Numerical Algorithms for Predicting Sediment Slides in Water Reservoirs

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ABSTRACT
Numerical algorithms are presented for predicting sediment slides in reservoirs during drawdown of the water levels. The methods are a part of a simulation model to compute flushing of sediments from water reservoirs. The slide computation is based on a visco-elastic Bingham model for the soil with a depth averaged limit equilibrium approach to find the location of the slide. The groundwater level and its effect on the slide are also computed. A differential equation is solved to find the horizontal location of the slide area. The vertical location of the slide is also computed from a differential equation together with an empirical formula for the maximum slide depth. After computing the slide plane, the Navier-Stokes equations for a very viscous fluid are solved to compute the movement of the material. Stopping the movement is also computed based on classical force considerations similar to Bishop's simplified method with Coulomb's friction law. The numerical model is tested on a 1g physical model test of a 3.8 meter high slope in a water tank where the water level is drawn down. The computed and measured slides start at the same time and follow the same formation pattern and size. The final computed geometry of the slide also fits well with a part of the measurements.

KEYWORDS: Sediments, reservoir, limit equilibrium model, slide movement, Navier-Stokes equations

INTRODUCTION
Reservoir sedimentation is today one of the main problems for the sustainable use of clean hydropower energy. A hydropower plant will need a reservoir to store water and to make the intake work properly. The turbulence and velocity of the river water entering the reservoir will decrease. The sediments in the river water will therefore settle and cause a reduction of reservoir storage capacity over time. It is estimated that 1% of the volume in the water reservoirs worldwide are lost annually due to sedimentation (Mahmood, 1987). The most common and inexpensive way of removing the sediments is by free-surface flushing. The water level at the dam is lowered and the water depth in the reservoir decreases while the velocity increases. The resulting high shear stress erodes the sediments that are flushed out of the reservoir. The geometry of the reservoir bed is often complex and contains cohesive sediments. The channel that
forms during flushing will erode vertically and laterally and cause geotechnical slides. The slides are important physical processes in the erosion, and affect the amount of sediments that are flushed out and thereby the effectiveness of the operation. A numerical model that computes the flushing must therefore also calculate the sediment slides. This is the focus of the current article.

Figure 1: Bodendorf Reservoir in Austria during flushing. The power plant is seen in the background and sediments sliding out into the river are seen in the foreground

Two and three-dimensional numerical models with simple sand slide algorithms have earlier been used to compute reservoir flushing. Jia et al. (2013) computed flow through the reservoir of the Three Gorges Dam, including effects of increased fluid density due to high sediment concentrations. The results were verified with velocity profiles measured in the field. Khosronejad (2008) computed sediment release from a square laboratory reservoir and compared the bed elevation changes with results from a physical model study. Flushing in more realistic complex hydropower reservoir geometries were modeled by Haun and Olsen (2012a,b). The results were compared with erosion patterns measured in laboratory experiments from The Norwegian Hydrotechnal Laboratory (Haun and Olsen, 2012a) and field measurements from the Angostura reservoir in Costa Rica (Haun and Olsen, 2012b). Simpler angle of repose slide algorithms were then used, where cohesion and groundwater effects were not taken into account. Haun et al. (2012) also computed flushing of the Bodendorf reservoir in Austria, but did not obtain very good results of the bed elevation changes. One possible problem could be lateral sand slides during the flushing. Such slides were observed at the Bodendorf reservoir in 2006 (Fig. 1). Maot et al. (2006) emphasized the slides so much that they only used a 1D model for the computation of the water and sediment flow in the flushing canal. A more complex method derived from the Bishop (1955) was used to analyse the slides. Maot et al. (2006) also did a very detailed field analysis of the cohesive parameters of the sediment deposits, which gave a good basis for the computation of the slides.

A large number of numerical methods exist for computing slides in soil. Stiansson et al. (2011) divided the approach in two paths: one is to compute the forces in the ground using a
three-dimensional Finite Element stress-strain model (Potts et al., 1997; Pastor et al., 1990; Li, 2007; Lane and Griffiths, 2000; Berligen, 2007, Lyamin and Sloan, 2002a,b; Krabbenhof et al., 2005; Li et al., 2010). Lane and Griffiths (2000) and Li (2007) made design charts for slopes using this method. A plasticity model was used and the glide plane was computed directly (Zienkiewicz et al., 1975; Krabbenhoft and Lyamin, 2012). The stress-strain computation can also be combined with a level set method is used to find the slide plane (Stolatska et al. 2001). In geotechnical literature the stress-strain computation is often called a Finite Element method, although the stress-strain equations can also be discretized using a Finite Volume method (Demirdic and Martinovic, 1993).

The other main path computing the slide location is the limit equilibrium model, where the ground is divided into vertical slices and the forces on each element is calculated (Jia et al., 2012; Lam and Fredlund, 1993; Stiansson et al., 2011; Xie et al., 2003; Loehr et al., 2007; Hovland, 1977; Wright et al., 1973). Viratjandr and Michalowski (2006) computed the safety factors for the slope of an earthfill dam during drawdown, and made design charts to assess this problem. The advantage of the Limit Equilibrium model is that it is simpler and takes less computational time that a stress-strain analysis. However, Gylland et al. (2011) points to the advantages of using a finite element method that a more complex topography can be taken into account together with easier incorporation of spatial variations in soil properties. The sediment slides in reservoirs are usually of limited horizontal magnitude, and the soil parameters are homogenous relative to the landslides Gylland et al. (2011) computed.

A disadvantage with the 3D Limit Equilibrium Model is that the geometry of the slide is approximated as a part of an ellipsoid or other pre-described surface (Wei et al., 2009). This may be a good approximation for many land slides, but the reservoir slides may be shaped differently. In a reservoir the cohesive banks will follow the river and have a larger magnitude in the horizontal direction normal to slide path. The assumption that they follow a pre-described 3D ellipsoidal shape may therefore cause inaccuracies. Another problem is that it is difficult to predict in advance where in the reservoir the slides are formed. There will most likely be a large number of slides located throughout the banks and the steepest part of the reservoir. An alternative approach to computing the slide plane geometry and its location is therefore suggested in the current study.

The 3D stress-strain analysis model and the Limit Equilibrium approach are designed to compute the safety factor of a slope, and they also predict the location of the glide plane. However, the computation of the movement of the slide requires additional algorithms. The most used approach is to extend the 3D stress-strain analysis with an elasto-plastic model (Demirdzic et al., 1993). This can be done with finite elements or finite volumes. An alternative method was used by Usuoka et al. (1998), who used the Navier-Stokes equations to compute the lateral spreading of soil liquefied by an earthquake. Uzuoka et al. (1998) included four cases in their paper: Three slides due to earthquakes and one slope failure where construction machinery was believed to trigger the slide. Hengbin et al. (2013) also used the approach of Usuoka et al. (1998) to compute land slides. The method used in the current paper also builds on solving the Navier-Stokes equations to predict the sediment slide movement.

The sand slide model was included into a computer program that calculates water and sediment flow in a reservoir (Haun and Olsen, 2012a,b). This model solved the Navier-Stokes equations in three dimensions together with the convection-diffusion equation for sediment concentration. A finite volume method was used for the discretization on a three-dimensional unstructured grid. The bed levels of the grid were computed from echo soundings of the
bathymetry, so procedures for making the computational grid and the geometry was therefore available for the current study.

One of the reasons for choosing the Navier-Stokes solution instead of an elastic-plastic model was that our sediment transport program already had such a solver for computing the water flow field in the reservoir. It was therefore convenient to also use this solver for the soil movement. Then the glide plane could be computed with the less complex Limit Equilibrium model instead of solving the stress-strain equations.

TEST CASE: JIA ET AL. (2009)

The use of laboratory experiments is very useful in validating numerical models. A laboratory experiment is usually more instrumented than what is possible in the field, enabling more parameters to be used in the verification. Also, better information about initial conditions and soil parameters are most often available.

There exist several cases where centrifuges have been used (Dewoolkar et al., 2003) to model slope failures, but also some experiments have been done at 1g. One of the physically largest laboratory studies on sand slides was carried out by Jia et al. (2009). This was chosen as a test case for the current study. The experiment was done in a 15 long, 5 meter wide and 6 meter high tank. A soil mixture of 12 % sand, 80 % silt and less than 5 % clay was filled to the level 6 meter on the left side of the box, and to 2 meters on the right side. In between, there was soil forming a straight slope at 1:1. A longitudinal profile is given in Fig. 2. The soil was allowed to settle for two months before the experiment started, causing consolidation of the material. Jia et al. (2009) measured the internal friction material of the soil to be 30 degrees, and the cohesion to be 1000 Pa. These data were used as input for the numerical model.

The test was done in two parts: First, water level was slowly filled in the tank to the top of the soil level. This was done over a period of 8 days. The filling caused a 0.2 m lowering of the top of the soil, from 6 meters to 5.8 meters. Also, the slope angle was reduced from 45 to 33 degrees. Then the water level was kept above the top of the soil for three days, before it was drawn down over a period of 3.7 hours, with a speed of 1 m/hour. The soil slide occurred during the drawdown phase. Figure 2 shows the ground level before and after the slide. It also shows three slide planes observed in the laboratory study. The laboratory tank had five windows at the side of the flume, enabling the observations of the slide planes. The first slide occurred after 0.7 hours, while the other two took place later. The upper part of the slope surface shows three distinctive layers, corresponding to the three slides. The average angle of the slope at the upper part of the layer after the drawdown is 22 degrees, reduced from the initial 33 degrees by the slides.
Figure 2: Longitudinal profile from the experiment of Jia et al. (2009) showing the ground level before and after water drawdown, together with the slide planes.

Figure 3 shows the ground level before and after the drawdown, without the slide planes. Also, notations are given for the areas between the curves. The slide causes erosion of material at the upstream part of the slope, which is deposited downstream. From the figures given by Jia et al. (2009), it is possible to compute the areas between the curves, $A_1$ for the eroded area and $A_2$ for the deposited area. The ratio of $A_2/A_1$ is a factor 3. This means that the soil that deposits in the downstream part of the slope has a much higher water content than at the erosion area. The porosity increases from 50 % to 83 %. This water entrainment process will cause changes in the soil properties, such as effective density, cohesion and possibly also other parameters. To model the water entrainment process and the changes in the downstream earth parameter would require a transport model for the porosity, together with a model for the water entrainment into the soil. This model would need to be calibrated with measurements of the soil parameters after the slide occurred. This information is not given by Jia et al. (2009). Due to the increased complexity and the lack of data, the process is not modeled in the current study. The computed ground elevation at the lower part of the slope will therefore not be correct. For the purpose of estimation of sediment slides in reservoirs, it will still be useful to be able to assess the volume of sediments moved and an estimate of the ground slope upstream of the slide.
Figure 3: Longitudinal profile of the ground levels before and after the slide. The areas between the curves are a measure of how much the slide eroded ($A_1$) and deposited ($A_2$).

### NUMERICAL MODEL

The numerical model is made up of several sub-modules. First, the groundwater level needed to be computed. Then the location of the slide and its magnitude in the horizontal direction was calculated. The depth of the slide was computed afterwards, and finally, the slide movement.

#### Groundwater model

The groundwater level, $z_r$, was computed by a 1D approach with depth-averaged cells by solving the following equation:

$$\frac{\partial z_r}{\partial t} = \frac{\partial}{\partial x} \left( K \frac{\partial z_r}{\partial x} \right)$$  \hspace{1cm} (1)

$K$ is the hydraulic conductivity and $x$ is a space dimension. A finite volume method was used for the discretization of the equation. The transient term on the left side of the equation was solved with a Crank-Nicholson method. The term on the right side was discretized with a central scheme. The water level in the tank was used as a boundary condition for the computation.

One question was how the groundwater level in the slide changed during the soil movement. In the current study, the level was moved downwards with the soil. The soil was assumed have a linearly distributed vertical velocity from the computed value at ground level to zero at the glide plane. The vertical soil velocity at the ground level was calculated by solving the Navier-Stokes equations, as described later. Also, the groundwater level was not set lower than the neighbor cell in the flow direction.
Computing the horizontal location of the slide

The Limit Equilibrium Model is based on a 1D grid of vertical columns. The force, \( F \), on each column in the direction following the ground is computed according to a simplified Bishop (1955) approach:

\[
F = (1 - p) A_z (\rho - \rho_w) g h \tan(\theta) + A_z \rho_w h_w \tan(\psi) - (1 - p) A_z (\rho - \rho_w) g h \tan(\phi) - A_z \tau_c / \sin(\theta)
\]

(2)

The area of each cell normal to the vertical direction is denoted \( A_z \) and the distance from the ground surface to the glide plane is denoted \( h \). The soil porosity is given as \( p \). The distance from the glide plane to the groundwater level is denoted \( h_w \). The buoyancy of the submerged particles is taken into account by subtracting the water density, \( \rho_w \), from the particle density, \( \rho \) (Janbu, 1970). The slope of the glide plane is denoted \( \theta \) and \( \phi \) is the friction slope. The cohesive shear strength is denoted \( \tau_c \). The gravitational acceleration is denoted \( g \). Eq. 2 has four terms. The two driving forces are the decomposed gravity of the soil particles, given as the first term on the right side of Eq. 2. The second term is the groundwater forces, which is also a driving force. The third and fourth terms are due to friction and cohesion, according to Coulomb's law. These two forces stabilize the soil.

The Limit Equilibrium Model is often used to compute a safety factor for a potential slide. In the current work, the safety factor in itself is not of interest, only the location of the slide plane. This is reflected in the form of Eq. 2, which does not include a safety factor.

Eq. 1 does not take intercolumn forces into account. The classical limit equilibrium model sums up the forces on all columns and the intercolumn forces are eliminated in order to compute a safety factor for the slope. In the current case, the safety factor is not of interest. To predict the extension of the slide, the intercolumn forces have to be taken into account. This is done by using Hook's law

\[
F_w = A_w \sigma_w = A_w E \epsilon_w = A_w E (\delta_w - \delta_p) / \Delta x
\]

(3)

The two columns are here called \( W \) (west) and \( P \), \( A_w \) is the area of the surface between the columns, \( E \) is the module of elasticity, and \( \Delta x \) is the distance between the centers of the two columns.

In the following, the column \( P \) is considered, which has two neighbours in the two directions east (E) and west (W). The sum of the forces on one element is then a combination of Eq. 2 and Eq. 3:

\[
F_w - F_E = F_s
\]

(4)

This gives the following equation for \( \delta_p \):
where

\[ a_w = E A_w / \Delta x \]  

(6)

\[ a_p = a_w + a_E \]  

(7)

The current study assumes a two-dimensional situation, a unit width can is chosen in the third direction. Then the areas in Eq. 2, 3 and 6 can be replaced by the slide depth, \( h \) and the horizontal lengths of the columns, \( \Delta x \).

Equations 2 and 3 describe forces in the horizontal direction following the ground level, and need to be combined into one equation. There are two unknown in the equation: \( h \) and \( \delta_x \). Ideally, the \( \delta_x \) should be removed and the equation should be solved with respect to \( h \). In the current approach, the \( h \) is instead partly eliminated by dividing the whole equation by \( h \). This gives the following equations for the source terms and the \( a_w \) coefficient:

\[
F_s = (1 - p) \Delta x (\rho_s - \rho_w) g \tan(\theta) + \Delta x \rho_w g \tan(\psi) - (1 - p) \Delta x (\rho_s - \rho_w) g \tan(\phi) - \Delta x \tau_c / (h \sin(\theta))
\]  

(8)

\[
a_w = E / \Delta x
\]  

(9)

Some simplifications has been done going from Eq. 5 to 8. It is assumed that the groundwater height is the same as the height of the slide. Furthermore, the slide depth, \( h \), is not completely eliminated from Eq. 8, as it is still included in the cohesive term. Initially, \( h \) is set equal to the cohesive height, \( H_c \) defined as:

\[
H_c = \frac{10 \tau_c}{[\rho_s (1 - p) + \rho_w p] g}
\]  

(10)

Eq. 10 formula is based on a vertical force balance for an almost vertical soil wall with only cohesive material. When a slide develops, \( h \) in Eq. 8 is set equal to the maximum of \( H_c \) and the computed slide depth in the previous iterations. The computation of the slide depth is described later.
A simplification in the current method is that the ground angle is used for $\theta$ in Eq. 8 instead of the angle of the glide plane. This gives a more stable solution, as this angle will change less over time depending on if the slide is moving or not.

Given that the slope is negative in the positive $x$-direction, the $\delta_x$ values in Eq. 5 should always be positive. However, the negative cohesion term will also give negative $\delta_x$ values far away from the slide. To avoid this, the solver is incorporated with a condition that $\delta_p$ is always larger than 0. This is simply done by adding the following function to Eq. 5:

$$\delta_p = \max \left[ 0, \left( \frac{a_w \delta_w + a_e \delta_e + S}{a_p} \right) \right]$$ (11)

Solving this equation gives positive values for the columns where there are movement of the material and zero values for columns with no movement. The approach is tested on a slope which initially is steeper than the angle of repose and the material has no cohesion. This results in a slide length that is equal to the bank height multiplied with the tan($\phi$).

The soil in the slide will have lower strength after it has started to move than in the initial condition. One of the soil parameters could be altered to reflect this. For loose sand on a slope, the steepest inclination will be equal to the friction angle even if there are small slides. Therefore, the friction angle should not be altered during the slide. The only way of reducing the shear strength of the soil is therefore to reduce the cohesion. The simplest way is to multiply the cohesive terms in Eq. 2 and 8 with a factor between 0 and 1 in the soil where slides have occurred. The value of 0.2 was chosen in the current study. The value is discussed later in the article.

**Vertical extension of the slide**

The vertical extension of the slide also needs to be determined before computing the soil movement. The slide shape is often assumed to follow a part of a circle (Bishop, 1955) or an ellipsoid. At least it should be a smooth curve that approximates these shapes. In the current study, the slide depth is computed by solving another convection-diffusion equation for the level, $z$, of the slide plane in each of the 2D cells that has positive $\delta_x$ values:

$$z_p = 0.5z_w + 0.5z_e + S$$ (12)

The boundary conditions are fixed values at the start and end of the slide, equal to the ground level. The source term, $S$, is negative and has to be determined. One method is to compute the safety factors for the slide planes with two guesses of $S$, the second being larger than the first. If the second guess gives a lower safety factor, the source term is further increased. If the safety factor is higher, the source term is decreased. The iterations are done until the changes in safety factor are below 0.1%. However, the procedure was prone to instabilities.

An alternative method was to compute the maximum depth of the slide with an analytical formula, and then change $S$ so that the computed maximum emerges. The method to change the source term is the same as when using the safety factor. This method was much more stable. However, some empirical formula for the depth was needed. The most straightforward method was to say that the slide depth was a constant times the slide length, $L$: 
\[ H = kL \]  

Looking at the soil physics, the slide plane will be close to straight for non-cohesive material. The depth will then be zero. For completely cohesive material with zero friction angle, the most extreme slide plane will be a semi-circle, with the slide depth equal half the length. The parameter \( k \) will therefore vary between 0 and 0.5, depending on soil friction factor and cohesion. The following formula can be made for \( k \):

\[ k = \frac{1}{2} \left( \frac{H_c}{H_c + H_f} \right)^A \]  

\( H_c \) has been defined in Eq. 10. The \( H_f \) is a friction height [m], defined as:

\[ H_f = L \tan(\phi) \]  

\( A \) is a constant to be determined empirically. The laboratory study by Jin et al. (2009), Fig, 19, gives the slide depth as 1 meter with a slide length of 6 meters. This means \( k \) is 1/6. Jin et al. (2009) measured \( \tau_c \) to be 1000 Pa, and the friction angle \( \phi \) to be 30 degrees. Based on this we can compute \( A \) to be 0.56 for the current case.

Note that the horizontal location of the slide is not altered by solving Eq. 12, only the depth of the slide will be computed. The solution of Eq. 9 will always give a parabolic shape of the slide plane, and the magnitude of the source term, \( S \), will only affect the depth of the slide. Unrealistic slide plane shapes will therefore not be computed (Chug, 2002).

As stated earlier, there are some approximations in deriving Eq. 8 from Eq. 2, causing some reduction in the accuracy of the method. Therefore the safety factor is computed after the slide geometry is determined. If the safety factor is above 1, the slide is stopped. The safety factor is based on the classical simplified Bishop method with Eq. 2.

**Soil movement – Navier-Stokes equations**

After a slide plane is computed, and it is determined that the safety factor is below 1, the movement of the soil must be computed. The algorithms described in the current paper were coded in the SSIM 2 model (Olsen, 2013). The soil slide was assumed to move according to a very viscous fluid, where the velocities could be modelled by the Navier-Stokes equations:

\[ \frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} - g = \frac{\partial}{\partial x_j} \left( \nu \frac{\partial U_i}{\partial x_j} - P \delta_{ij} \right) \quad i=1,2 \]  

\( U \) is the velocity of the soil, \( x \) is a space dimension, \( t \) is time, \( g \) is the gravity, which only works in the vertical direction, \( \nu \) is the viscosity, \( P \) is the pressure and \( \delta \) is the Kronecker delta.
The equations were solved using a finite volume method, using a first-order upwind method to compute the convective term. The pressure was computed with the SIMPLE method (Patankar, 1980). In the current study, the water surface movement was computed by a volume of fluid method (Haun et al., 2011). The index \( i \) in Eq. 16 is for the horizontal and vertical direction in the current study.

The shear friction, \( \tau \), in the slide plane can be computed as a sink term in Eq. 16. From a numerical point of view, the most stable approach is to compute the shear stress as a viscosity times the velocity gradient, and include it in the viscous term which is the first term on the right side of Eq. 16. This means that the friction is taken into account by the viscosity. This approach requires a relationship between the shear friction and viscosity. Following earlier work of Uzoka et al. (1998), Moriguchi et al. (2009 and Hengbin et al. (2013), the soil was computed as a visco-elastic Bingham fluid in the current study. Complex relationships between these parameters exist (O’Brien and Julien, 1998), but a simple model was chosen in the current study:

\[ \frac{d}{dt} \frac{\tau}{U/i} = \frac{\partial}{\partial z} \left( \frac{\tau}{U/i} \right) \]

The shear stress was computed based on Columbs law:

\[ \tau = \tau_c + \tan(\phi)(1 - p)[(\rho_s - \rho_w)gh_w + \rho_sgh(h - h_w)] \]  

The velocity gradient was computed as a velocity divided by half the height of the cells bordering the shear plane. The velocity, \( U \), was computed as an average value in the slide cells, \( U_0 \), and to this value, an acceleration term from Newtons 2nd law was added:

\[ U = U_0 + \Delta t F_i / m \]

The mass of the soil and water in the slide cells is denoted \( m \), \( F_i \) is the gravitational force on the slide in the slide direction and \( \Delta t \) is the time step in the computation. \( F_i \) was computed from a Safety Factor (SF) obtained from Bishops (1955) simplified method:

\[ F_i = \tau(1 - SF) \sum A_z \]

The viscosity computed by Eq. 17 during the slide was around 200-1000 kPa sec.
This viscosity was used in the cells bordering the slide plane. For the cells above, a viscosity of 10 times this value was used in order to model the higher stiffness of the soil above the glide plane.

**Computational grids**

The algorithms described in the current paper were coded in the SSIIM 2 model (Olsen, 2013). This model had an existing 3D grid for the water in the reservoir. In the current study, only a 2D slice is computed, so the water flow grid will also be two-dimensional (Fig. 4). The
additional grid requirement for the current study was a 1D grid for the computation of the groundwater level and the horizontal and vertical location of the slide plane. Also, a 2D grid for the soil movement was needed. Figure 4 shows all these three grids.

**Figure 4:** Computational grids. A is a depth-averaged grid used to compute the groundwater level and the horizontal strain. B is a 2D grid for the computation of the slide movement. C is the grid for the computation of the water flow in the river adjacent to the sliding bank. Grid C is not used in the current study, except to give boundary values for the groundwater model.

As the sediment slide module was an addition to the already existing model, grids A and B were made in one block, with a no-slip boundary condition between them (marked with a red line in Figure 8. The grids were regenerated every time there was a movement in the water level in the reservoir, slide plane or the ground level. Note that the grid has completely vertical lines, so the grid movements meant only vertical adjustments of the cells in addition to variation of the number of cells in the vertical direction.

The existing model to compute the water flow and sediment movement in the river uses Grid C in Fig. 4. This model will for example compute erosion at the outside of a bend (Fischer-Antze et al., 2008), increasing the height and steepness of the slope. The coupling of the two models has not yet been tested.

**RESULTS**

The computations were done in the same order as the experiment: First the groundwater level was computed during the phase where water was filled in the tank. As the groundwater level was one of the main driving forces for the slide to form and move, it was important to assure that the numerical model could compute this correctly. The second part was the computation of the slide movement during the drawdown phase.
Groundwater levels during filling

The experiment by Jia et al. (2009) started with filling water in the tank with the soil. The water level was initially at the base of the slope, at 2 meters. During 96 hours it was raised to 5.8 meters. The computation was done with a time step of 10 seconds, with 34560 time steps.

![Computed groundwater levels during filling.](image)

The key to getting correct results for the groundwater level is the choice of the hydraulic conductivity coefficient. Jia et al. (2009) conducted laboratory measurements of the coefficient and found it to be between $4.2 \times 10^{-6}$ and $6.4 \times 10^{-6}$ m/s, with an average value of $5.3 \times 10^{-6}$ m/s. All these three values were tested in our numerical model, and $4.2 \times 10^{-6}$ m/s gave the best results, as seen in Fig. 5. This value was therefore used in the computation of the slide movement.

During the filling, the crest of the slope settled, so it was 0.2 meters below its original level. The angle of the slope also decreased. None of these changes were taken into account in the numerical model. It was assumed that the geometry was similar to the initial condition during the filling.

Slide movement

The slide movement was computed with the algorithms, coefficients and initial values described earlier in the paper. It was assumed that the initial level of the top of the slope was at 5.8 meters, as reported by Jia et al. (2009), and that the initial slope was 33 degrees. The initial water level was at the crest of the slope, and was drawn down at a rate of 1 m/hour.

The observed slide geometry is sketched in Fig. 2. Three slides were observed following each other, giving rise to three flat areas at the top of the slide geometry (Fig. 2). The first of the slides started after 2500 seconds. Figure 6 and 7 shows results from the numerical model. The first slide computed to emerge after 2100 seconds (Fig. 7), shortly before the observed slide. The length of the slide was similar to what was observed in the laboratory. The computed depth of the slide was also similar to what was observed, but this only shows that Eq. 12-14 was implemented correctly. Fig. 7 shows three computed slides that are located similar to what was observed in the physical model. However, the slide formation in the numerical model was more complex than this. The
slide did not move continuously over time. They emerged for some minutes, moved the slope and then disappeared. Then later a new slide emerged. Also, sometimes smaller slides emerged between the times of the larger slides in Fig. 7.

**Figure 6:** Computed ground level at the end of the drawdown of the water level, compared with the measured ground level before and after the drawdown.

The final ground elevation geometry is given in Fig. 6. The eroded volume corresponds well with the measurements. The deposited volume is much larger than what was modeled, but this is due to water entrainment. This process was not modeled numerically.
Figure 7: Velocity vectors for the slides. The upper figure is after 2100 seconds, the middle after 5700 seconds and the lower after 7800 seconds. The red lines indicate boundaries between the grids, and the blue line is the water level in the reservoir and the groundwater level. The original ground level is also indicated with a green line.

Looking in detail at the erosion area in Fig. 6, the measured ground level shows a more edgy pattern than the results from the computation. The edges are due to the three distinct slides observed in the physical model. At the top of the slide, the soil is relatively stiff and does not change in shape as it does further downstream. Looking at Fig. 7, there are also some edges in the numerical results after 5700 and 7800 seconds. However, they vanish at the final result. The
reason is that the elevation changes are computed in the middle of a vertical cell, while the actual movement of the grid takes place in the intersection between the grid lines. An interpolation from the center of the cell to the grid lines has to be done, and when this is done many times, the edges disappear.

**DISCUSSION**

The purpose of a numerical model is to predict a geotechnical situation and how it will develop in the future. Then the input parameters can then not always be calibrated. It is therefore an advantage if the model has as few calibration parameters as possible, and clear guidelines for how they can be found. The current model includes physical input parameters that can be found by laboratory analysis: angle of repose, cohesion, density and initial porosity. However, one of the model calibration parameters used in the current study are related to the changes in the soil characteristics as it fluidizes. This is the reduction factor for the cohesion in the soil when the slide moves. The factor is a user-specified between 0 and 1. The value 0.2 was used in the current study. To assess how the factor affects the final results, a parameter sensitivity study was carried out with variation in the parameter. The results are shown in Fig. 9. A factor of 0.1 will also give a fairly similar result as 0.2. Choosing extreme values like 0.0 or 1.0 will give somewhat poorer results.

Another question is how the viscosity is computed in the current model. Initially in the study, a constant viscosity model was tested, where the user specified the value. Interestingly, the main effect of the viscosity was its direct effect on the speed of the slide. If a too high value was used, the slide would move so slow that it did not reach the final geometry. The computed erosion would be too small. If a too low viscosity was specified, the slide would move in a series of short rapid movements, with no movements in between. This is also how the currently computed slide moves. The paper from the experiment (Jia et al., 2009) stated that the observed vertical movement was 0.25 mm/s, but not if this was a constant velocity or an average of several short-lasting slides.
Figure 8: Parameter test for the reduction factor in the cohesion during the slide movement. The ground level at the start of the experiment is shown together with computed ground levels at the end of the simulation.

An alternative to Eq. 17-20 for computing the viscosity is to assume a velocity of the soil in the slide and estimate a shear stress in the shear plane. And then to use Eq. 17 to compute a constant viscosity that is used during the whole computation. If we assume the slide moves with the drawdown of the water level in the tank, the vertical component would be 1 m/hour or 0.28 mm/s. Assuming a slope of 30 degrees, the horizontal velocity would be 0.48 mm/s. The bulk density of the soil/water in the slide would be around 1800 kg/m³. If the thickness of the slide is 1 meter, the shear stress on the slide plane can be computed from:

\[
\tau = \rho gh \tan(\theta) = 10 \text{kPa}.
\]  

(21)

Assuming the glide plane is 5% of the slide depth, \( \Delta z \) would be approximately 50 mm. This gives a velocity gradient \( \Delta U/\Delta z \) of 0.01 sec\(^{-1}\) and a viscosity of 1 MPa sec. This is a bit higher than what was predicted with the other procedure, but it explains why the computed slide does not move continuously.

The advantage of Eq. 17-20 is that the method does not require a user-specified viscosity.

CONCLUSIONS

A numerical model for prediction of sediment slides in reservoirs is presented. The model is able to find the location of slides in a given ground geometry and predict the slide length. The model includes a new formula for the depth of the slide, based on its length and the cohesion and friction angle of the soil. The model also replicates changes in groundwater levels and takes this information into account when computing the slide. The model is capable of computing the slide movement and the location of the ground after the slide, in the eroded areas. This is based on formulas for computing the effective viscosity of the slide. The results from the numerical model
compare well with some of the results of experiments from a physical model study, with respect to slide location and horizontal and vertical magnitude. The changes in the ground level at the upper part of the slide are also reasonably well computed. But the model has not been able to compute the water entrainment into the moving slide and the geometry of the soil deposited after the slide.

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