The Calibration Algorithm for Installation Error of the Strap-down Measurement-While-Drilling System

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ABSTRACT

Rotary steerable drilling system is a Mechatronics tool that generated by the development of directional drilling. The drill string attitude should be measured in real-time in the rotary steerable drilling process. So the strap-down measurement system in this paper was developed, installing the triaxial accelerometer and triaxial fluxgate sensors to the center of the drill string, which can measure the drill string attitude real time. But installation error is a big problem because it is impossible to guarantee the coordinate axis are orthogonal and installation is centered exactly. Therefore, the calibration mathematical model is established in this paper and proposed a numerical fitting method to calculate the calibration coefficients which compared with orthogonal method. The results show that orthogonal method and curve-fitting method has little difference in calculating the calibration coefficients. However, the curve fitting method is simple in programming and the instruments used for correction will be able to meet the requirements even having lower accuracy than orthogonal method as required. So the curve fitting method is more suitable for engineering applications. Final error calculation results show that the calibration model described in this paper can significantly reduce the measurement errors which caused by the installation error, and achieve the engineering application's goals.

KEYWORDS: Calibration Algorithm, MWD, Strap-down, Rotary steerable, Installation error
NOMENCLATURE

\[ G: \text{ Acceleration of gravity } (m / s^2) \]
\[ B: \text{ Earth's magnetic field strength } (\mu T) \]
\[ I: \text{ Inclination } (°) \]
\[ A_m: \text{ Azimuth } (°) \]
\[ G_x: \text{ Gravity component in the x-axis direction of the coordinate system of the instrument} \]
\[ G_y: \text{ Gravity component in the y-axis direction of the coordinate system of the instrument} \]
\[ G_z: \text{ Gravity component in the z-axis direction of the coordinate system of the instrument} \]
\[ B_x: \text{ Geomagnetic field in the x-axis direction of the coordinate system of the instrument} \]
\[ B_y: \text{ Geomagnetic field in the y-axis direction of the coordinate system of the instrument} \]
\[ B_z: \text{ Geomagnetic field in the z-axis direction of the coordinate system of the instrument} \]
\[ A_x, A_y, A_z: \text{ Triaxial accelerometer output voltage } (V) \]
\[ F_x, F_y, F_z: \text{ Triaxial fluxgate output voltage } (V) \]
\[ K_m: \text{ The accelerometer mathematical model coefficients } (i=x, y, z; m=x, y, z) (V \cdot s^2 / m) \]
\[ K_{Ax}, K_{Ay}, K_{Az}: \text{ Triaxial accelerometer calibration coefficients } (V \cdot s^2 / m) \]
\[ L_{Fx}, L_{Fy}, L_{Fz}: \text{ Triaxial fluxgate calibration coefficient } (V / \mu T) \]
\[ I_{Ax}, I_{Ay}, I_{Az}: \text{ Triaxial accelerometer installation angle } (°) \]
\[ T_{Ax}, T_{Ay}, T_{Az}: \text{ The phase of the installation angle } (°) \]
\[ \text{Bias}_{Ax}, \text{Bias}_{Ay}, \text{Bias}_{Az}: \text{ The bias of the accelerometer sensors } (m / s^2) \]
\[ \text{Bias}_{Fx}, \text{Bias}_{Fy}, \text{Bias}_{Fz}: \text{ The bias of the fluxgate sensors } (\mu T) \]

INTRODUCTION

Directional drilling which can dramatically reduce the cost and time of drilling operations is the technology of directing a wellbore along a predefined trajectory. In recent years, the development of directional well drilling technologies has gained more attention. Especially, the most important thing is that the drill string position, including the inclination (deviation from the vertical direction) and azimuth (deviation from the north direction in the horizontal plane) [1], should be provided in the drilling process, which is called measurement while drilling (MWD). Current MWD surveying is performed along the well path at stationary survey stations. That is to say, usually in drilling engineering, the bottom drill string attitude (inclination and azimuth) measurement is carried out in the case of the drill string does not rotate. But with the development of drilling technology, continuous measurement of well trajectory becomes increasingly important. It also has become essential in rotary steerable systems.

Rotary steerable system [2-5] is a Mechatronics tool generated by the development of directional drilling, which can drill more economical and smooth borehole. However, to measure the posture while the drill string rotation is one of the technical difficulties, since this is completely different from the current MWD surveying method [6-7].

This paper developed a set of strap-down measurement system to achieve the functionality of rotary steerable drilling [8]. The measurement system incorporates three-axis magnetometers and three-axis accelerometers arranged in three mutually orthogonal directions [9], [10]. The magnetic surveying instrument is installed inside the nonmagnetic drill collar which is usually designed for Monel metal. So the measurement system will avoid external magnetic interferences [11].
Aboelmagd Noureldin [12] proposed that the nonmagnetic drill collar is too expensive and developed a measurement system using fiber optic gyroscopes (FOG) instead of fluxgate sensor [13]. Taking into account the gyroscope has low reliability, this paper focuses on the magnetic surveying tools which have a wider range of applications.

In practical cases, even if carefully try to make the three-axis accelerometers and three-axis fluxgates orthogonal, it is impossible to guarantee the coordinate axis are orthogonal and installation is centered exactly. This will bring about the final solution error regardless of whatever the solution method. Therefore, if want to improve the accuracy measurement, we must develop a compensation algorithm to make sensors centered and mutually orthogonal, that is, from the mathematical model of the system, design the corresponding algorithm to solve out the installed error, and to accurately calculate the drilling string attitude.

Mathematical model

As show in the Fig.1, assume that acceleration of gravity as \( G \), Earth's magnetic field strength as \( B \), \( G_x, G_y, G_z \) are respectively defined as measurement signals of triaxial accelerometers on the x, y, z axis, and \( B_x, B_y, B_z \) are respectively defined as measurement signals of triaxial fluxgate on the x, y, z axis. Under the certain sample frequency, measurement signals can be expressed as time series.

Calculated deviation angle “\( I \)” and azimuth “\( A_m \)” as follows:

\[
I = \arctan\left(\frac{\sqrt{G_x^2 + G_y^2}}{-G_z}\right)
\]

\[
A_m = \arctan\left(\frac{G(B_yG_x - B_xG_y)}{B_z(G_x^2 + G_y^2) - G_z(B_xG_x + B_yG_y)}\right)
\]

In this paper we try to establish the algorithm model of error compensation since the installation error cannot be avoided and the calibration parameters will be obtained through the laboratory experiments.

First of all, assuming that \( Ax, Ay, Az \) were output voltage of the accelerometer before the establishment of mathematical model, so the relationship between the respective components of the gravity and the output voltage as shown in the following formula.

\[
\begin{bmatrix}
Ax \\
Ay \\
Az
\end{bmatrix} =
\begin{bmatrix}
K_{xx} & K_{xy} & K_{xz} \\
K_{yx} & K_{yy} & K_{yz} \\
K_{zx} & K_{zy} & K_{zz}
\end{bmatrix}
\begin{bmatrix}
G_x \\
G_y \\
G_z
\end{bmatrix} +
\begin{bmatrix}
Ax_0 \\
Ay_0 \\
Az_0
\end{bmatrix}
\]

where \( K_{Ax}, K_{Ay}, K_{Az} \) indicated triaxial accelerometer calibration coefficient.
Then assume that $I_{Ax}$, $I_{Ay}$, $I_{Az}$, and $T_{Ax}$, $T_{Ay}$, $T_{Az}$ indicate triaxial accelerometer installation angle and the phase of the installation angle. The following formula can be obtained.

$$
\begin{bmatrix}
    Ax \\
    Ay \\
    Az
\end{bmatrix}
= \begin{bmatrix}
    K_{Ax} \times \cos I_{Ax} & K_{Ax} \times \sin I_{Ax} \times \cos T_{Ax} & K_{Ax} \times \sin I_{Ax} \times \sin T_{Ax} \\
    K_{Ay} \times \sin I_{Ay} \times \sin T_{Ay} & K_{Ay} \times \cos I_{Ay} & K_{Ay} \times \sin I_{Ay} \times \cos T_{Ay} \\
    K_{Az} \times \sin I_{Az} \times \cos T_{Az} & K_{Az} \times \sin I_{Az} \times \sin T_{Az} & K_{Az} \times \cos I_{Az}
\end{bmatrix}
\times
\begin{bmatrix}
    \gamma x \\
    \gamma y \\
    \gamma z
\end{bmatrix}
+ \begin{bmatrix}
    Ax_0 \\
    Ay_0 \\
    Az_0
\end{bmatrix}
$$

(4)

Obviously, $K_{xx} = K_{Ax} \times \cos I_{Ax}$, $K_{xy} = K_{Ay} \times \sin I_{Ax} \times \cos T_{Ax}$, $K_{xz} = K_{Az} \times \sin I_{Ax} \times \sin T_{Ax}$.

The accelerometer calibration coefficient as follows (unit: V/G):

$$
\begin{bmatrix}
    K_{Ax} \\
    K_{Ay} \\
    K_{Az}
\end{bmatrix}
= \begin{bmatrix}
    \sqrt{(K_{xx} \times G)^2 + (K_{xy} \times G)^2 + (K_{xz} \times G)^2} \\
    \sqrt{(K_{yx} \times G^2 + (K_{y} \times G)^2 + (K_{z} \times G)^2} \\
    \sqrt{(K_{zx} \times G^2 + (K_{zy} \times G)^2 + (K_{zz} \times G)^2}
\end{bmatrix}
\times \begin{bmatrix}
    1/G
\end{bmatrix}
$$

(5)

The bias of the sensors can be obtained (unit: G):

$$
\begin{bmatrix}
    \text{Bias}_{Ax} \\
    \text{Bias}_{Ay} \\
    \text{Bias}_{Az}
\end{bmatrix}
= \begin{bmatrix}
    1/K_{Ax} & 0 & 0 \\
    0 & 1/K_{Ay} & 0 \\
    0 & 0 & 1/K_{Az}
\end{bmatrix}
\times
\begin{bmatrix}
    Ax_0 \\
    Ay_0 \\
    Az_0
\end{bmatrix}
$$

(6)

The coefficient in Equation (4) can be calculated respectively as follows:

$$
\begin{align*}
\cos I_{Ax} &= \frac{K_{xx} \times G}{K_{Ax} \times G} \\
\sin I_{Ax} \times \cos T_{Ax} &= \frac{K_{xy} \times G}{K_{Ax} \times G} \\
\sin I_{Ax} \times \sin T_{Ax} &= \frac{K_{xz} \times G}{K_{Ax} \times G} \\
\sin I_{Ay} \times \sin T_{Ay} &= \frac{K_{yx} \times G}{K_{Ay} \times G} \\
\cos I_{Ay} &= \frac{K_{yy} \times G}{K_{Ay} \times G} \\
\sin I_{Ay} \times \cos T_{Ay} &= \frac{K_{yz} \times G}{K_{Ay} \times G} \\
\sin I_{Az} \times \cos T_{Az} &= \frac{K_{zx} \times G}{K_{Az} \times G} \\
\sin I_{Az} \times \sin T_{Az} &= \frac{K_{zy} \times G}{K_{Az} \times G}
\end{align*}
$$

(7) (8) (9) (10) (11) (12) (13) (14)
\[
\cos I_{x} = \frac{K_{x} \times G}{K_{ax} \times G}
\]  

(15)

For the accelerometer installed in the x-axis, defined the \( \cos(AxPx), \cos(AxPy) \) and \( \cos(AxPz) \) are cosine values of angles between accelerometer sensitive axis and three axes of the instrument coordinate system.

\[
Ax = \left[ G_{x} \cdot \cos(AxPx) + G_{y} \cdot \cos(AxPy) + G_{z} \cdot \cos(AxPz) \right] \cdot K_{ax} + Bias_{ax} \cdot K_{ax}
\]

\[
= (G_{x} \cdot \cos(AxPx) + G_{y} \cdot \cos(AxPy) + G_{z} \cdot \cos(AxPz) + Bias_{ax}) \cdot K_{ax}
\]  

(16)

Obviously, \( G = \sqrt{G_{x}^{2} + G_{y}^{2} + G_{z}^{2}} \), \( \cos(AXPx) = \cos I_{ax} \), \( \cos(AXPY) = \sin I_{ax} \times \cos T_{ax} \), ...

Then we get the calibration mathematical model of accelerometer error as shown in Eq.17.

\[
\begin{bmatrix}
Ax \\
Ay \\
Az
\end{bmatrix} =
\begin{bmatrix}
K_{Ax} \times \cos AxPx & K_{Ax} \times \cos AxPy & K_{Ax} \times \cos AxPz \\
K_{Ay} \times \cos AyPx & K_{Ay} \times \cos AyPy & K_{Ay} \times \cos AyPz \\
K_{Az} \times \cos AzPx & K_{Az} \times \cos AzPy & K_{Az} \times \cos AzPz
\end{bmatrix}
\begin{bmatrix}
Gx \\
Gy \\
Gz
\end{bmatrix} +
\begin{bmatrix}
K_{Ax} \times Bias_{Ax} \\
K_{Ay} \times Bias_{Ay} \\
K_{Az} \times Bias_{Az}
\end{bmatrix}
\]  

(17)

Similarly, we get the calibration mathematical model of fluxgate error as shown in equation (18).

\[
\begin{bmatrix}
Fx \\
Vy \\
Fz
\end{bmatrix} =
\begin{bmatrix}
L_{Fx} \times \cos FxPx & L_{Fx} \times \cos FxPy & L_{Fx} \times \cos FxPz \\
L_{Vy} \times \cos VyPx & L_{Vy} \times \cos VyPy & L_{Vy} \times \cos VyPz \\
L_{Fz} \times \cos FzPx & L_{Fz} \times \cos FzPy & L_{Fz} \times \cos FzPz
\end{bmatrix}
\begin{bmatrix}
Bx \\
By \\
Bz
\end{bmatrix} +
\begin{bmatrix}
L_{Fx} \times Bias_{Fx} \\
L_{Vy} \times Bias_{Fy} \\
L_{Fz} \times Bias_{Fz}
\end{bmatrix}
\]  

(18)

wherein,

\[
B=\sqrt{Bx^{2} + By^{2} + Bz^{2}}
\]  

(19)

So for each sensor, need to determine 5 calibration parameters. For example, for \( Ax \), need to calculate the \( K_{Ax} , Bias_{Ax} , \cos AxPx , \cos AxPy , \cos AxPz \); for \( Fx \), need to calculate the \( L_{Fx} , Bias_{Fx} , \cos FxPx , \cos FxPy , \cos FxPz \).

**Experimental methods**

Designed the experimental instrument that can be put in arbitrary position in the three-dimensional space (as shown in Fig.1 on the left side), and using non-magnetic materials to ensure that the fluxgate sensors will not be disturbed.
Figure 1: Experimental equipment and location coordinates

First using the orthogonal method to calibrate the installation error. Determine the 24 positions as shown in Table 1, correction parameters can be obtained when the inclination and azimuth values for each point have been calculated. As show in Fig.2 on the right side, point A represents the number 2 in Table 1.

As an example of $AX$, obtain the following formula by Equation (17) and Table 1.

$$K_{Ax} \times \cos(Ax_{Px}) \times G_x = \frac{1}{8}(Ax_1 + Ax_3 + Ax_{16} + Ax_{24} - Ax_1 - Ax_3 - Ax_{20} - Ax_{22})$$  \hspace{1cm} (20)

$$K_{Ax} \times \cos(Ax_{Px}) \times G_y = \frac{1}{8}(Ax_2 + Ax_8 + Ax_{12} + Ax_{18} - Ax_2 - Ax_8 - Ax_{14} - Ax_{16})$$  \hspace{1cm} (21)

$$K_{Ax} \times \cos(Ax_{Px}) \times G_z = \frac{1}{8}(Ax_{11} + Ax_{15} + Ax_{17} + Ax_{23} - Ax_9 - Ax_{13} - Ax_{17} - Ax_{21})$$  \hspace{1cm} (22)

and because $\sqrt{G_x^2 + G_y^2 + G_z^2} = G$,

$$K_{Ax} \times \cos(Ax_{Px}) \times G = \sqrt{(K_{Ax} \times \cos(Ax_{Px}) \times G_x)^2 + (K_{Ax} \times \cos(Ax_{Px}) \times G_y)^2 + (K_{Ax} \times \cos(Ax_{Px}) \times G_z)^2}$$  \hspace{1cm} (23)

Obtained $K_{Ax} \times \cos(Ax_{Py}) \times G$, $K_{Ax} \times \cos(Ax_{Pz}) \times G$ and $K_{Ax} \times \text{Bias}_{Ax}$ in the same method.

As we know, $\cos^2(Ax_{Px}) + \cos^2(Ax_{Py}) + \cos^2(Ax_{Pz}) = 1$, then:

$$K_{Ax} \times G = \sqrt{(K_{Ax} \times \cos(Ax_{Px}) \times G)^2 + (K_{Ax} \times \cos(Ax_{Py}) \times G)^2 + (K_{Ax} \times \cos(Ax_{Pz}) \times G)^2}$$  \hspace{1cm} (24)

Finally we get five calibration coefficients of Ax: $K_{Ax}$, $\text{Bias}_{Ax}$, $\cos(Ax_{Px})$, $\cos(Ax_{Py})$, $\cos(Ax_{Pz})$. 
<table>
<thead>
<tr>
<th>No.</th>
<th>Sensor shaft</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>north east down</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2(A)</td>
<td>east south down</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>south west down</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>west north down</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>north west up</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>east north up</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>south east up</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>west south up</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>up east north</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>east down north</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>down west north</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>west up north</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>up west south</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>west down south</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>15</td>
<td>down east south</td>
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<tr>
<td>16</td>
<td>east up south</td>
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<td></td>
</tr>
<tr>
<td>17</td>
<td>up north west</td>
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<td></td>
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<tr>
<td>18</td>
<td>north down west</td>
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<td></td>
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<td>down south west</td>
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<td>21</td>
<td>up south east</td>
<td></td>
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<tr>
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<tr>
<td>23</td>
<td>down north east</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>north up east</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

But using orthogonal method for the instrument required of the calibration system is not only high precision but also complex in structure. It is difficult for practical application, so we developed the method of data fitting. Specific steps are as follows.

Let the instrument fixed in one position (a fixed well inclination and azimuth) and rotated 360°. Sample one data when the instrument rotated 45° (error: ±1°) and will get 8 sampling data when the instrument rotated 360°. Using numerical fitting theory which based on the orthogonal trigonometric we can get the sensors output voltage curve when the instrument rotated 360°. Then we can calculate the calibration coefficients for each sensor.

Calculation methods are described with Ax and Fx as an example. According to Eq. 17 and Eq.18, Ax and Fx are presented as:

\[
Ax = (Gx \cdot \cos AxPx + Gy \cdot \cos AxPy + Gz \cdot \cos AxPz + Bias_{Ax}) \cdot K_{Ax} \tag{25}
\]

\[
Fx = (Bx \cdot \cos FxPn + By \cdot \cos FxPy + Bz \cdot \cos FxPz + Bias_{Fx}) \cdot L_{Fx} \tag{26}
\]
and,

\[ Ax = K_{Ax} \cdot G \cdot \sin I \cdot \cos AxPx \cdot \cos T - K_{Ax} \cdot G \cdot \sin I \cdot \cos AxPy \cdot \sin T \]
\[ + K_{Ax} \cdot (-G \cdot \cos I \cdot \cos AxPz + \text{Bias}_{Ax}) \]  \hspace{1cm} (27)

Assume that:

\[ M = K_{Ax} \cdot G \cdot \sin I \cdot \cos AxPx \] \hspace{1cm} (28)
\[ N = -K_{Ax} \cdot G \cdot \sin I \cdot \cos AxPy \] \hspace{1cm} (29)
\[ P = K_{Ax} \cdot (-G \cdot \cos I \cdot \cos AxPz + \text{Bias}_{Ax}) \] \hspace{1cm} (30)

If the inclination is unchanged, \( M, N, P \) are constants. Brought into Eq. 17, \( Ax \) and \( Fx \) are presented as:

\[ Ax = M \cdot \cos r + N \cdot \sin r + P \] \hspace{1cm} (31)
\[ Fx = m \cdot \cos r + n \cdot \sin r + p \] \hspace{1cm} (32)

wherein:

\[ m = L_{Fx} \cdot [(B_h \cdot \cos A \cdot \cos I - B_v \cdot \sin I) \cdot \cos FxPx - B_h \cdot \sin A \cdot \cos FxPy] \]
\[ n = L_{Fx} \cdot [-B_h \cdot \sin A \cdot \cos FxPx + (B_h \cdot \cos A \cdot \cos I + B_v \cdot \sin I) \cdot \cos FxPy] \]
\[ p = L_{Fx} \cdot [(B_h \cdot \cos A \cdot \sin I + B_v \cdot \cos I) \cdot \cos FxPz + \text{Bias}_{Fx}] \]

Eq.31-32 are the output of accelerometer and fluxgate mathematical model, so, in order to achieve higher fitting precision, selected the orthogonal trigonometric as basis functions to fit the curve of the output of each sensor.

Taken \( Ax \) as an example, assuming \( A_m = \alpha_o, I = d_1 \), \( Ax \) output is:

\[ Ax_1 = M_1 \cdot \cos r + N_1 \cdot \sin r + P_1 \] \hspace{1cm} (33)

Assuming \( A_m = \alpha_o, I = d_2 = d_1 + 90^\circ \), output is:

\[ Ax_2 = M_2 \cdot \cos r + N_2 \cdot \sin r + P_2 \] \hspace{1cm} (34)

Then
We can calculate the $d_1$, $d_2$, $K_{Ax}$, $Bias_{Ax}$, $cosAxPx$, $cosAxPy$, $cosAxPz$ as follows.

$$d_1 = \arctan(M_1/M_2)$$  \hspace{1cm} (36)$$
$$d_2 = d_1 + 90$$  \hspace{1cm} (37)$$
$$cosAxPx = 1 \cdot (\tan d_1 + 1) \cdot M_1 / (M_1^2 \cdot \tan d_1^2 + 2 \cdot N_1^2 \cdot \tan d_1 + M_1^2 + N_1^2 \cdot \tan d_1^2 + P_1^2 \cdot \tan d_1 + N_1^2 + P_1^2 \cdot \tan d_1 - 2 \cdot \tan d_1 \cdot P_1)$$  \hspace{1cm} (38)$$
$$cosAxPy = \frac{N_1}{M_1} \cdot cosAxPx$$  \hspace{1cm} (39)$$
$$cosAxPz = \frac{(P_2 - P_1) \cdot \sin d_1}{M_1 \cdot (\sin d_1 + \cos d_1)}$$  \hspace{1cm} (40)$$
$$K_{Ax} = \frac{M_1}{G \cdot \sin d_1 \cdot cosAxPx}$$  \hspace{1cm} (41)$$
$$Bias_{Ax} = \frac{P_1}{SF_{ax}} + G \cdot \cos d_1 \cdot cosAxPz$$  \hspace{1cm} (42)$$

The fluxgate calculation method is similar as above, obtained the following equations with $F_x$ as an example.

$$L_{Fx} \times \begin{bmatrix} B_h \cdot \cos \alpha_0 \cdot \cos d_1 - B_v \cdot \sin d_1 & -B_h \cdot \sin \alpha_0 \\ -B_h \cdot \sin \alpha_0 & -B_h \cdot \cos \alpha_0 \cdot \cos d_1 + B_v \cdot \sin d_1 \\ B_h \cdot \cos \alpha_0 \cdot \cos d_2 - B_v \cdot \sin d_2 & -B_h \cdot \sin \alpha_0 \\ -B_h \cdot \sin \alpha_0 & -B_h \cdot \cos \alpha_0 \cdot \cos d_2 + B_v \cdot \sin d_2 \end{bmatrix} \times \begin{bmatrix} \cos FxPx \\ \cos FxPy \\ \cos FxPz \\ Bias_{Fx} \end{bmatrix} = \begin{bmatrix} m_1 \\ n_1 \\ m_2 \\ n_2 \end{bmatrix}$$  \hspace{1cm} (43)$$
$$L_{Fx} \times \begin{bmatrix} B_h \cdot \cos \alpha_0 \cdot \sin d_1 + B_v \cdot \cos d_1 & 1 \\ B_h \cdot \cos \alpha_0 \cdot \sin d_2 + B_v \cdot \cos d_2 & 1 \end{bmatrix} \times \begin{bmatrix} \cos FxPx \\ \cos FxPz \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$  \hspace{1cm} (44)$$
$$\cos^2 FxPx + \cos^2 FxPy + \cos^2 FxPz = 1$$  \hspace{1cm} (45)$$
By (43), (44),(45) we obtain $L_{Fx}$, $Bias_{Fx}$, $cosFxPx$, $cosFxPy$, $cosFxPz$.

**RESULTS**

Using orthogonal method and the numerical fitting calibration method calculated the calibration coefficients respectively. The results obtained for comparison in the Table 2.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Numerical fitting</th>
<th>Orthogonal design</th>
<th>Coefficients</th>
<th>Numerical fitting</th>
<th>Orthogonal design</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{Ax}$ (V/G)</td>
<td>1.1060</td>
<td>1.1063</td>
<td>$L_{Fx}$ (V/uT)</td>
<td>0.0546</td>
<td>0.0544</td>
</tr>
<tr>
<td>$Bias_{Ax}$ (G)</td>
<td>0.0587</td>
<td>0.0567</td>
<td>$Bias_{Fx}$ (uT)</td>
<td>2.2123</td>
<td>2.2975</td>
</tr>
<tr>
<td>$cosAxPx$</td>
<td>0.9965</td>
<td>0.9965</td>
<td>$cosFxPx$</td>
<td>0.9920</td>
<td>0.9960</td>
</tr>
<tr>
<td>$cosAxPy$</td>
<td>0.0824</td>
<td>0.0832</td>
<td>$cosFxPy$</td>
<td>0.0880</td>
<td>0.0883</td>
</tr>
<tr>
<td>$cosAxPz$</td>
<td>−0.001</td>
<td>−0.0008</td>
<td>$cosFxPz$</td>
<td>−0.0155</td>
<td>−0.0156</td>
</tr>
</tbody>
</table>

Two calibration methods have a little difference in calculate the coefficients which shown in the Table 2. Using these coefficients to calculate the borehole inclination and azimuth as shown in Fig.2.

It can be seen that each error range is within the scope of the engineering allows. So the two calibration methods all achieving the purpose of the system calibration. But the numerical fitting method is easy to operate, and the calibration instrument has simple structure, even if the calibration instrument has lower precision than before, we can also obtain the very precise calculation coefficients like orthogonal method. Despite the error of the final result still exists, it has been greatly improved that achieve the requirements of field application.
CONCLUSION

(1), In Rotary steerable system, must establish a measurement system equipped with a triaxial fluxgate and triaxial accelerometer, but the installation error cannot be avoided and must be calibrated.

(2), Developed the calibration model which can well meet the requirements of field application. The final errors of the measurement of Inclination and Azimuth are small.

(3), Orthogonal method and curve-fitting method has little difference of calculating calibration coefficients, however, the curve fitting method is easy to operate and the calibration instrument has simple structure, even if the calibration instrument has lower precision than before, we can also obtain the very precise calculation coefficients like orthogonal method, more suitable for engineering applications.

REFERENCES


