum_{j=1}^{p}\lambda_j$. The number of components is determined according to the cumulative contribution rate [17]. The eigenvectors of l_i which is corresponding to λ_i (i=1, 2...7) is calculated and $\sum_{j=1}^{7} l_{ij}^2 = 1$, where l_{ij} represents a j component of vector. The eigenvectors as follows: l_1=（0.35，0.49，0.46，0.47，0.29，0.06，0.35），l_2=（0.59，-0.18，-0.25，-0.26，-0.32，0.18，0.59），l_3=（-0.06，0.01，-0.22，-0.13，0.53，0.81，-0.05）.

By

$$Z_i = \sum_{i=1}^{7} l_{i1} X_i \ , i, \ \ j = 1, 2, ..., 7. \ , \quad (7)$$

The principal component of i is obtained, seen from Table 4.

Table 4: Data of sample and Principal component

Cat. No.	Original variable $X_1\ X_2 \ldots\ldots X_p$	Principal constituent $Z_1\ Z_2 \ldots\ldots Z_p$
1	$x_{11}\ x_{21} \ldots\ldots x_{p1}$	$Z_{11}\ Z_{21} \ldots\ldots Z_{p1}$
2	$x_{12}\ x_{22} \ldots\ldots x_{p2}$	$Z_{12}\ Z_{22} \ldots\ldots Z_{p2}$
.	.	.
.	.	.
.	.	.
n	$x_{1n}\ x_{2n} \ldots\ldots x_{pn}$	$Z_{1n}\ Z_{2n} \ldots\ldots Z_{pn}$

Through SAS (Statistics Analysis System) to analyze the parameter, the extraction results are shown in table 5, table 6. SAS is a combination of software system, which is composed of a plurality of function module, the basic part of BASE SAS module. Using the program can complete all the work to be done, including statistical analysis, forecasting, modeling and Simulation of sampling. The output contains the statistics of the number of observations and variable.

Table 5: Correlation matrix of soil microstructure characteristic parameters

Code number of parameter	*a*	*b*	*c*	*d*	*e*	*f*	*g*
a	1.00	0.48	0.42	0.41	0.10	0.16	0.99
b		1.00	0.95	0.98	0.58	0.12	0.48
c			1.00	0.99	0.45	-0.18	0.42
d				1.00	0.53	-0.08	0.46
e					1.00	0.44	0.10
f						1.00	0.16
g							1.00

Table 6: Eigenvalue, Difference, Proportion, Cumulative

Z_i	a_{11}	a_{22}	a_{33}	a_{44}
1	3.91	2.42	0.56	0.56
2	1.49	0.16	0.21	0.77
3	1.33	1.05	0.19	0.96
4	0.28	0.28	0.04	1

a、*b*、*c*、*d*、*e*、*g*、*h* represent Area, Intercept H, Intercept V, Perimeter, Count, Anisotropy, Mean Chord respectively in table 1 and table 5. a_{11}、a_{22}、a_{33}、a_{44} in table 5 represent respectively the characteristic parameters of Eigenvalue, Difference, Proportion, Cumulative.

Table 7: Score of Principle Component

Code number	F_1	F_2	F_3	Conditions	The comprehensive evaluation index
1	0.61	0.26	0.44	-5°C, 300kPa	0.48
2	-0.07	0.86	-0.61	-5°C, 400kPa	0.03
3	-2.65	0.70	0.66	-5°C, 500kPa	-1.21
4	0.50	-1.04	2.09	-10°C, 300kPa	0.46
5	-0.08	0.21	-2.14	-10°C, 400kPa	-0.41
6	3.53	0.30	0.11	-10°C, 500kPa	2.06
7	0.86	1.59	0.13	-15°C, 300kPa	0.84
8	0.50	-2.54	-0.90	-15°C, 400kPa	-0.42
9	-3.18	-0.33	-0.13	-15°C, 500kPa	-1.87

The principle component feature vectors table (seen from the table 7) is obtained by eigenvectors. A sum of, the final evaluation value is obtained by the weighted sum of 3 main components, weight for the variance of each principal component contribution.

$$F_m = W_1Z_1 + W_2Z_2 + W_3Z_3. \qquad (8)$$

Relationship between the main components SEM and microscopic parameters

Seen from the three eigenvectors, the first principal component coefficients is very small in the heterogeneity of the absolute value, and the rest are positive. F_1 represents the influence on soil particles area; the second principal component is negative in the transverse and longitudinal length, perimeter, particle number, and coefficient area, heterogeneity, average chord length is positive, and the absolute value of heterogeneous anisotropic coefficient is small. Therefore, F_2 represents the effect degree on soil particles mellow. The third principal component in the area, the longitudinal length, perimeter, average chord length is negative, and the transverse length, grain number, heterogeneous sex is positive, what's more, the absolute value of coefficient of transverse length is very small. F_3 represents the effect on heterogeneous anisotropic.

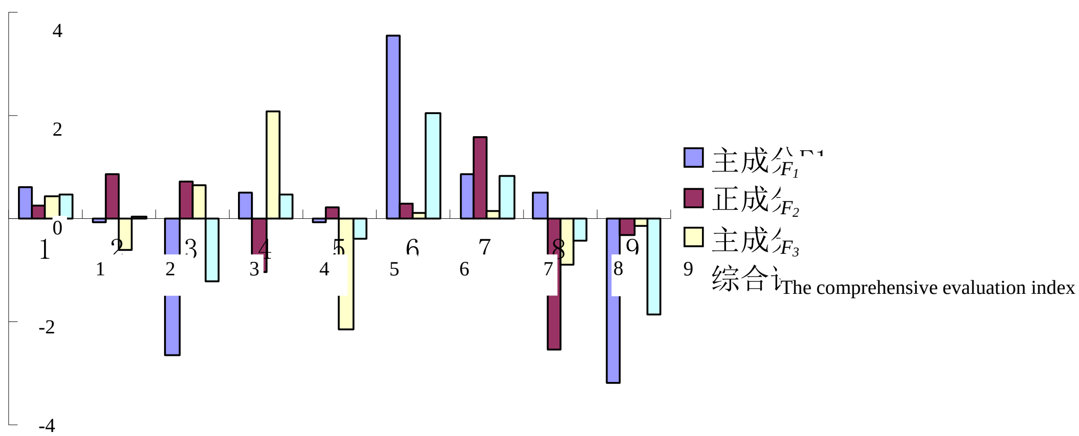

Figure 5: Score of principle component

Analysis with principal component

With temperature decreasing *c* increased first and then decreased, the angle of internal friction decreases as the decrease of temperature firstly and then increases.

From the Figure 5, soil particle area is maximum positive influence in the Condition of -5°C, and 500kPa in F_1. the higher scores of P_7 in F_2 shows that soil particles mellow degree is affected larger positive effect in lower confining pressure when the temperature is higher. The main components of F_3 showed that, soil particles in heterogeneous anisotropic has great positive impact in higher temperature. According to the comprehensive evaluation score, as can be seen, the comprehensive evaluation value is the highest when the confining pressure is 500kPa and temperature is -10°C.

CONCLUSIONS

At constant temperature, the shear strength resistance of specimens increases with the increasing of confining pressure. At constant confining pressure, the shear strength resistance of specimens increases with decreasing temperature. The decrease of temperature and the increase of confining pressure have little effect on the number of soil particles, but the effect on the other parameters of soil particles, while the parameters between form situation that one is higher and the other is lower.

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