Experimental and Theoretical Investigations on the Deformation Characteristic of Frozen Silt in Underground Engineering

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ABSTRACT

A series of triaxial compressive tests of frozen silt has been conducted at -8°C. The nonlinear characteristic of the stress-strain curves for both axial and volumetric strains prior to failure was also analyzed. The constitutive equation for frozen silt is developed within the framework of the endochronic theory of plasticity. By defining intrinsic time and the softening and hardening function, then introducing them into endochronic equation, the constitutive equation of frozen silt is derived to reproduce the elastoplastic process according to experiment data and intrinsic time equation. The numerical schemes are examined for efficiency in the modeling the deformation process under triaxial compression. It is found that the endochronic theory, in which two groups of internal state variables are used to describe the mechanical features, could well predict the complicated deformation characteristic of frozen soil.

KEYWORDS: Frozen silt; Endochronic theory; Constitutive model; Plasticity

INTRODUCTION

With the development of engineering activities relating to frozen soil, such as building highways, railways and tunnels, more and more problems were inevitably encountered during constructions in cold region and underground [1-2]. The mechanical characteristics of frozen soil are the major engineering property, which is an important point to ensure engineering safety and building expenditure reduction. The important research objective of the mechanical analysis is the safety of construction and operation for frozen soil engineers by predicting the inelastic deformation process. The goal can be achieved through better design and calculation.
methods and by better control of the parameters of the deformation processes based on a deeper knowledge of the phenomena that accompany the deformation and the relationships between the mechanical properties of the materials and the conditions of deformation. The constitutive relationship is used to describe the deformation behavior of materials under external load, and in formulation of complex problems the mathematical modeling has become a powerful tool. The theoretical modeling of the constitutive relation of frozen soils is rather difficult because of the complicated mechanical properties of frozen soil. The researches have shown that the frozen soil change its size or shape in a way that is not reversible when load is applied. The concept of elasto-plastic deformation was at the heart of many engineering structures of frozen soil. To describe the elasto-plastic deformation behavior of frozen soil, a number of constitutive models for the frozen soil material have been proposed by researchers over the last decades. Traditional constitutive models of soil materials can be divided into two groups: one is experience model obtained by fitting experimental data, which aim at improving fitting precision; the other is theoretic models, which can be obtained based on Drucker’s stability postulate and plastic potential theory. Based on a large number of experimental results, Wang et al [3] proposed a hyperbola function to describe the stress-strain relationship of frozen soil. Considering that the experience hyperbola function cannot reflect the strain softening behavior of frozen soil, Lai et al [4] proposed an improved Duncan-Chang model to describe deformation behavior. The improved Duncan-Chang model can describe not only the strain softening behavior but also the strain hardening behavior of frozen soil. The experience constitutive models are obtained by fitting experimental data and aim at improving fitting precision, which make the physical meaning of the methods is indeterminate. Shoop et al [5] proposed a modified Capped Drucker-Prager plasticity model to describe the thaw-weakened frozen soil. By analyzing the results of triaxial tests on frozen soil and drawing the streamlines and potential lines, Lai et al [6] proposed an ellipse plastic potential surface according to the Drucker’s postulate and the associated the plastic flow rule was adopted. It is generally known that the plastic flow rule of soil materials is not associated. In fact, the associated flow rule is usually used in these models, which makes the proper description on the deformation of frozen soils difficult. Recently, it has become clear that plasticity of many materials, especially geological materials, is not associated with the yield surface by means of a normality rule and Drucker's postulate. In order to overcome the defect of associated plastic flow rule, Yang et al [7] presented a non-associated constitutive model, in which the ellipse function is taken as the yield surface, the plastic potential surface depends on the same arguments but a non-associated flow rule is used. There are many papers on the experimental investigation of subsequent yield surfaces, but the general equation of surfaces has not been found all along. The plastic flow theory is considerably perfect in form but is difficult to concretize. The research on constitutive model is still an important subject in the current development of mechanical characteristic of frozen soils.

The endochronic theory reflects the plastic characteristic of materials by means of memory integrals, expressed in terms of memory kernels. The theory of plasticity did not use the notion of loading surface, it made no distinction between active and passive processes of deformation
to describe the most behavior features of the mechanical behavior and suggest physical interpretations in terms of damage, micro-cracking, grain rearrangements and internal friction [8-9]. The endochronic constitutive theories, which have met with great success in modeling geological materials exhibiting strain-softening, pressure sensitivity and inelastic dilatation, provides a unified point of view to describe the elasto-plastic behavior of material [10-11]. In this study, an extensive laboratory program to fully characterize the mechanical behavior of the frozen soil was completed. The test results and the constitutive model are presented. The material parameters in the constitutive model are calibrated for samples of frozen silt by fitting the model to reproduce data from triaxial compression experiments. Using the proposed constitutive model and computational algorithms, the compaction experiment is analyzed numerically.

**ENDOCHRONIC PLASTICITY THEORY**

Endochronic theory was first proposed by Valanis [12-13] to circumvent the complexities difficulties of classical plasticity theory, which need to establish suitable yield criterion and flow rules [14-15]. The equations of the endochronic theory of plasticity have the forms

\[ s_{ij} = 2 \int_{0}^{z_s} \rho_s(z_s - z_s') \frac{\partial e^p_{ij}}{\partial z_s'} dz_s' \]

\[ \sigma_{kk} = \int_{0}^{z_h} \rho_h(z_h - z_h') \frac{\partial e^p_{kk}}{\partial z_h'} dz_h' \]

where \( s_{ij}, e^p_{ij} \), and are deviators of the stress and plastic deviator strain tensors, \( d e^p_{ij} = d e_{ij} - d s_{ij} / 2G \), \( d e_{ij} = d e_{ij} - d e_{kk} \delta_{ij} / 3 \); \( \sigma^p_{kk} \) and \( e^p_{kk} \) are components of the spherical stress and strain tensors, \( d e^p_{kk} = d e_{kk} - d \sigma^p_{kk} / 3K \); \( \rho_s(z_s) \), \( \rho_h(z_h) \) are some heredity functions;

The intrinsic time determines the positive function which depends on the measure of plastic deformation. Various equations may be proposed for determining \( z \), but the following equation has proved to be more suitable,

\[ dz_s = \frac{d \xi_s}{f_s(\xi_s, \sigma_y)} \]

\[ dz_h = \frac{d \xi_h}{f_h(\xi_h, \sigma_y)} \]

in which \( f_s(\xi_s, \sigma_y), f_h(\xi_h, \sigma_y) \) are material hardening functions.
By scaling the intrinsic time, these functions cause hardening or softening plastic behavior as a function of confining pressure and plastic strain of material. The time scales \( \xi_s, \xi_h \), which depend on material deformation and the history effects, are independent of elapsed time. In references [13,16], the time scales are defined in plastic strain space. In order to facilitate the study of constitutive model, the intrinsic time measures are defined in terms of deviatoric plastic strain and volumetric plastic strain,

\[
d\xi_s = (de^{p}_{ij} : de^{p}_{ij})^{1/2}
\]

\[
d\xi_h = \|d\varepsilon^{p}_{ij}\|
\]

in which \( de^p_{ij} \) is deviatoric plastic strain increment tensors, \( \| \) represents the Euclidean norm.

One class of endochronic models has been developed by Valanis and Fan [13], in which the kernel function is assumed with a finite sum of exponentially decaying function as follows,

\[
\rho(z) \approx \sum_{r=1}^{m} C_r e^{-\alpha z}
\]

where \( C_r, \alpha \) are material constants dependent on the mechanical properties of the material.

Eq. (1) and Eq. (2) are given by integral form constitutive relation, which is not convenient to calculate in practical application. In order to facilitate the calculation, the follow forms are derived according to the Leibniz’ differential rule from eqns. (1) and (2)

\[
dS_{ij} = 2Gde^{p}_{ij} - \alpha S_{ij} dz
\]

\[
d\sigma_{kk} = 3Kde^{p}_{kk} - \beta \sigma_{kk} dz
\]

where \( G \) is the elastic shear modulus; \( K \) is the elastic bulk modulus; the increment of the deviatoric stress tensor \( dS_{ij} \) is determined based on the known input increment of the deviatoric strain tensor \( de^p_{ij} \), the calculated increment of intrinsic time \( dz \) and the known deviatoric stress tensor \( S_{ij} \) at current state.

Under the triaxial compression condition, the following equations can be obtained from Eq. (5) and Eq. (6),

\[
d\xi_s = \frac{\sqrt{3}}{2} (de_i - \frac{dS_i}{2G})
\]

\[
d\xi_h = \left| d\varepsilon_v - \frac{d\sigma_{nn}}{K} \right|
\]

Substituting the incremental form, an effective numerical algorithm for implementing the endochronic theory can be derived based on Eq.8 and Eq.9.
TEST CONDITIONS AND RESULTS

In this paper, the soil used in test was silt and Particle size distribution is given in Fig.1.

When mixed thoroughly with water content of 12.8% by weight and compacted, the silt had an optimum dry density of 1.81 g/cm³. The cylindrical specimen of frozen silt with 6.18 cm in diameter and 12.5 cm in height was prepared in a split mold, in which the silt was compacted at the optimum moisture content. The specimen was quickly frozen in order to prevent the formation of ice lenses because of water migration. Then, the specimen was placed into the triaxial pressure cell of MTS low temperature testing machine at a temperature of -8°C and kept constant for 24h. After the confining and axial pressures had been applied for 5 minutes, the triaxial shear tests began, and shear strain rate was $2.09 \times 10^{-2}$ mm/s. The typical stress-strain curves for triaxial compression tests at -8 °C are shown in Fig. 2 and Fig. 3.

**Figure 1:** Particle size distribution of frozen silt
The test results show that the nonlinearity in the stress-strain curves was evident for both axial and volumetric strain prior to failure was also observed. With regard to this study, the relevant deformation behavior when subjected to compressive loads under different confining pressures can be divided into the following regions. The slope of the stress–axial strain curve is usually found to increase in the early stages of loading due to closure of pre-existing cracks. Thereafter both the slopes of the stress–axial strain and the stress–lateral strain curves remain relatively constant defining the range of elastic deformation. With the further increase of axial strain, the slope of the stress-strain curves gradually decreases under low confining pressures.

The experimental results also indicate that the stress-strain curves of frozen silt are significantly affected by confining pressures. The frozen silt presents strain softening in shear
process under low confining pressures, but presents strain hardening phenomenon under high confining pressures. The slope of axial strain-stress curve of frozen silt is initially positive, but gradually decreases to positive under low confining pressure. The volumes of frozen silt only reduce with increasing in axial strain under high confining pressures. However, the volumes of frozen silt always reduce to a critical value with increasing of axial strain, but with a further increasing in axial train, the volumes expand under low confining pressures.

COMPARISON AND DISCUSSION OF THE THEORETICAL AND EXPERIMENTAL RESULTS

The purpose of the present study is to analyze the effectiveness of the use of endochronic theory for studying the elasto-plastic deformation of frozen silt. In order to demonstrate a part of the wide range of problems that can be solved by the present formulation, we have illustrated the effective performance of the elasto-plastic endochronic model in the following content.

Elastic parameters

The $K$-$G$ model is chosen to calculate the elastic part of strain in this paper. In the following, the procedure for determining the elastic bulk modulus and shear modulus is described. After the confining and axial pressures had been applied, the triaxial compressive loading-unloading-reloading test began. The elastic shear modulus $G$ can be calculated from $q-\varepsilon_s$ curves described by results under different confining pressures and shown in Fig. 4.

Figure 4: The triaxial compressive loading-unloading-reloading curve of frozen silt
It can be seen that the stress–strain curve appears the clear closed-hysteresis loops. Every closed-hysteresis loop is composed of loading and unloading curves. Unloading of any cycle is always started from envelope stress-strain curve and terminated at a particular value of plastic strain at zero stress level.

The plastic strain increases as number of cycle increases. The shape of unloading curve is greatly influenced by the plastic strain. With the increase of loading-unloading times, the plastic strain gradually increases and the hysteresis loop.

We proposed the following parabolic function to express the relationship between shear modulus $G$ and confining pressures $\sigma_3$.

$$G = G_0 + \eta_g p_a e^{m\sigma_3/p_a}$$  \hspace{1cm} (14)

where $G_0=1825.6$, $\eta_g=44.6$; $m=0.0388$, $p_a=0.10133$Pa

In elastic stage, the relationship between shear elastic modulus $G$ and the volumetric elastic modulus $K$ can be expressed as

$$K = \frac{2G(1+\nu)}{3(1-2\nu)}$$  \hspace{1cm} (15)

where $\nu$ is Poisson’s ratio, $\nu = 1/3$ was used in fitting test data. For many frozen soil, values between $\nu = 0.25$ and $0.4$ may be appropriate (He et al., 1999).

The volumetric elastic modulus can be calculated and expressed as follows:

$$K = K_0 + \eta_k p_a e^{m\sigma_3/p_a}$$  \hspace{1cm} (16)

where $K_0=4462.6$, $\eta_k=109.2$

**Hardening and softening function**

Although the yield surface has not been explicitly assumed in endochronic theory, introducing delta function to the kernels results implicitly in that concept.

Using the elastic moduli calculated by Eq. (14) and Eq. (16), the total strain can be split into the elastic part and the plastic part. Since the total deviatoric strain increment is $\varepsilon_{sv} = \varepsilon_{s} - \varepsilon_{r} / 3$, where $\varepsilon_s$ is the equivalent shear strain. Thus the plastic volumetric strain is $\varepsilon_{sv} = \varepsilon_{s} - \varepsilon_{r}$, and the plastic shear strain is $\varepsilon_{sv} = \varepsilon_{s} - \varepsilon_{r} / 3 - \varepsilon_{r}$. In order to include the hardening effect of deviatoric loading on the behavior of material and the effect of the plastic strain history effects, for application, the following specific forms for the
dependence of deviatoric plastic strain and volumetric plastic strain history functions on $\xi_s$ and $\xi_h$ are proposed

\begin{align}
  f_s(\xi_s) &= a_{1s} \left[1 - \exp\left(-\frac{\xi_s}{t_{1s}}\right)\right] + a_{2s} \left[1 - \exp\left(-\frac{\xi_s}{t_{2s}}\right)\right] + f_{s0} \\
  f_h(\xi_h) &= a_{1h} \left[1 - \exp\left(-\frac{\xi_h}{t_{1h}}\right)\right] + a_{2h} \left[1 - \exp\left(-\frac{\xi_h}{t_{2h}}\right)\right] + f_{h0}
\end{align}

in which $f_s(\xi_s)$, $f_h(\xi_h)$ are softening or hardening functions related to deviatoric plastic strain and volumetric plastic strain, respectively; $a_{1s}$, $a_{2s}$, $t_{1s}$, $t_{2s}$, $f_{s0}$, $a_{1h}$, $a_{2h}$, $t_{1h}$, $t_{2h}$, $f_{h0}$ are material constants related to the deviatoric plastic strain and volumetric plastic strain.

The material function represents the effects of micro-structural changes of material due to accumulation of the plastic strain history. In fact, the intrinsic time scale is not only related to the irreversible deformation of the material but also to the loading condition. The following form can be used to reflect the effects of confining pressure and plastic strain history,

$$f(\xi, \sigma_3) = f(\xi) f(\sigma_3)$$

where $f(\xi, \sigma_3)$ is the function related to the hardening and softening behaviors of the material undergoing deformation evolution; $f(\sigma_3)$ is the function of confining pressure.

In this paper, the function $f(\sigma_3)$, which are related to the deviatoric and volumetric strains respectively, are given as follows

\begin{align}
  f_s(\sigma_3) &= k_{s1} \left(\frac{\sigma_3}{p_a}\right)^2 + k_{s2} \frac{\sigma_3}{p_a} + k_{s3} \\
  f_v(\sigma_3) &= k_{v1} \left(\frac{\sigma_3}{p_a}\right)^2 + k_{v2} \frac{\sigma_3}{p_a} + k_{v3}
\end{align}

where $k_{s1}$, $k_{s2}$, $k_{s3}$, $k_{v1}$, $k_{v2}$, $k_{v3}$ are material constants dependent of mechanical properties.

**Numerical simulation**

The stress-strain states of frozen soil specimens under triaxial compression are investigated for different confining pressures. The endochronic constitutive equations presented above together with the numerical integration and consistent linearization schemes have been used to evaluate the capability of the model in the simulation of frozen soil compaction process. All the parameters involved in this model can be determined using the conventional hydrostatic and triaxial compression tests. In the simulation of triaxial loading
test under high confining pressure and during the harden stage of the stress-strain curve, the following values are taken for material properties $a_{s_1} = 4.3$, $t_{s_1} = 0.00424$, $a_{s_2} = 107.9$, $t_{s_2} = 0.04253$, $f_{s_0} = 0.16$, $a_{v_1} = 3.4$, $t_{v_1} = 0.0114$, $a_{v_2} = 150$, $t_{v_2} = 0.03025$, $f_{v_0} = 0.155$, $\alpha = 2000$, $\beta = 2400$, $k_{s_1} = 9 \times 10^{-5}$, $k_{s_2} = -0.0129$, $k_{v_1} = 1.1$, $k_{v_2} = -3 \times 10^{-5}$, $k_{v_3} = 0.0061$, $k_{s_4} = 1.3345$. During the soften stage of the stress and strain curve, the parameters $a_{s_1} = 2.46$, $a_{s_2} = 61.66$, $f_{s_0} = 0.16$, $a_{v_1} = -3.67$, $a_{v_2} = -162$, $f_{v_0} = -0.167$. Some representative results are shown in Fig. 5a,b.

![Figure 5a](image1.png)

**Figure 5a:** Comparisons of model predictions and test results for frozen silt

![Figure 5b](image2.png)

**Figure 5b:** Comparisons of model predictions and test results for frozen silt

The figures depict the theoretical and experimental change in stress $\sigma_1 - \sigma_3$ in relation to the axial strain $\varepsilon_a$ and volumetric strain $\varepsilon_v$ with different confining pressures. It is
demonstrated that the calculated results are in good agreement with the experimental data. Since this model does not require establishing complicated yield criterion and all the parameters can be determined using the conventional hydrostatic, triaxial compression tests, which demonstrates the potential of endochronic theory to predict the deformation of frozen soil.

**SUMMARY AND CONCLUSION**

A series of triaxial compressive tests of frozen silt has been conducted at -8°C. The results show that the nonlinearity of the stress-strain curves is evident for both axial and volumetric strains prior to failure. The slope of the stress–axial strain curve linearly increases in the early stages of loading due to closure of pre-existing cracks. Then, both the slopes of the stress–axial strain and the stress–lateral strain curves gradually decrease with the increase of axial strain. A point is reached where the stress– strain curve becomes decrease with further increase of axial strain.

To describe the deformation behavior of frozen soil using sophisticated mathematical techniques, an elasto-plastic constitutive model has been established to define the stress–axial strain and volume change characteristics based on endochronic plasticity theory. The deformation responses stated in terms of endochronic equations were obtained. Finally, the numerical schemes were examined for efficiency in the modeling compaction process of frozen soil. The results clearly indicate that the endochronic theory in the context of elasto-plasticity can be effectively used for simulating deformation for frozen soil.

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