Comprehensive Assessment of Post-Earthquake Collapse Risk based on Fuzzy Evidence Theory

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ABSTRACT
The post earthquake collapse is one of the important causes of the mountain town houses damaged and traffic congestion, it has important significance to carry on the scientific and reasonable assessment to take preventive measures accordingly. Aiming at the complexity of post earthquake collapse formation mechanism and the uncertainty of evaluation index, according to evidence theory and fuzzy set theory, a risk assessment method of collapse disaster can be taken into account interaction of multiple indicator has been proposed. In this method, the element of evidence space is firstly considered as fuzzy sets. Through the fuzzy probability to describe expert advice, which has combined with expert advice to get fuzzy sets to determine the probability distribution function. Then by the synthesis rules of evidence theory function, the trust degree function of post earthquake collapse risk can be obtained, thus the collation based on the risk levels of classification has been established. Finally, examples were provided to validate and prove its scientific and reasonable, which has provide a new way for post earthquake collapse risk evaluation.

KEYWORDS: post earthquake collapse; evidence theory; synthetic rules; fuzzy subset; comprehensive evaluation

INTRODUCTION
The post earthquake collapse is a sudden strong, the typical serious secondary damage disasters, due to the strong earthquake collapse of sudden and destructive, it caused heavy losses of destroyed roads, railways and buildings, to human life and property caused heavy losses. Therefore, carrying out the assessment of earthquake collapse disaster and taking disaster
prevention measures can improve the ability of mountainous urban earthquake disaster prevention and lay a solid foundation for sustainable development\[1-5\]. At present, the domestic and foreign scholars have carried out relatively abundant exploration research for the collapse of an earthquake disaster risk evaluation technology. There are many corresponding research methods, mainly including expert judgment method and fuzzy mathematics method and statistic analysis, etc\[6\]. However, because of the multivariate complexity of collapse disaster influence factors and formation mechanism, it makes the research method have certain limitation, such as the weight value determined by expert judgment method has certain subjectivity; Fuzzy mathematics method is not in conformity with the measurement principles such as "additivity", "not responsible for the world" and "polarity", the principle of "choose big or small" also has some deficiencies; Statistical analysis is often dependent on the disaster history data results in the decrease of its accuracy and precision, etc. therefore, it is necessary to explore a collapse of an earthquake risk assessment method which has comprehensive consideration of multiple factors interact with each other.

The Evidence theory is put forward by the Dempster, improved and promoted by his students Shafer, so it also called D-S evidence theory\[7-9\]. The theory based on the general Bayesian method, described the inaccuracy of the evidence from different angles. Its advantage lies in the representation, measurement uncertainty and combination, but when it faced with imprecise, incomplete fuzzy information, the evidence theory will become very complicated. And fuzzy set theory has the advantage of using possibility theory to deal with imprecise, fuzzy data; it can make up for the defects of evidence theory\[8\]. So by using the fuzzy evidence theory, combined with the membership functions and fuzzy subset, the paper has analyses the influence factors of affect collapse hazard, and studied the earthquake collapse hazard evaluation.

**THE BASIC PRINCIPLE OF FUZZY EVIDENCE THEORY**

Evidence theory\[7-9\] is a kind of information fusion technology, which has used the D-S synthetic rules to fuse the reliability of different evidence bodies. It is called "recognition framework" based on the reliability of synthetic resulting from the fusion sample space, and it is expressed by the \( \Theta \), which is made up of some mutually exclusive proposition complete set.

**Definition 1**: Assuming that gives basic probability assignment \( m \) to proposition \( A: 2^\Theta \rightarrow [0,1] \), and meet two conditions: ① \( m(\Theta) = 0 \); ② \( \sum_{A \subseteq \Theta} m(A) = 1 \), \( \forall A \subseteq \Theta \), \( m(A) \) is the probability distributive function of identification framework \( \Theta \), if \( m(A) > 0 \), then \( A \) is the focal element of evidence.

**Definition 2**: For the proposition \( A \), defined function \( Bel: 2^\Theta \rightarrow [0,1] \) is the belief function of \( \Theta \).

\[
Bel(A) = \sum_{B \subseteq A} m(B) \quad \forall A \subseteq \Theta
\]

The belief function \( Bel(A) \) express the all support degree of proposition \( A \), It can determine it each other with the mass function \( m(A) \) and belief function \( Bel(A) \), therefore, in this paper the reliability of the proposition \( A \) is determined with the mass function \( m(A) \). Generally speaking, it is often use the fuzzy language to describe the evidence of evidence space, so. This paper
introduces the fuzzy subset to evidence space to gain mass functions. Assumption: $E = \{e_1, e_2, ..., e_n\}$ is an evidence space, and $\Theta = H = \{H_1, H_2, ..., H_m\}$ is a framework of identification. The mapping is given: $u_F : E \rightarrow [0,1], \ e \rightarrow u_F(e)$, then the fuzzy subset $\tilde{F}$ of evidence space is determined by the mapping $u_F, u_F$ is called the membership function of $\tilde{F}$, $u_F(e)$ is called the membership degree of $\tilde{F}$, The entire collection of fuzzy subsets of $E$ denoted as $\tilde{F}(E)^{[8]}$.

Definition 3: $w_i$ is the weight of evidence $e_i$ in the evidence space, then the fuzzy probability of fuzzy event $\tilde{F}$:

$$P(\tilde{F}) = \sum_{i=1}^{n} u_F(e_i) w_i$$  (2)

Definition 4: let $\tilde{F}_1, \tilde{F}_2 \in \tilde{F}(E)$, define the operation:

$$\tilde{F}_1 \cup \tilde{F}_2 : u_{\tilde{F}_1 \cup \tilde{F}_2} (e) = u_{\tilde{F}_1}(e) \lor u_{\tilde{F}_2}(e)$$  (3)

Definition 5: let mapping $T: \tilde{F}(E) \rightarrow 2^\Theta$, for $\forall H_j \in 2^\Theta$, $\exists \tilde{F}_i \in \tilde{F}$ s.t. $T(\tilde{F}_i) = H_j$ ($i = 1, 2, ..., n; j = 1, 2, ..., m$), $T(P)$ Can be defined $T(P): 2^\Theta \rightarrow [0, 1]$, and it should be satisfied with the conditions as follows:

$$T[p](H_j) = \begin{cases} 0 & H_j = \Phi \\ \frac{1}{M} P(\bigcup_{\tilde{F}_i : T(\tilde{F}_i) = H_j} \tilde{F}_i) & H_j \neq \Phi \end{cases}$$  (4)

$$M = \sum_{H_j \in 2^\Theta \atop \tilde{F}_i \in \tilde{F}(E) \atop T(\tilde{F}_i) = H_j} p(\bigcup_{\tilde{F}_i : T(\tilde{F}_i) = H_j} \tilde{F}_i),$$ the $T[p](H_j)$ is one of mass function of $H$, therefore, the fuzzy subsets are introduced into the evidence body space, and then we can get the final evaluation results by the formula of D-S evidence synthesis as follows$^{[11]}$.

$$m(A) = [m_1 \oplus m_2 \oplus ... \oplus m_n](A)$$

$$= \begin{cases} 0 & A = \Phi \\ \sum_{A \cap H_j = \phi} m_i(A_i) m_2(B_j) m_3(C_k) ... & A = \Phi \\ 1 - \sum_{A \cap H_j = \phi} m_i(A_i) m_2(B_j) m_3(C_k) ... & A \neq \Phi \end{cases}$$  (5)
ASSESSMENT OF POST EARTHQUAKE COLLAPSE RISK BASED ON FUZZY EVIDENCE THEORY

THE HAZARD ASSESSMENT INDEX SYSTEM OF POST EARTHQUAKE COLLAPSE

It is very important to select the assessment index in the hazard assessment of geological hazard, the quantity and quality of evaluation index is directly related to the accuracy of the evaluation\[12\]. Due to the complexity and many of factors influencing the earthquake collapse risk, and combined with the relevant research of earthquake collapse risk index system, this paper finally select 11 index as evaluation objects, denote it respectively with $C_i, C_j, ..., C_{11}$, meanwhile, in order to evaluate the risk of post earthquake collapse, through analyses of actual situation and expert opinion, and combine with the collapse standard of reference[12], the valuation value of evaluation index and the risk rating standard of post earthquake collapse were determined (see table 1).

**Table 1:** The evaluation classification of post-earthquake collapse risk

<table>
<thead>
<tr>
<th>The risk assessment system of the earthquake collapse</th>
<th>Risk level</th>
</tr>
</thead>
<tbody>
<tr>
<td>evaluation factors</td>
<td>evaluation index</td>
</tr>
<tr>
<td>topography and geomorphology $B_1$</td>
<td>slope gradient $C_1$</td>
</tr>
<tr>
<td></td>
<td>height of the slope $C_2$</td>
</tr>
<tr>
<td></td>
<td>size of dangerous rock $C_3$</td>
</tr>
<tr>
<td>geological structure $B_2$</td>
<td>formation lithology $C_4$</td>
</tr>
<tr>
<td></td>
<td>slope structure type $C_5$</td>
</tr>
<tr>
<td>hydrological conditions $B_3$</td>
<td>vegetation coverage $C_6$</td>
</tr>
<tr>
<td></td>
<td>rainfall effect $C_7$</td>
</tr>
<tr>
<td></td>
<td>effect of groundwater weathering $C_8$</td>
</tr>
<tr>
<td>other factors $B_4$</td>
<td>earthquake intensity $C_9$</td>
</tr>
<tr>
<td></td>
<td>damage to the highway $C_{10}$</td>
</tr>
</tbody>
</table>
APPLYING THE METHOD OF AHP TO CONFIRM THE INDEX WEIGHT

After the hierarchy of risk evaluation index of post earthquake collapse is established, in order to quantify the evaluation index. Firstly, the classification of collapse risk is regarded as target layer; therefore, we could treat the hierarchy structure model as project hierarchy by the evaluation index of layer. Using the method of AHP to calculate the weight of evaluation indexes at all levels, moreover, the Various indicators at the same level compare each other for the importance on a layer of indicators, which is been imitated quantitative by the "9 scale table", As shown in table 2. Thus, a constructing judgement matrix of AHP is formed, it is one of the key steps to construct decision matrix when obtaining index weight by hierarchy process, in this paper, the concrete construct decision matrix is omitted for the influence of length. Finally, each stage index system weight is determined by the "mediation method[13]."

<table>
<thead>
<tr>
<th>scale</th>
<th>relatively regular</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>the index I is the equally important with index J</td>
</tr>
<tr>
<td>3</td>
<td>the index I is little important than index J</td>
</tr>
<tr>
<td>5</td>
<td>the index I is obviously important than index J</td>
</tr>
<tr>
<td>7</td>
<td>the index I is more important than index J</td>
</tr>
<tr>
<td>9</td>
<td>the index I is extremely important than index J</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>the adjacent judgment of value respectively</td>
</tr>
</tbody>
</table>

DETERMINE THE BASIC TRUST FUNCTION AND MASS FUNCTIONS SYNTHESIS

The score of evaluation index is given by the experience of several experts, combined with the weight by the AHP. Thus, we can get the fuzzy membership probability of evaluation index from each expert. At the same time, the unknown information is merged into the identification framework $\Theta$, the map defined is used to get the basic reliability distribution value under of fusion result, and then, every body of evidence can be fused on the Basis of evidence synthesis rules, the mass function of the index synthetic is obtained, finally, we can determine the risk level of earthquake collapse by the risk identification framework.

THE FLOW CHART OF THE EARTHQUAKE COLLAPSE RISK BASED ON FUZZY EVIDENCE THEORY

Based on the risk evaluation index system of the post earthquake collapse, this article take the set of risk grade as identification framework, and also take the evaluation index as evidence body, thus, the mass function can be obtained. It is the aim of evaluating comprehensive the collapse with the fusion results. The basic diagram is shown in figure 1:
In order to describe the applicability and effectiveness of the fuzzy evidence theory, on the basis of out-site inspection and detailed analysis of geology data, the author take the collapse point BT54 in the hanchuan county of Sichuan province as example for evaluating. Firstly, the classification and grading of each evaluation index of BT54 can be obtained by the characteristics of fuzzy evidence theory, the evaluation set is $H = \{H_1, H_2, H_3, H_4\}$, which is correspond to the high risk, medium risk, weak risk and no risk. All of the focal element are single-hypothesis set, that is to say, the evaluation conclusion is only come from the elements of $\Theta$. secondly, aimed at the 11 evaluation factors of the rule hierarchy $B = \{B_1, B_2, ..., B_n\}$, according to 11 evidences come from observed data, therefore, the evidences space $E = \{e_1, e_2, ..., e_11\}$ can be determined. And we can take the $E = \{e_1, e_2, ..., e_11\}$ as the universe, which has Contain finite element. Thus, the $F_1, F_2, ..., F_{11}$ can be defined as the fuzzy subset of universe $E$. but the $F_{i+1}$ is better than $F_i$, these fuzzy subset can be expressed relatively accurate the judgment of expert, the membership function is shown in figure 2 ~ figure 5. Then, the relative importance of the multiple comparison of upper target by using the "9 scale method". According to the calculation results: $\omega = \{0.07, 0.03, 0.09, 0.24, 0.23, 0.01, 0.03, 0.05, 0.02, 0.15, 0.08\}$. As can be seen from the above 11 index weight, formation lithology and slope structure type are the most important factors affecting the collapse risk, next came the earthquake intensity, the next is the other Influence factors, and The evaluation results is basically tallies with practical investigation. Then, in view of the criterion layer $B_1$, $P(F) = \{0.03, 0.26, 0.12, 0.12\}$ is obtained by the formula (2). Due to the judging set $H = \{H_1, H_2, H_3, H_4\} = \{I, II, III, IV\}$, the mass function of rule hierarchy can be obtained by the define the mapping $T$, By the formula (4): $T|p|(H_1) = P(F_1 \cup F_2) = 0$. Similarly, the mass function of rule hierarchy $B_2, B_3, B_4$ also can be determined, the Mass functions synthesis are shown in table 3.
\[
T(F_j) = \begin{cases} 
H_1 & j = 1 \\
H_2 & j = 2 \\
H_3 & j = 3, 4 \\
H_4 & j = 5, 6 
\end{cases}
\]

\[
T[p(H_j)] = \begin{cases} 
0 & j = 1 \\
0.28 & j = 2 \\
0.42 & j = 3, 4 \\
0.30 & j = 5, 6 
\end{cases}
\]

Figure 2: The membership functions of \(B_1\)

Figure 3: The membership functions of \(B_2\)

Figure 4: The membership functions of \(B_3\)

Figure 5: The membership functions of \(B_4\)
Table 3: Combination of index mass function

<table>
<thead>
<tr>
<th>The index of guidelines layer</th>
<th>$H_1$</th>
<th>$H_2$</th>
<th>$H_3$</th>
<th>$H_4$</th>
<th>$\Theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>0</td>
<td>0.28</td>
<td>0.42</td>
<td>0.30</td>
<td>0</td>
</tr>
<tr>
<td>$B_2$</td>
<td>0</td>
<td>0.18</td>
<td>0.32</td>
<td>0.38</td>
<td>0.12</td>
</tr>
<tr>
<td>$B_3$</td>
<td>0</td>
<td>0.13</td>
<td>0.31</td>
<td>0.46</td>
<td>0.10</td>
</tr>
<tr>
<td>$B_4$</td>
<td>0</td>
<td>0.22</td>
<td>0.44</td>
<td>0.34</td>
<td>0</td>
</tr>
<tr>
<td>synthetic</td>
<td>0</td>
<td>0.06</td>
<td>0.52</td>
<td>0.42</td>
<td>0</td>
</tr>
</tbody>
</table>

By the synthesis of formula (5): $m(H) = \{0, 0.06, 0.52, 0.42\}$, It can be determined the confidence level of collapse point belongs to the dangerous risk small is 0.52, the confidence level belongs to medium risk is 0.06, and the confidence level belongs to no risk is 0.42, therefore, we can determine that the risk level of BT54 is Level III by the confidence maximum principle. In addition, In order to verify the scientificity and rationality of this method, a comparative analysis of the risk level of collapse point calculated by the suggested method, the results in literature [14] and the actual situation of post earthquake is presented in table 4, we can see that the evaluation results of two kinds of evaluation methods are consistent with the actual situation besides W33, the evaluation method based on fuzzy evidence theory are in complete accord with actual situation. The reason of that is the fuzzy evidence theory combines the advantages of the fuzzy sets theory and evidence theory, which has considered the importance of expert experience and combined the internal relation between the evaluation indexes. Therefore, the evaluation result can accurately reflect the situation of the earthquake collapse.

Table 4: The actual comparison table of each collapse

<table>
<thead>
<tr>
<th>the collapse point</th>
<th>unascertained measurement theory</th>
<th>fuzzy evidence theory</th>
<th>actual situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>BT54</td>
<td>III</td>
<td>III</td>
<td>III</td>
</tr>
<tr>
<td>BT09</td>
<td>III</td>
<td>III</td>
<td>III</td>
</tr>
<tr>
<td>BT49</td>
<td>II</td>
<td>II</td>
<td>II</td>
</tr>
<tr>
<td>BT66</td>
<td>III</td>
<td>III</td>
<td>III</td>
</tr>
<tr>
<td>BT80</td>
<td>III</td>
<td>III</td>
<td>III</td>
</tr>
<tr>
<td>W17</td>
<td>II</td>
<td>II</td>
<td>II</td>
</tr>
<tr>
<td>W33</td>
<td>II</td>
<td>III</td>
<td>III</td>
</tr>
</tbody>
</table>

CONCLUSION

The factors of affecting the earthquake collapse of risk assessment are complicated. Which have include both internal factors and external factors, and the collapse of the dangers of nonlinear mapping relationship between these factors is extremely complex. In view of the
disadvantage of present evaluation method of the earthquake collapse mechanism, the paper probes into the earthquake collapse risk by using the theory of fuzzy evidence theory. This method can not only overcome the single index evaluation and expert scoring method on the result of the earthquake collapse risk assessment, but also avoid the difficulties of lack of incomplete information data. Its application in earthquake collapse risk evaluation, which has improved the accuracy and validity of evaluation result, and it also has provided a new way of thinking for the earthquake collapse risk assessment.

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