ABSTRACT

Based on the finite element optimization, according to the additional parameters identification of cantilever retaining wall system, a new method is proposed. First of all, the simplified analysis model of cantilever retaining wall system is established. At the same time, assumed that the horizontal resistance coefficient and the added mass of the element area of soil show parameter is nonlinear distribution, the theoretical formulas of the additional parameters of the soil are derived. Then, three combination optimization methods and four objective functions are taken into comprehensive consideration. Through the comparison the results of analysis, the optimal combination scheme is selected. Finally, a numerical simulation of cantilever retaining wall is studied to evaluate the effectiveness of the proposed method. The results indicate that combining with the “zero order method and first order method” and the objective function-based acceleration response has the highest accuracy. Therefore, an effective theoretical model is provided for the health diagnosis of cantilever retaining wall system.

KEYWORDS: Cantilever retaining wall system, Additional parameters, Modal parameters, Acceleration response, Finite element optimization.

INTRODUCTION

At present, the modal analysis has been widely used in the structural damage identification and mechanical fault diagnosis, etc., which is provide a new method of the health diagnosis on the retaining wall. However, there are two main problems in the modal analysis of the retaining wall: on the one hand, how to simplify the cantilever retaining wall system to be more in line with the actual situation; on the other hand, how to simulate the impact of the soil on the modal characteristics, namely how to reasonably determine the additional parameters of soil. Hence, these two issues are discussed in this paper.

To date, numerous physical parameter identification methods have been presented [1~12]. The combined extended Kalman filtering technique and global weighted iteration approach of Hoshiya et al. [1, 2] is adopted to determine the stiffness and damping ratio of three degrees shear
structure. Later, a new method is developed to identify the elastic modulus of elastic plane plate that is combined with the extended Kalman filtering technique and the finite element method [3]. Ren et al. [4] proposed the multi-scale parameter Kalman filter method which is combined orthogonal wavelet transform and Kalman filter method. The theoretical analysis and numerical simulations indicate that the identification of system parameters in multi-scale is more efficient than in single scale. Jiang et al. [5] developed a physical parameter identification algorithm based on modes, frequencies and empirical genetic algorithm. Zhou et al. [6] applied the genetic annealing hybrid algorithm to determine the physical parameter of the frame structure on elastic foundation. Wang et al. [7] introduced the multi-objective differential evolution algorithm in which the dynamic response and natural frequencies can be used to detect the physical parameter within a Truss bridge. Levin et al. [8] provided a algorithm in which the frequency response function was used as a objective function based on simulated annealing algorithm and genetic algorithm. The improved genetic algorithm is used to identify physical parameter of the large structure system by Koh et al. [9]. Perea et al. [10, 11] presented the multi-objective function optimization algorithm by two steps, which natural frequency error and modal assurance criterion is defined as the objective function, respectively.

In this study, an additional parameters identification methodology is presented which allows using modal parameters and dynamic response measurements within a model updating procedure by using finite element optimization algorithms. The numerical finite element modal analysis is conducted as well to verify the validity of the parameters detection algorithm.

THE SIMPLIFIED ANALYSIS MODEL OF CANTILEVER RETAINING WALL SYSTEM

Simplified analysis model

This paper aims to identify additional parameters of soil through the finite element optimization algorithms. Therefore, a simplified analytical model of the cantilever retaining wall system is established, in order to conduct modal analysis. Following basic assumptions are made to the simplified analysis model: ① stiffness of the cantilever retaining wall base plate is larger than upright plate, therefore ignore the impact of the base plate, the bottom of the upright plate as solid connected; ② cantilever retaining wall as a thin plate element, calculate the physical parameters of the retaining wall structure after discrete; ③ the soil is simulated as simplified to added stiffness and added damping, and soil behind retaining wall which is attached to and moves together with the retaining wall is reduced to the added mass and is concentrated at the nodes; ④ full contact between the retaining wall and soil. The simplified analysis model is shown in Fig. 1.
Figure 1: Simplified analysis model of cantilever retaining wall system

Theoretical analysis of additional parameters

Assuming the horizontal resistance coefficient and the added mass of the unit area with depth appear dual parameters nonlinear distribution [12], namely:

\[ K(z) = K_0 z^s \] (1)

\[ M(z) = M_0 (H - z)^s \] (2)

where \( K_0 \), \( t \) is the horizontal resistance coefficient parameters to be determined, respectively, \( M_0 \), \( s \) is undetermined parameters for the added mass of unit area, \( z \) is the depth of the calculation point (m), \( H \) is retaining wall height (m).

To solve expression of added parameter, according to the solution method of Du et al. [13] for the ideological foundation stiffness matrix, that is assigning element reaction force of wall-soil interaction to the four nodes of the element according to the internal force equilibrium conditions. Obtained by a separate element in Figure 1, each node of the added stiffness of the element \( i \) is:

\[ k_i^1 = k_i^2 = \frac{b \int_{z_s}^{z} K_0 z' (z - z_s) dz}{2a} = \frac{bK_0}{2a(t+1)(t+2)} \left[ z_4^{t+2} - (t + 2)z_4 z_4^{t+1} + (t+1)z_4^{t+2} \right] \] (3)
\[ k_i^j = k_4^j = \frac{b\int_{z_4}^{z_1} K_0 z'(z_1 - z) dz}{2a} = \frac{bK_0}{2a(t+1)(t+2)} \left[ z_1^{t+2} - (t+2)z_1 z_4^{t+1} + (t+1)z_4^{t+2} \right] \] (4)

where \( z_1, z_4 \) is the coordinates of the z-direction of the cell \( i \) node 1 and node 4, respectively, \( a, b \) is the cell length, other symbols have the same meaning as before.

Similarly, the added mass of each node of the element \( i \) can be obtained as follows:

\[ m_i^j = m_2^j = \frac{bM_0}{2a(s+1)(s+2)} \left[ (H - z_4)^{s+2} - (H - z_1)^{s+1} (H - (2 + s)z_4 + (1 + s)z_1) \right] \] (5)

\[ m_i^j = m_4^j = \frac{bM_0}{2a(s+1)(s+2)} \left[ (H - z_1)^{s+2} - (H - z_4)^{s+1} (H - (2 + s)z_1 + (1 + s)z_4) \right] \] (6)

After the discrete of cantilever retaining wall system, based on the finite element principle “element lumped”, added stiffness of the soil and added mass on node \( i \) can be expressed as:

\[ k_{si} = \sum k_{si}^e \quad m_{si} = \sum m_{si}^e \] (7)

According to Rayleigh damping, the added damping of the node \( i \) can be described as by Eq. (7):

\[ c_{si} = \alpha_s k_{si} + \beta_s m_{si} \] (8)

where \( \alpha_s, \beta_s \) is the mass damping coefficient and stiffness damping coefficient, respectively.

So, as already explained, the problem of additional parameters identification becomes simply the updating of the model parameters to minimize the difference between a mathematical model and the real behaviors of the actual cantilever retaining wall system. Namely, how to reasonably determine the parameter of \( K_0, \ t, \ M_0, \ s, \ \alpha_s, \ \beta_s \) is the very crucial problem of additional parameters identification.

**FINITE ELEMENT OPTIMIZATION THEORY**

**Mathematical model of optimization design**

In this paper, the research of the parameters identification is based on the finite element (FE) software ANSYS, the optimization problem can be expressed as in ANSYS:

\[
\begin{align*}
\min \quad & F = f(X) \\
\text{s.t.} \quad & X = \{x_1, x_2, \ldots, x_n\} \\
& x_i^l \leq x_i \leq x_i^u \\
& g_i(X) \leq g_i^* \quad (i = 1, 2, \ldots, m_1) \\
& h_i(X) \leq h_i \quad (i = 1, 2, \ldots, m_2) \\
& w_i(X) \leq w_i^* \quad (i = 1, 2, \ldots, m_3)
\end{align*}
\] (9)
where $F$ is objective function; $x_i$ is design variables, namely identifying parameters, $x_i^l$ and $x_i^u$ is the lower limit and upper limit of design variable; $g_i$, $h_i$ and $w_i$ is state variable of different boundary conditions, superscript $l$ and superscript $u$ represents the lower limit and upper limit of state variable; $m_1 + m_2 + m_3$ is the total number of state variables.

**Objective functions**

The simplest objective function of the FE model updating in structural dynamics is the differences between the numerical and experimental Eigen frequencies. The sum of squared relative differences has usually been used:

$$
F_1(X) = \sum_{i=1}^{n} \left( \frac{\omega_i - \omega_i^\sim}{\omega_i} \right)^2 \quad X \in (\Theta)
$$

(10)

in which $\omega_i$ and $\omega_i^\sim$ are the numerical and corresponding experimental (EXP) frequencies to $i$th mode, respectively; and $n$ refers to the number of identified Eigen frequencies that are used in the updating process.

In the same way, modal shape, $\varphi$, at different locations of the retaining wall might also be used to define the objective function (OBJ):

$$
F_2(X) = \sum_{i=1}^{n} \sum_{j=1}^{Num} \left( \varphi_{ij} - \varphi_{ij}^\sim \right)^2 \quad X \in (\Theta)
$$

(11)

where $\varphi_{ij}$ and $\varphi_{ij}^\sim$ is the $i$th numerical and corresponding experimental coefficient of modal vector with $Num$ degrees of freedom for mode $j$, respectively.

Suitable combinations of the frequencies and modal shape have also been used as the objective function (OBJ):

$$
F_3(X) = w_1 \sum_{i=1}^{n} \left( \frac{\omega_i - \omega_i^\sim}{\omega_i} \right)^2 + w_2 \sum_{i=1}^{n} \sum_{j=1}^{Num} \left( \varphi_{ij} - \varphi_{ij}^\sim \right)^2 \quad X \in (\Theta)
$$

(12)

where $w_1$ and $w_2$ is the weight of frequency and modal shape, respectively.

However, the use of dynamic response measurements is also feasible to be used:

$$
F_4(X) = \sum_{i=1}^{n} \sum_{j=1}^{L} \left( y_i(j) - y_i^\sim(j) \right)^2 \quad X \in (\Theta)
$$

(13)

where $y_i(j)$ and $y_i^\sim(j)$ is the numerical and corresponding experimental dynamic response values of $j$ moment on the node $i$, respectively; and $L$ is the length of the measured data.
The choice of the suitable objective function is a difficult task for a parameter identification procedure. Objective functions formulated according to the results of different test conditions should be checked in order to reduce the uncertainty in the damage detection procedure. Therefore, the identification results of different objective function would be analyzed to select the suitable objective function.

Optimization method

ANSYS offers a variety of optimization methods and optimization tools: random search method, zero order method and first order method. Based on the characteristics of optimization tool and optimization method, three combination optimization methods were used: random search and zero order method (method 1), random search and first order method (method 2), zero order method and first order method (method 3), respectively.

In a word, the flow diagram of added parameter identification shown in Figure 2

---

**Figure 2:** The flow diagram of added parameter identification
NUMERICAL SIMULATION STUDY

Finite element model

The analysis is based on the newly-built high-fill subgrade cantilever retaining wall on Jinxing Avenue in Chongqing. It is made by the C25 concrete and HRB335 steel bar with the thickness of 35mm concrete cover, setting a settlement joint of each 12m. The structure is shown as in Fig.3 and the parameters of main materials are listed in Table 1.

![Figure 3: The size range of retaining wall system (unit: cm)](image)

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic modulus E(MPa)</th>
<th>Poisson ratio μ</th>
<th>Density ρ(kg/m³)</th>
<th>Internal friction angle φ(°)</th>
<th>Cohesion C(KPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinforced Concrete</td>
<td>2.8×10⁴</td>
<td>0.2</td>
<td>2500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soil</td>
<td>288</td>
<td>0.3</td>
<td>1800</td>
<td>31</td>
<td>3.8</td>
</tr>
</tbody>
</table>

A whole FE model (fig. 4) and simplified FE model (fig. 5) of retaining wall system are established respectively. The analysis results of the whole FE model and the simplified FE model is used as the experimental and numerical results, respectively. The results of two models are used as the basic data of the FE model updating.
Figure 4: The whole FE model mesh  
Figure 5: The simplified FE model mesh

Results analysis of additional parameters identification

Table 2~5 show the results of additional parameters using the different combination optimization method, respectively.

Table 2: Identification results of added parameter based on objective function $F_1$

<table>
<thead>
<tr>
<th>parameter</th>
<th>method</th>
<th>Scheme 1 (method 1)</th>
<th>Scheme 2 (method 2)</th>
<th>Scheme 3 (method 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{so} (\times 10^7)$</td>
<td>2.156</td>
<td>2.211</td>
<td>2.172</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>1.483</td>
<td>1.501</td>
<td>1.904</td>
<td></td>
</tr>
<tr>
<td>$M_{so}$</td>
<td>55.808</td>
<td>59.320</td>
<td>52.487</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>0.574</td>
<td>0.577</td>
<td>0.736</td>
<td></td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>5.020</td>
<td>14.701</td>
<td>13.119</td>
<td></td>
</tr>
<tr>
<td>$\beta_s (\times 10^{-4})$</td>
<td>3.994</td>
<td>1.025</td>
<td>3.806</td>
<td></td>
</tr>
<tr>
<td>$F_1$</td>
<td>$3.596 \times 10^{-7}$</td>
<td>$6.923 \times 10^{-6}$</td>
<td>$1.202 \times 10^{-9}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Identification results of added parameter based on objective function $F_2$

<table>
<thead>
<tr>
<th>parameter</th>
<th>method</th>
<th>Scheme 4 (method 1)</th>
<th>Scheme 5 (method 2)</th>
<th>Scheme 6 (method 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{so} (\times 10^7)$</td>
<td>2.338</td>
<td>2.338</td>
<td>2.196</td>
<td></td>
</tr>
<tr>
<td>$t$</td>
<td>1.752</td>
<td>1.752</td>
<td>1.854</td>
<td></td>
</tr>
<tr>
<td>$M_{so}$</td>
<td>57.688</td>
<td>57.688</td>
<td>50.478</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>0.629</td>
<td>0.629</td>
<td>0.704</td>
<td></td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>14.586</td>
<td>14.586</td>
<td>5.020</td>
<td></td>
</tr>
<tr>
<td>$\beta_s (\times 10^{-4})$</td>
<td>1.445</td>
<td>1.445</td>
<td>1.006</td>
<td></td>
</tr>
<tr>
<td>$F_2$</td>
<td>$5.516 \times 10^{-6}$</td>
<td>$5.516 \times 10^{-6}$</td>
<td>$1.715 \times 10^{-7}$</td>
<td></td>
</tr>
</tbody>
</table>
Table 4: Identification results of added parameter based on objective function $F_3$

<table>
<thead>
<tr>
<th>method parameter</th>
<th>Scheme 7 (method 1)</th>
<th>Scheme 8 (method 2)</th>
<th>Scheme 9 (method 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{so} \times 10^7$</td>
<td>2.126</td>
<td>2.391</td>
<td>2.177</td>
</tr>
<tr>
<td>$t$</td>
<td>1.909</td>
<td>1.729</td>
<td>1.868</td>
</tr>
<tr>
<td>$M_{eo}$</td>
<td>51.526</td>
<td>58.101</td>
<td>56.918</td>
</tr>
<tr>
<td>$s$</td>
<td>0.701</td>
<td>0.742</td>
<td>0.654</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>5.001</td>
<td>5.451</td>
<td>14.979</td>
</tr>
<tr>
<td>$\beta_s \times 10^{-4}$</td>
<td>1.002</td>
<td>1.840</td>
<td>3.370</td>
</tr>
<tr>
<td>$F_3$</td>
<td>$4.451 \times 10^{-7}$</td>
<td>$2.748 \times 10^{-5}$</td>
<td>$1.077 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Table 5: Identification results of added parameter based on objective function $F_4$

<table>
<thead>
<tr>
<th>method parameter</th>
<th>Scheme 10 (method 1)</th>
<th>Scheme 11 (method 2)</th>
<th>Scheme 12 (method 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{so} \times 10^7$</td>
<td>2.246</td>
<td>2.185</td>
<td>2.172</td>
</tr>
<tr>
<td>$t$</td>
<td>1.782</td>
<td>1.831</td>
<td>1.875</td>
</tr>
<tr>
<td>$M_{eo}$</td>
<td>57.531</td>
<td>58.128</td>
<td>58.623</td>
</tr>
<tr>
<td>$s$</td>
<td>0.598</td>
<td>0.607</td>
<td>0.625</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>12.365</td>
<td>11.472</td>
<td>11.284</td>
</tr>
<tr>
<td>$\beta_s \times 10^{-4}$</td>
<td>4.136</td>
<td>3.578</td>
<td>3.256</td>
</tr>
<tr>
<td>$F_4$</td>
<td>$6.623 \times 10^{-2}$</td>
<td>$1.691 \times 10^{-2}$</td>
<td>$8.587 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table 2, 3, 4 and 5 shows the results of the damping coefficient has relatively large discretion only based on the modal parameter because the damping coefficient has little influence on the modal parameter, and its is relatively stable based on the acceleration time history response due to the damping coefficient has great influence on the acceleration response. The result of additional stiffness and additional mass is relatively stable by various combination methods.

In order to choose the optimal parameter identification result. First of all, Based on the objective function minimization principle, can obtain the corresponding optimization scheme of each objective function is scheme 3, scheme 6, scheme 9 and scheme 12, respectively. Secondly, the four objective functions of the optimization scheme are calculated, respectively. Finally, through comparing to the objective function, the optimal scheme is obtained. The result is shown in Table 6.
Table 6: Comparison optimization scheme

<table>
<thead>
<tr>
<th>OBJ</th>
<th>Scheme 3</th>
<th>Scheme 6</th>
<th>Scheme 9</th>
<th>Scheme 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>1.202×10^{-9}</td>
<td>1.108×10^{-4}</td>
<td>3.035×10^{-8}</td>
<td><strong>6.312×10^{-10}</strong></td>
</tr>
<tr>
<td>F2</td>
<td>5.852×10^{-5}</td>
<td>1.715×10^{-7}</td>
<td>7.435×10^{-8}</td>
<td><strong>1.147×10^{-8}</strong></td>
</tr>
<tr>
<td>F3</td>
<td>5.852×10^{-5}</td>
<td>1.110×10^{-4}</td>
<td>1.077×10^{-7}</td>
<td><strong>1.210×10^{-8}</strong></td>
</tr>
<tr>
<td>F4</td>
<td>2.705×10^{-2}</td>
<td>0.186</td>
<td>1.581×10^{-2}</td>
<td><strong>8.587×10^{-3}</strong></td>
</tr>
</tbody>
</table>

Through Table 6, can know that the result is the best based on the acceleration time history response, and the result is the poorest based on the modal shape.

CONCLUSION

According to the theoretical analysis and numerical simulation analysis, the following conclusions are obtained:

In order to analyze the modal characteristics of the cantilever retaining wall system, this paper presents a simplified analytical model of the cantilever retaining wall systems. Assumed that the horizontal resistance coefficient and the added mass of the element area of soil show parameter is nonlinear distribution, the theoretical formulas of the additional parameters of the soil are derived.

Three combination optimization methods and four objective functions are taken into comprehensive consideration. The result, when the third method and the fourth objective function are combined, has the highest accuracy. The results of the damping coefficient has relatively large discretion only based on the modal parameter because the damping coefficient has little influence on the modal parameter

ACKNOWLEDGEMENTS

This work is financially supported by the fund of National Engineering and Research Center for Highways in Mountain Area (The project No.: gsgzj-2013-05) and the Natural Science Foundation of China (Grant No: 51027004, 50878218).

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