Influence of Under-reamed Pile Groups Arrangement on Tensile Bearing Capacity using FE Method

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ABSTRACT

Under-reamed (Belled) piles are useful methods to increase the bearing capacity in deep foundations. They have some enlarged bases in their stems. They are suited for soils which are often subjected to considerable ground movements due to seasonal moisture variations. Also group arrangement is an effective parameter in application of these piles. In this paper using finite element software PLAXIS 3D Foundation, the group arrangement is investigated in order to reach the maximum bearing capacity. First the numerical model is verified, then some groups including 3, 5 and 7 piles with spacing/diameter (S/D) ratios of 4 to 12 for triangle, square and pentagon plan are subjected to tensile loading. The optimum S/D ratio which is related to the angle between piles in plan is obtained 9 for triangle and 5 for pentagonal arrangement. As a result pentagon arrangement comparing to square arrangement have 16% increases in bearing capacity in the same area with the same number of piles. Triangle arrangement is also more efficient than a symmetric square arrangement with the similar area.

KEYWORDS: Under-reamed Pile group, Arrangement, Angles, Tensile loading, Finite element method

INTRODUCTION

Under-reamed piles are bored cast in-situ concrete piles having one or more bulbs formed by enlarging the pile stem. They are used to increase tip strength of compressive piles and bearing capacity of tensile piles, so they have advantages over uniform diameter piles. The bulbs can be provided at desired depths where substantial bearing or anchorage is available. These piles like other types of bored piles can be used under situations where the vibrations and noise caused by driving of piles are to be avoided or strata of adequate bearing are so deep that they are difficult to reach by driven piles [1]. Bearing capacity of piles is different when they are working together as a group. Group effect is related to the spacing of group piles. There are varieties of equations to calculate ultimate bearing capacity of piles [1], [4], [5]. Piles and pile groups are considered in many researches. Indians were among the first researches on under-reamed piles. One of the researches on under-reamed pile capacity is done by Sharma [1] in a handbook for under-reamed pile design (1978). He mentioned that when tensile load is applied, the bulb in pile stem acts as a moment and increase potential of bearing. Gupta and Sandaram (1986) [16], in a study on clayey and silty soils, found when soil moisture around the bulb is near the liquid limit, the force causing failure in the pile is enough lower than calculated quantities and it's because of soil resistance reduction around the bulb and as a result tension disordering in this zone. Makarchian and Peyman Khademi (1388) [17], in a laboratory study on effective parameters of uplift force in sandy soil found that the most effective items on increasing tensile bearing capacity is respectively as follow: type of the pile, embedment length, relative density of sand and pile diameter. Zahra Mardani, et al. (2013) [18], numerically studied on tensile bearing capacity of under-reamed pile using finite element method and results was shown that under-reamed piles have a greater bearing capacity comparing to normal piles with uniform stem with the same volume and length. Nabil Esmael [9] worked on axial load tests in 2001, in the research the behavior of bored pile groups in cemented sands was examined by a field testing program at a site in South Surra, Kuwait, Test results on single piles was indicated that 70% of the ultimate load was transmitted in skin friction that was uniform along the pile shafts. C. Y. Lee's research [10] was about settlement and load distribution analysis of under-reamed piles (2007), also elastic behavior of under-reamed piles in homogeneous soils was presented in the research. The modified boundary element method was used to obtain parametric solutions of under-reamed piles under axial loading. Consideration is given to the effect of pile slenderness ratio, pile-soil relative stiffness, bulb diameter and bearing stratum on response of under-reamed piles. The characteristic of load distribution along the pile length was also studied. It was found that the most efficient means of reducing under-reamed pile settlement occurs when the enlarged base is resting on stiffer soil stratum. In 2011, K. Watanabe, et al. [11], worked on static axial reciprocal load tests of cast-in-place nodular concrete pile and nodular diaphragm wall. As a result of load tests, the nodular cast-in-place concrete pile and nodular diaphragm wall have large tension and compression resistance. The tension resistance at the nodular part and under-reamed part shows a large value. Srilakshmi and Kurian [12] studied on geometrical features of under-reamed piles using finite element method by ANSYS software (2010), maximum spacing between two piles was reached 1.75 m for bulb diameter of 0.75 m, stem diameter of 0.3 m and pile depth of 3.5 m. Hamed Niroumand, et al. [13] studied on uplift capacity of enlarged base piles in sand (2012). It was observed that the ultimate uplift capacity is depended on the relative undrained/drained shear strength of cohesionless soil and the depth ratio of embedment and soil thickness ratio. Dilip Kumar, et al. [14], worked on uplift capacity of pile by finite element method (2004), in the research the load carrying capacity of an individual pile was found more than that of a pile in a group in case of piles under uplift load and of varying cross-section. Yun-gang Zhan, et al. [15], modeled vertical bearing capacity of piles by ABAQUS (2012), results showed that the confirmed relation between axial force, shaft resistance, self-weight of pile, base resistance, and vertical bearing capacity can be used to study the mutual effects of shaft and base resistance based on numerical result.

In a review of last researches, it seems that they are done all about a specific arrangement, specific type of groups, and symmetric arrangement in plan or an individual under-reamed pile. Still no comprehensive research is done on group arrangement or spacing of under-reamed piles in group. In this research 4 types of pile group arrangements are considered containing 3, 5 and 7 piles. They are compared with each other to find out an optimized pattern to reach the maximum bearing capacity in saturated sandy soil. Figure 1 shows these arrangements.

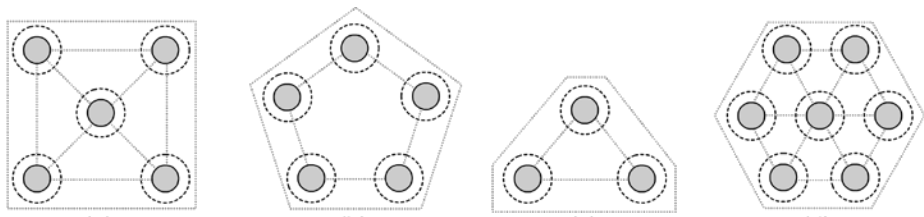


Figure 1: Pile group arrangements; (a) Square, (b) Pentagon, (c) Triangle, (d) Hexagon

MODELING VALIDATION

The modeling is validated according to existing equations, an illustrated example which is manually calculated in a handbook of under-reamed pile foundations prepared by Devendra Sharma, M. P. Jain, Chandra Prakash (1978) [1], is first compared to a finite element model containing the same conditions of the example. Results are shown the same results in the both of numerical model and the example of under-reamed pile. Equation 1 shows the formula which is used in this example [1] and figure 2 shows a comparison between the results.

$$Q_u = A_p.\left(\frac{1}{2}D.\gamma.N_\gamma + \gamma.d_f.N_q\right) + A_a\left[\frac{1}{2}D_u.n.\gamma.N_\gamma + \gamma.N_q.\sum d_r\right] + \frac{1}{2}\pi.D.\gamma.K\tan\delta\left(d_1^2 + d_f^2 + d_n^2\right) \quad (1)$$

where Q_u (kg) is ultimate bearing capacity, A_p (cm^2) is cross sectional area of pile stem at toe level and it is equal to: $(\pi/4).D^2$, D (cm) is stem diameter, A_a (cm^2) is cross sectional area of pile at bulb level and its equal to: $(\pi/4).(D_u^2 - D^2)$, D_u (cm) is under-reamed bulb diameter, Nγ , Nq are bearing capacity factors depending upon the angle of internal friction φ, given in existing tables in foundation engineering books [1], [8], n is number of under-reamed bulbs, d_r (cm) is depth of the center of different under-reamed bulbs below the ground level, d_f (cm) is depth of below ground level, K is earth pressure coefficient (usually taken 1.75 for sandy soils), δ is the angle of wall friction (may be taken equal to the angle of internal friction φ), d_1 (cm) is depth of the center of first under-reamed bulb, d_n (cm) is depth of the center of the last under-reamed bulb.

To working out the uplift loads from the formula, there will be no contribution by the pile toe as the direction of load application is reversed. Thus the first term of the formula will not occur. However the ultimate loads by this method may be found slightly on higher side. For determining the safe loads in uplift the recommended factor of safety is 3. And if the pile is a single bulb pile, the third term is neglected [1]. Figure 2 shows a comparison between the model and the example of an under-reamed pile under tensile loading.

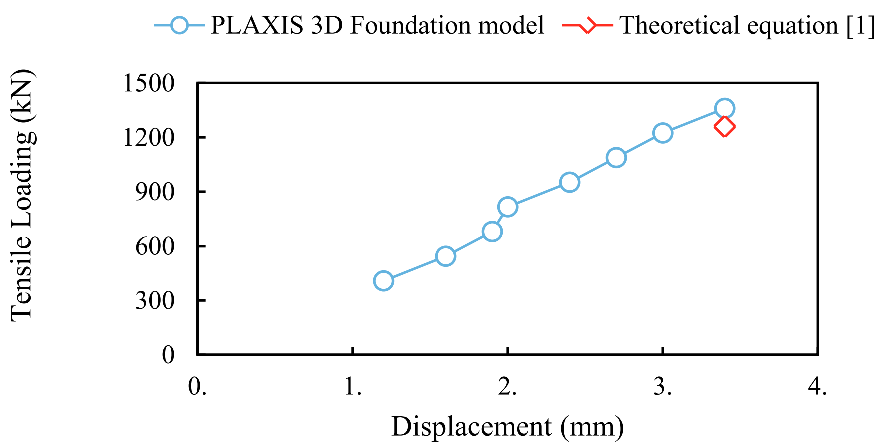


Figure 2: Comparison between the FE model (by PLAXIS 3D Foundation) and the example based on equation, for an individual under-reamed pile.

In the other validation, theories of designing uplift capacity of enlarged base pile that are derived from researches conducted by Majer (1955), Balla (1961), Meyerhof and Adams (1968), Ovesen (1981), Dickin and Leung (1990, 1992) and so on gathered in a research by Hamed Niroomand, et al. [13] is used. The research is done on sand; amendment on the design method has been made from time to time by taking into account more factors to produce more complete and accurate design. Generally, enlarged base pile gain uplift capacity through skin resistance between the pile shaft and the soil and bearing on top of the base [13], the net ultimate resistance of a pile subjected to uplift forces:

$$Q_{un} = Q_{ug} - W \quad (2)$$

where Q_{un}= Net uplift capacity, Q_{ug} = Gross uplift capacity, W = Effective weight of pile. The net uplift capacity of enlarged base pile can be calculated from formula generated by Meyerhof and Adams (1968) and Das Seeley (1975) [14];

$$Q_{un} = B_q \,.\, A_q \,.\, \gamma \,.\, L \quad (3)$$

where B_q= uplift factor, A_q= Area of bottom of enlarged base, γ = Unit weight of soil above the bottom of the bell, L = length of the pile.

$$B_q = 2\left(\frac{L}{D_b}\right) K_u \tan\varphi \left[m\left(\frac{L}{D_b}\right) + 1\right] + 1 \quad (4)$$

where K_u = Uplift coefficient (0.9 for soil friction angle from 30º to 45º) φ = angle of friction of soil, m = coefficient of shape factor that is based on soil friction angle φ, according to tables by Meyerhof and Adams (1968) [14]. Embedment ratio L/D_b is also from other table based on φ [14]. Properties of pile and soil which are applied to the model in this research are as follow; Base diameter: 1 m, pile length: 5 m, φ = 35, γ = 17kN/m^3. Figure 3 shows the comparison between model and the theoretical method on an individual under-reamed pile in sandy soil. The net ultimate resistance of pile using the theory is 1010 kN, while the number in the model is reached to 966 kN (about 4% reductions).

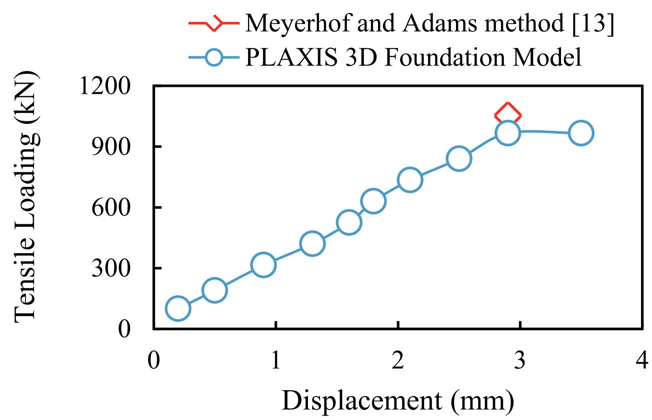
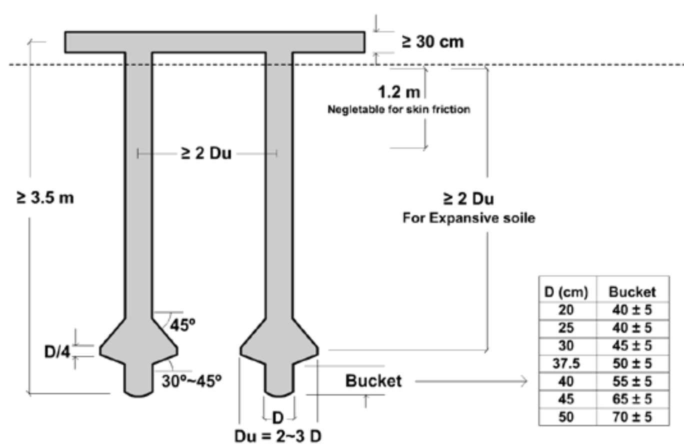


Figure 3: Comparison between Meyerhof method [13] and PLAXIS model.

NUMERICAL MODELING METHODOLOGY

Modeling process is done using Plaxis 3D Foundation, which is finite element based software [7]. As shown in figure 4, there are some considerations for piles and pile caps specifications, the modeling process in this research is done under these standards [1], [2], [3], [6]. In the models applied in this research all stem diameters are 0.5 m, maximum depth of piles are 6.2 m, single under-reamed bulb is located at the end of the pile stem and the maximum diameter of bulb is 1.4 m, the bucket length is 0.7 m. Thickness of the caps are 1 m and they are located on the floor not in the soil. Plaxis 3D Foundation has no option to model an under-reamed pile, so it should be considered in different layers (as shown in figure 6).



D (cm)	Bucket
20	40 ± 5
25	40 ± 5
30	45 ± 5
37.5	50 ± 5
40	55 ± 5
45	65 ± 5
50	70 ± 5

Figure 4: Schematic section of considerations for under-reamed piles in group [1].

Modeling process is started by making one pile models and then groups containing uniform stem piles and under-reamed piles. Modeling is continued by making groups containing 4 and 5 piles with different arrangements, and spacing/diameter (S/D) ratios of 6 to 12. Groups of 5 piles are arranged in square shape with a central pile. General conditions of models are considered the same for all of them, and outputs variables such as ultimate bearing capacities, displacements, etc. are recorded step by step in a spreadsheet for next analyses and comparisons. Results for square arrangement are discussed elaborately in the next section. Soil and pile specifications, material modes (an option for behavior of material in PLAXIS software), stiffness characteristics and interfaces are shown in table 1. Material modes are considered linear elastic and Mohr-Coulomb for piles and soil respectively. The given soil and pile properties in table 1 are the default values which are set for the used materials.

Table 1: Materials Specifications, considered in the FE models (Plaxis 3D Foundation)

Specification		**Pile**	**Soil**
Material model		Linear elastic	Mohr Coulomb
Material properties	γ (kN/m^3)	24	17
	γ_{sat} (kN/m^3)	24	20
Stiffness	E (kN/m^2)	2.920×10^7	1.5×10^4
	ν (nu)	0.2	0.3
Strength	C (kN/m^2)	-	0
	φ (Phi)	-	32^o
	ψ (dailation)	-	2^o
Interface	Rigidity (R_{inter})	Rigid (1)	0.7

Dimension of 3D models is a 20x20 square in plan and 15 m of depth. Caps are allowed to have about 10 m space to the boundaries. There are roller fixities on the boundaries. Figure 5 shows a 3D mesh generated by Plaxis 3D Foundation, its necessary to generate 3D mesh before calculation phase [7]. Figure 6 shows model outputs for square and triangle arrangements. As figure 6 shows, there is more stress at the opposite direction of loading in the bulbs zone. For square arrangement the extreme value for this loading phase is 1.85×10^3 kN/m^2. 15 nodes element is used to generate the mesh, and generated meshes are refined around the piles to increase calculation precision. The caps which are shown in figure 6 are just schematically generated by Plaxis software; in fact they have 1 m thickness as mentioned.

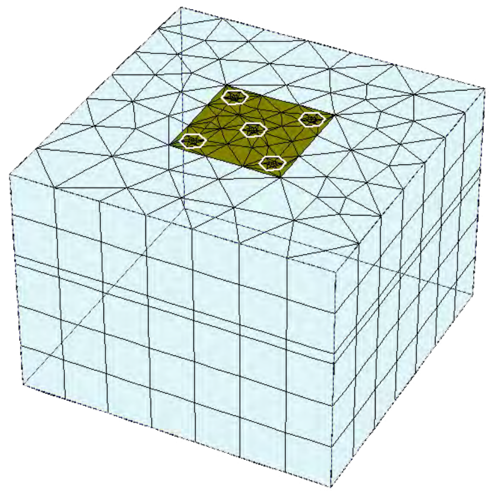


Figure 5: 3D Mesh for 5 pile square arrangement, generated by Plaxis 3D Foundation software

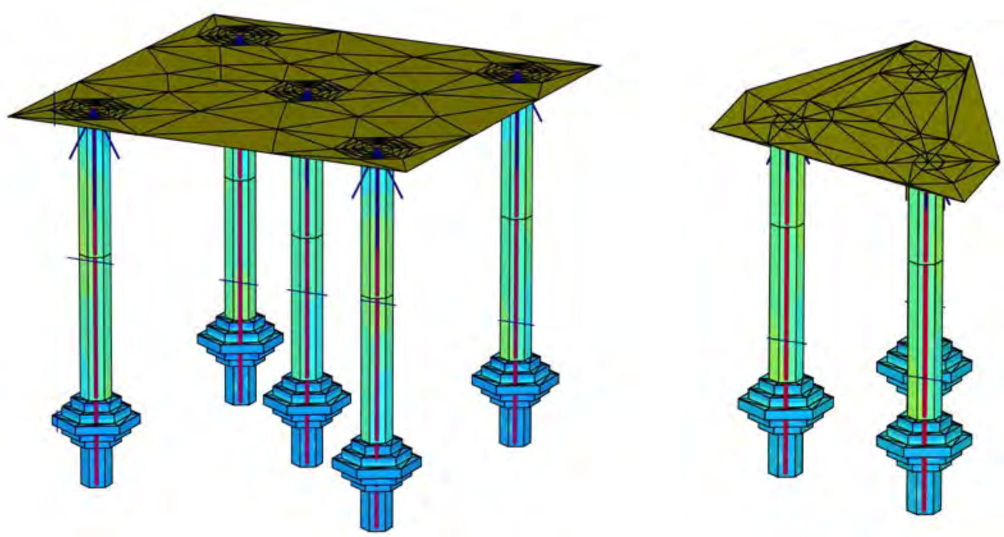


Figure 6: Shading views of stress values for Square and Triangle arrangements, generated by PLAXIS 3D Foundation software

Then next set of models with pentagon plan geometry are arranged to complete the modeling, rest of the values were recorded precisely. Also a set of triangle models (containing 3 piles), and hexagon models (containing 6 and 7 piles) are made as the finishing stage of modeling. Results for pentagon, triangle and hexagon arrangements are discussed elaborately in the next sections.

PARAMETRIC ANALYSIS

Quantitative parameters of the models are compared together to derive a relation between these parameters in order to find out the best arrangement pattern. Table 2 shows S/D ranges for some types of arrangements modeled in this research, also area ranges and concrete volume ranges are shown for the groups. Area ranges are the efficient areas that is measured center to center of piles in plan, and volumes are totally measured that is included the volume of pile caps. Figure 7 shows a comparison of geometry between Square, Pentagon, Triangle and Hexagon arrangement, angle variations are shown in this figure for every arrangement, and areas is also shown for each arrangement in order to compare their efficiency.

Table 2: Range of Spacing/Diameter (S/D) ratio in model series for 3, 5 and 7 piles groups

Arrangement	Number of piles	Range of S/D ratio	Area range (m^2)	Volume (m^3)
Square	5	6 - 16	9 - 64	35-110
Pentagon	5	3 - 10	4 - 43	25-93
Triangle	3	3 - 10	1 - 11	9-19
Hexagon	7	8 - 12	41.5 - 93	79-160

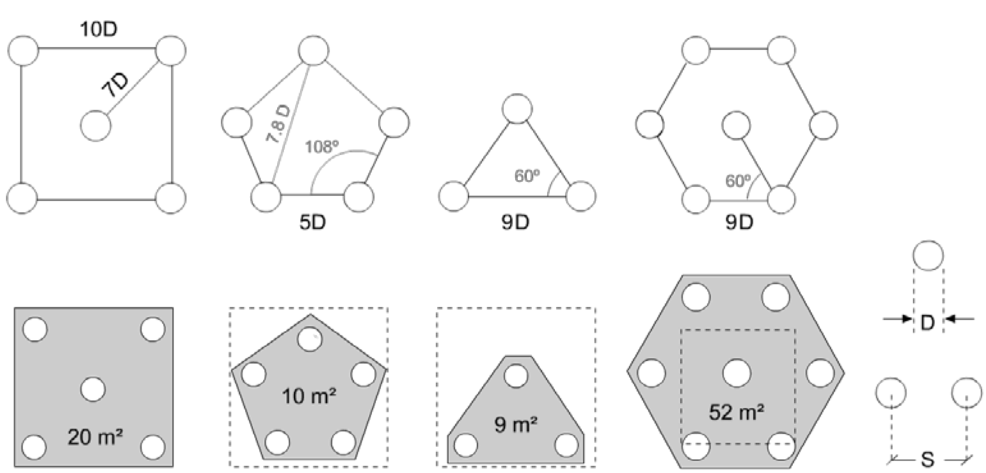


Figure 7: Comparison of geometry between Square, Pentagon, and Triangle and Hexagon arrangements.

RESULTS AND DISCUSSIONS

Figure 8 shows shading view of total displacements for a pentagon model output. For all the arrangements, increasing the space between piles, their bearing capacities are increased gradually. After a particular spacing which is the optimum spacing, bearing capacities are decreased. Figure 9 shows this fact for pentagon arrangement. Outputs are shown that piles which are located at the optimum arrangement are working together uniformly well. So each pile could withstand more loads in the optimum arrangement (Figure 10), that's why every single pile in pentagon at optimum arrangement could be subjected to about 230 kN and in the square arrangement every pile could withstand just about 150 kN. Pentagonal arrangement in comparison with square arrangement of 5 piles, could be subjected to bigger loads up to 16%, also in a same loading condition in the comparison it could save 40% of materials and 46% of area. Another comparison is applied for triangular arrangement containing 3 piles and square arrangement containing 4 piles that is shown the triangular arrangement 23% more efficient in bearing capacity at equal area and more efficient in area (28%) at equal loading condition. Hexagonal arrangement is appeared not an efficient arrangement in relation to area, it is compared to a simple square arrangement of 6 piles and with a symmetric hexagonal arrangement of 6 piles, it could be subjected to just ¼ of square arrangement loading in unit area and ⅓ of loading quantity of symmetric hexagonal arrangement. Pentagonal arrangement is also compared to the symmetric hexagonal arrangement of 6 piles, pentagon is appeared still well than hexagon and the amount of loading per unit area is increased to 90%. Figure 9 shows different shading views of vertical displacement in output program for two under-reamed piles next together in pentagon arrangement at spacing/diameter (S/D) ratio of 5, 7 and 10. The extreme value of displacement at S/D ratios of 5 is 5.48 mm which is shown in red color. Also the failure zone could be visible by shear strain shading output in Plaxis 3D Foundation. Figure 10 shows a comparison of displacement or stress shading between pentagon and square arrangements in plan.

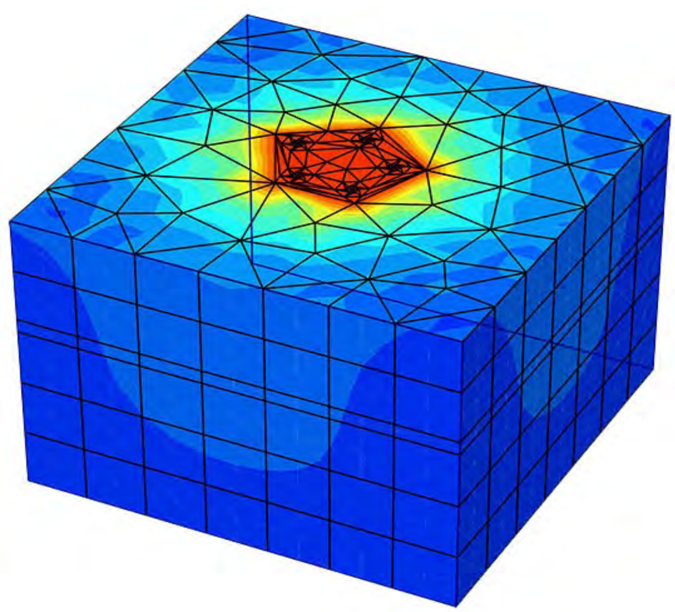


Figure 8: Shading view of total displacements for pentagon arrangement, generated by Plaxis 3D Foundation software

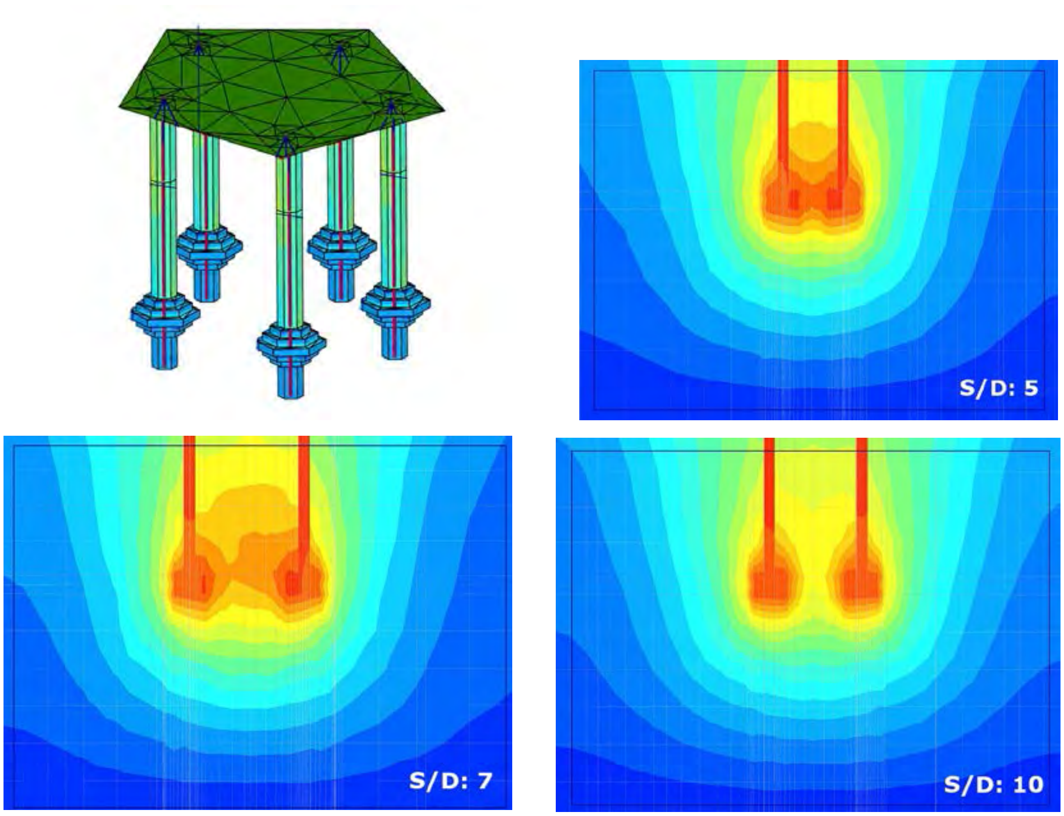


Figure 9: Shading view for vertical displacements in loading phase, pentagon arrangement

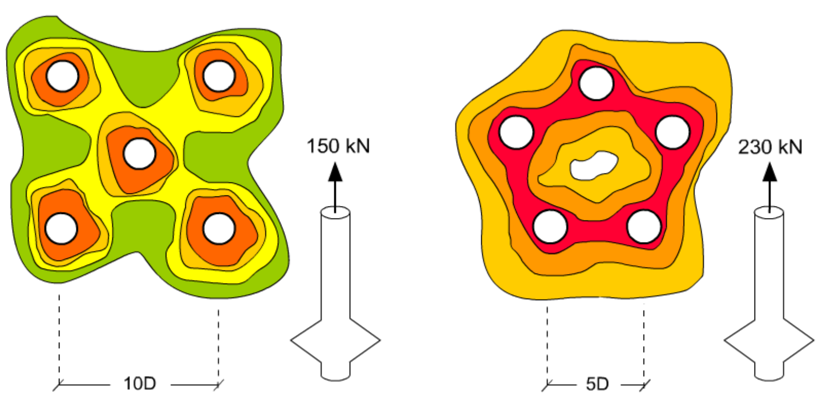


Figure 10: Group effect of optimum pentagon arrangement compared to square arrangement.

Uplift vs. Tensile loading curves are shown in figure 11 for every arrangement, failure criterion for maximum uplift recorded in this figure is when the soil behavior become plastic. Figure 12 shows tensile bearing capacities corresponding to the S/D ratio, and Figure 13 shows area efficiency of the arrangements, bearing capacities in figures 12 and 13 are net values which means the weight of pile caps are not taken into account. As Figure 12 shows, pentagonal arrangement is reached the maximum capacity in S/D ratio of 5. The diagram shows that pentagon is reached the optimum arrangement in lowest S/D ratio in whole curves. Triangular, Hexagonal and square arrangements are reached to the optimum point at S/D ratios about 9 to 10. The optimum S/D ratios for the arrangements are shown in table 3. Pentagon has the smallest number of S/D ratio between them.

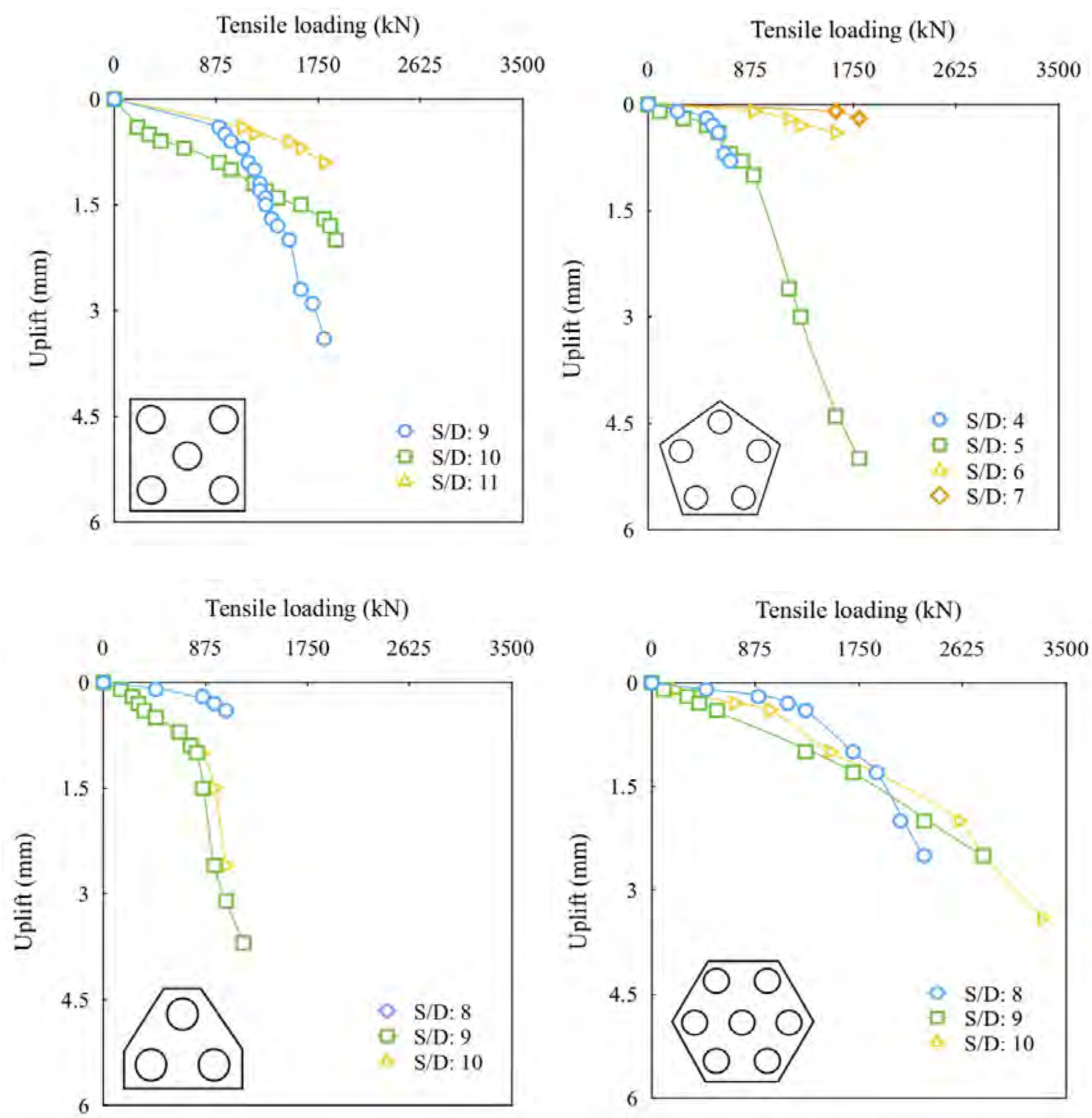


Figure 11: Tensile loading vs uplift for various arrangements at different S/D ratios.

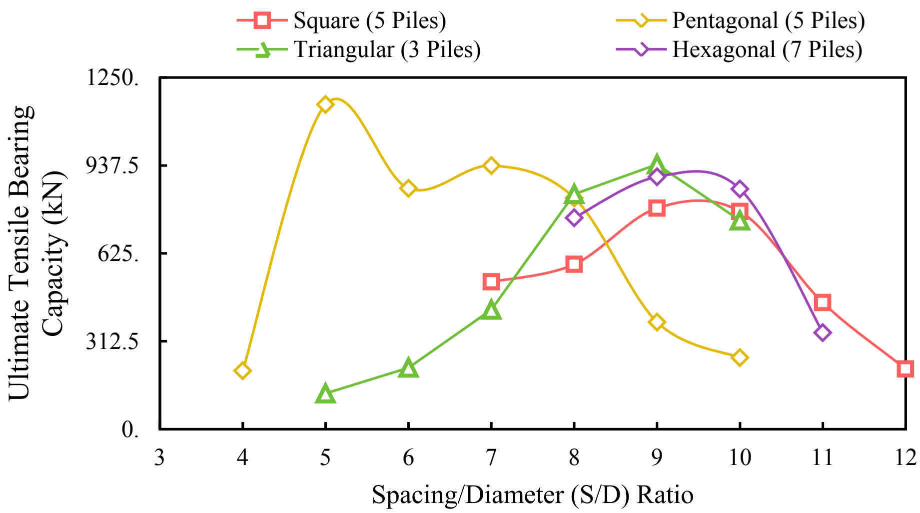


Figure 12: Ultimate tensile bearing capacity for different S/D ratios of under-reamed pile groups.

Table 3: Optimum Spacing/Diameter (S/D) ratio for 3, 5 and 7 piles groups

Arrangement	Number of piles	Optimum S/D ratio
Square	5	10
Pentagon	5	5
Triangle	3	9
Hexagon	7	9

Their area efficiencies are compared in Figure 13. Pentagonal arrangement is reached to more tensile bearing capacities in smaller areas, for example to reach 800 kN, pentagon needs just 15 square meter, while in this area the square group could withstand just about 500 kN. Comparing to hexagonal arrangement with seven piles, pentagon arrangement is still better from the area efficiency standpoint. In this diagram also triangular arrangement is shown as an efficient arrangement in small areas, it’s also works better than square arrangement with 4 and 5 piles in the applied models. In this figure the ultimate bearing capacity is measured without weight of pile caps and the areas measured by a center to center boundary of piles in plan. In a review of Figure 13, it’s obvious that hexagonal arrangement for 7 piles is not efficient at all comparing to even smaller number of piles in pentagonal arrangement.

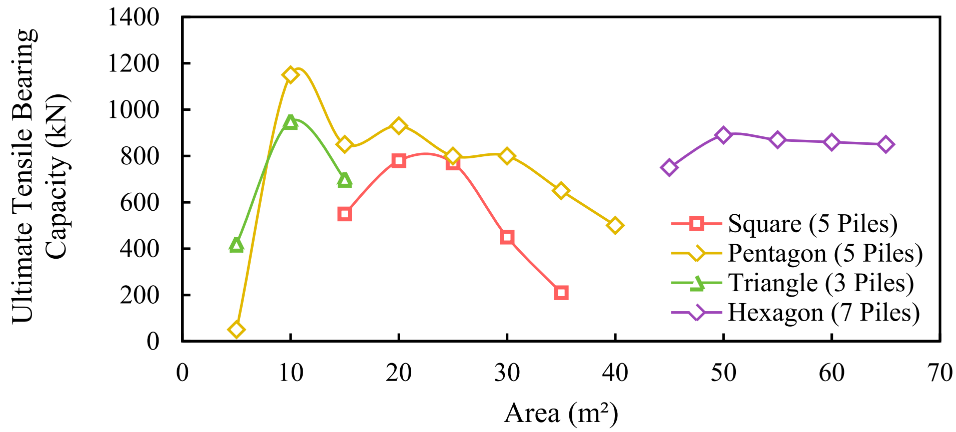


Figure 13: Area efficiency for different arrangements of under-reamed pile groups in sand.

In a comparison between pentagon and square with equal number of piles, pentagon shows a smaller S/D ratio, and smaller areas to reach larger bearing capacities. Figure 14 shows a comparison between square and pentagon arrangements, also result for pentagon arrangement containing normal piles (uniform stem) is shown in this figure at end right column. Volume values are measured considering pile caps. Comparison between these models couldn't be generalized for bigger groups of them because the group effect would be different for various extended groups.

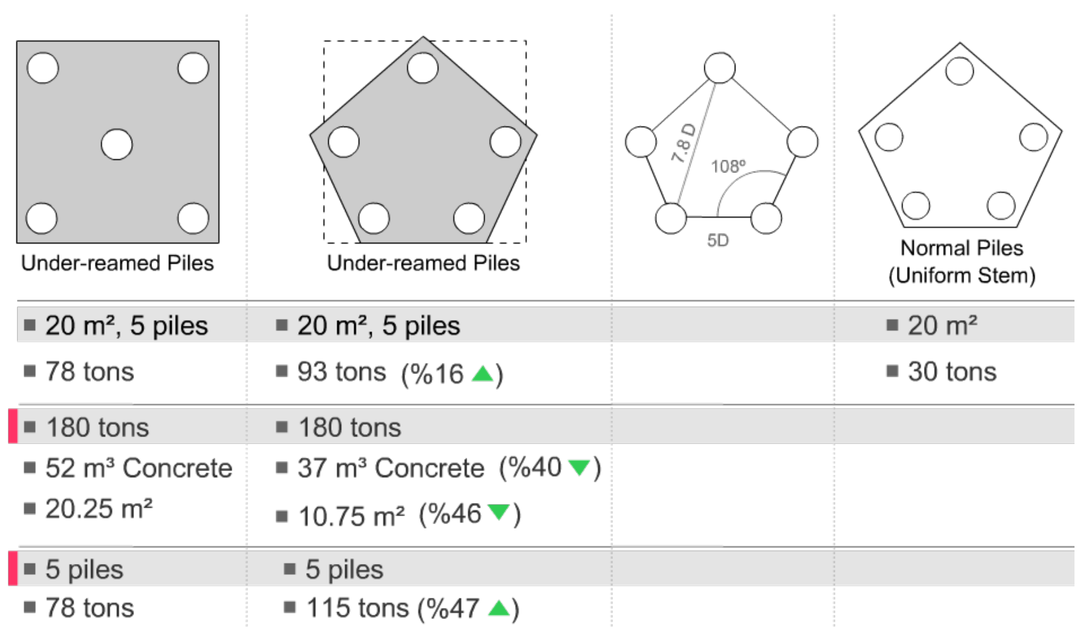


Figure 14: Comparison between square and pentagon arrangements in sandy soil.

Figure 15 shows a comparison between volumes or concrete consumption for all groups. Bearing capacities in figure 15 are net values which mean the weight of pile caps are not taken into account. Pentagonal arrangement is reached to the highest tensile bearing capacity among the set of the models at concrete volume value less than square and hexagon arrangements, so concrete consumption is decreased in pentagonal arrangement (47 percent) comparing to square arrangement.

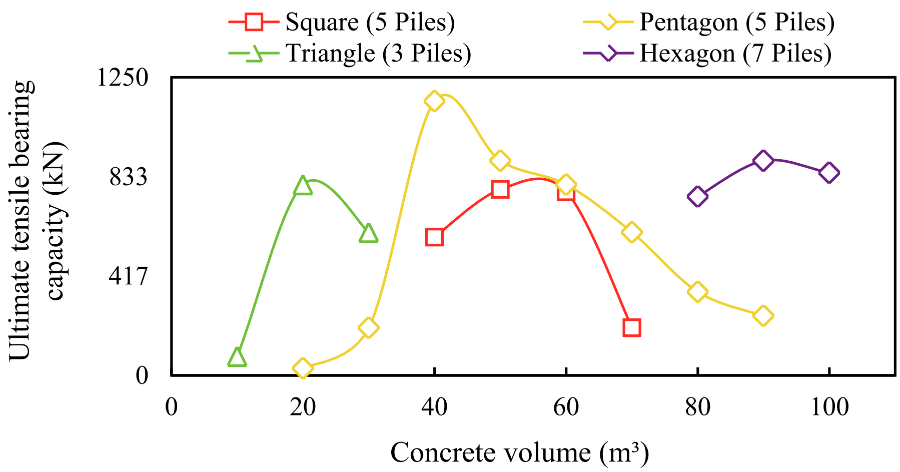


Figure 15: Comparison of concrete volumes between some groups of under-reamed piles.

CONCLUSIONS

In this research some series of models containing under-reamed pile groups with different arrangements and spacing between piles which are subjected to tensile loading, are applied using finite element method. They are compared together and analyses contain some results as follows:

1. For all the arrangements, increasing the space between piles, their bearing capacities are increased gradually. After a particular spacing which is the optimum spacing, bearing capacities are decreased.
2. Optimum S/D ratios were obtained 10, 5, 9 and 9 respectively for square arrangement (containing 5 piles), pentagon arrangement (containing 5 piles), triangle arrangement (containing 3 piles) and hexagon arrangement (containing 7 piles) of under-reamed pile groups in saturated sandy soil. This number is related to angles between piles in plan.
3. Ultimate bearing capacity for pentagonal arrangement is increased more than 16 percent comparing to square arrangement with the same number of piles and in the same area. Also Pentagon could be applied in much lower concrete consumption than square arrangement (more than 40%). So pentagon arrangement appeared more efficient than square arrangement for a group with 5 under-reamed piles in saturated sandy soil. So it is able to withstand greater loads in smaller areas, it causes less operation volume in project process.
4. Triangular arrangement can be subjected to higher loading quantities up to 23% comparing to symmetric square arrangement with 4 piles. To achieve a certain amount of bearing capacity comparing to square arrangement, area is decreased 28% in triangular arrangement.

Asymmetry may sometimes improve tensile bearing capacity potentials of an under-reamed pile group.

5. In the applied models in this research, pentagonal and triangular arrangements had very good results for tensile bearing capacity per area unit or concrete usage compared to other types of arrangements with similar number of piles in the same areas.

REFERENCES

1. Devendra Sharma, M. P. Jain, Chandra Prakash (1978) "Handbook on Under-reamed and Bored compactoin pile foundation", Central Building Research Institute, Roorkee, 78 pages.
2. Michael Tomlinson and John Woodward (1977) "Pile Design and construction practice", 5th Edition, Taylor and Francis Publication, 551 pages.
3. Shamsher Prakash, Hari D. Sharma (1990) "Pile Foundation in Engineering Practice", John Wiley Publication, 721 pages.
4. Reed L. Mosher, William P. Dawkins (2000) "Theoretical Manual for Pile Foundations", US Army Corps of Engineers, Engineer Research and Development Center, 92 pages.
5. Ken Fleming, Austin Weltman, Mark Randolph and Keith Elson (2009) "Piling Engineering", 3rd Edition, Taylor and Francis Publication, 398 pages.
6. Elwyn E. Seelye, et al. (1945) "Data Book for Civil Engineering, Design" John Wiley and sons, Inc. London Chapman and Hall, Limited.
7. R.B.J. Brinkgreve, et al. (2006), "Plaxis 3D Foundation manual, Version 1.5", Delft University of Technology & PLAXIS bv, The Netherlands.
8. Braja M. Das (1983) "Principfles of Foundation Engineering", seventh edition, Global engineering publication.
9. Nabil F. Esmael (2001) "Axial Load Tests on bored piles and pile groups in cemented sands", Journal of geotechnical and geoenvironmental engineering, 766-773.
10. C. Y. Lee (2007) "Settlement and Load Distribution Analysis of Under-reamed piles", ARPN journal of engineering applied science, Vol. 2, No. 4, 35-40.
11. K. Watanabe, H. Sei, T. Nishiyama and Y. Ishii (2011) "Static Axial Reciprocal Load Test of Cast-In-Place nodular concrete pile and nodular diaphragm wall", Geotechnical Engineering Journal of the SEAGS & AGSSEA, Vol 42, No. 2, 11-19.
12. Nainan P. Kurian and G. Srilakshmi (2010) "Studies on Geometrical Features of Underreamed piles by the finite element method", Journal of Karunya University, Volume 2, No.1
13. Hamed Niroumand, Khairul anuar Kassim, et al. (2012) "Uplift Capacity of Enlarged Base Piles in Sand", Electronic Journal of Geotechnical Engineering (EJGE), Vol. 17, Bund R, P. 2721-2737.
14. Dilip Kumar Maharaj, J. Gayatri and D. Jayanthi (2004) "Uplift Capacity of Pile by finite element method", Electronic Journal of Geotechical Engineeing (EJGE), Vol 8, Bundle D.

15. Yun-gang Zhan, Hong Wnag and Fu-cen Liu (2012) "Modeling Vertical Bearing Capacity of Pile Foundation by Using ABAQUS", Electronic Journal of Geotechnical Engineering (EJGE), Vol. 17, Bund. L, P. 1855-1865.

16. Gupta S, Sundaram R. (1986)"For which soil conditions are Under-reamed pile foundations superior to other foundations system", Expert paper, proc IGC,Vol II, pp. 131-133.

17. Makarchian M., Khademi P. (2010) "Laboratory evaluation of effective parameters on pile uplift force in sandy soil", 8th National Congress of Civil Engineering, Shiraz University, Shiraz, Iran.

18. Zahra Mardani, Hamid Alielahi, S. M. Mortazavi (2013) "Numerical Evaluation of tensile bearing capacity of Under-reamed pile group in granular soil using finite element method", 1st International congress of civil engineering, architecture and sustainable urban development, Islamic Azad University of Tabriz, Tabriz, Iran. (In Persian)