The Precise and Efficient Numerical Model for Ground Penetrating Radar (GPR) Wave Propagation in Inhomogeneous Layered Structure

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ABSTRACT

Thickness and dielectric constants of pavement structure can be non-destructively obtained by inverse analysis of Ground Penetrating Radar (GPR) echo signal. Construction of GPR wave propagation model in layered structure is a key problem for inversion, and the media in simulation process are usually assumed to be homogeneous in the traditional way, but the actual road media are inhomogeneous and this assumption may lead to errors. The precise integration method (PIM) is developed to simulate GPR wave propagating in inhomogeneous layered structure in this paper. The numerical results show that simulated waveform calculated by inhomogeneous model is in an excellent agreement with measured signal. In addition, the proposed method can save significant CPU time compared with the traditional FDTD method.

KEYWORDS: Inhomogeneous media; layered structure; electromagnetic wave; precise integration method; numerical model

INTRODUCTION

Ground Penetrating Radar (GPR), as a kind of high resolution, non-destructive tool, has been widely used in quality detection of pavement structure. Accurate test of dielectric parameters of pavement media is the key to ensure precision of detection. There are two general ways to get the dielectric parameters of pavement media. One is the core-drilling test method, i.e. the dielectric parameters of pavement are assumed to be constant and the same as that of drilling core sample from pavement. This method cannot represent the dielectric properties of the whole pavement structure,
and it may lead to occur errors. The other is the inversion method, i.e. the dielectric parameters of pavement are obtained by back-calculation of GPR echo signal using optimization algorithm. This way is high precision, and widely used in GPR detection. One of the key steps for inversion based on the measured GPR signal is to construct an accurate and efficient numerical model of GPR wave propagating in pavement structure.

There are many ways to simulate GPR wave propagating in underground structures, such as transfer matrix method [1], ray tracing method [2], PSTD method [3], finite element method [4], time-domain finite difference (FDTD) method [5], and ADI-FDTD method [6]. Unfortunately, these methods are limited in their engineering applications due to some disadvantages. For example, The transfer matrix method is possible to cause numerical instability if media are lossy; the computational efficiency of FDTD method is limited to the Courant-Friedrichs-Lewy (CFL) stability condition; the ADI-FDTD method eliminates constraints of CFL condition, but a large time step will increase numerical dispersion. In addition, pavement media are usually assumed to be homogeneous in simulation process, but the actual road media are inhomogeneous. Homogeneous assumption may bring some errors.

Precise integration method (PIM) is an unconditionally stable and highly precise numerical algorithms. It can produce numerical results up to the precision of computer used, and is very suitable for solving wave propagation in layered structure [7] - [10]. In this paper, the PIM is developed to establish a numerical model of GPR wave propagation in inhomogeneous layered structure. The performance of the proposed model is verified by comparing FDTD method and measured signals.

**GOVERNING EQUATION**

![Figure 1: Diagram of GPR wave propagation in n-layer structure](image_url)

The process of the GPR electromagnetic wave propagation in n-layer structure is shown in Figure 1. Transmitting antenna emits electromagnetic pulse to the n-layer structure, and then the electromagnetic wave reflected and transmitted at the interface, finally, the reflected wave is received by receiving antenna. Considering the two-dimensional TE case, the Maxwell equations in frequency domain can be expressed as
\[
\frac{\partial H_y}{\partial z} = io\varepsilon \cdot E_x + \sigma \cdot E_z
\]
\[
\frac{\partial H_x}{\partial x} = io\varepsilon \cdot E_z + \sigma \cdot E_y
\]
\[
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial z} = -io\mu \cdot H_y - \sigma_m \cdot H_x
\]

where \( H \) is magnetic field, \( E \) is electric field, \( \varepsilon \) is dielectric constant, \( \sigma \) is conductivity, \( \sigma_m \) is magnetic conductivity, \( \mu \) is permeability, \( \omega \) is angular frequency, \( i \) is imaginary unit.

Substituting Eq. (2) into Eq. (3), we have
\[
\frac{\partial E_x}{\partial z} = \left( \frac{io\mu + \sigma_m}{i\omega\varepsilon + \sigma} \right) \cdot \frac{\partial^2 H_y}{\partial x^2}
\]
\[
\frac{\partial H_y}{\partial z} = \left( \frac{i\omega\varepsilon + \sigma}{i\omega\mu + \sigma_m} \right) \cdot E_x
\]

Frequency domain formulation of electric and magnetic fields in the plane problem can be written as
\[
E(x, z) = E(z) \cdot \exp(ikx)
\]
\[
H(x, z) = H(z) \cdot \exp(ikx)
\]

where \( k \) is wave-number.

Substituting Eq. (6) and Eq. (7) into Eq. (4) and Eq. (5) yields
\[
\frac{\partial E_x}{\partial z} = \left( \frac{io\mu + \sigma_m + \frac{k^2}{i\omega\varepsilon + \sigma}}{i\omega\mu + \sigma_m} \right) \cdot H_y
\]
\[
\frac{\partial H_y}{\partial z} = \left( \frac{i\omega\varepsilon + \sigma}{i\omega\mu + \sigma_m} \right) \cdot E_x
\]

Eq. (8) and Eq. (9) can also be expressed in the form of first-order ordinary differential equations as follows
\[
v' = T \cdot v
\]

where \( v = \left[ E_x, H_y \right]^T \), \( v' = \frac{\partial v}{\partial z} \), \( T \) is the \( 2 \times 2 \) matrix, the elements of which are
\[
T_{11} = T_{22} = 0, T_{12} = \sigma_m + i\omega\mu + k^2/(i\omega\varepsilon + \sigma) \text{ and } T_{21} = \sigma + i\omega\varepsilon.
\]
PRECISE INTEGRATION METHOD

For linear systems, the following relationship of the field values at the two ends of arbitrary interval \([z_a, z_b]\) stands

\[
\begin{align*}
E_b &= F E_a - G H_b \\
H_b &= Q E_a + P H_b
\end{align*}
\]  

(11)  

(12)

in which \(E_a\) and \(H_a\) are the electric and magnetic fields at \(z = z_a\), \(E_b\) and \(H_b\) are the electric and magnetic fields at \(z = z_b\), for the convenience of description, the subscript \(x\) and \(y\) of electric and magnetic fields are omitted. \(F, G, P\) and \(Q\) are all complex variables to be determined, which are only relevant with \(z_a\) and \(z_b\).

Assuming that \(E_a\) and \(H_a\) are given, and differentiating Eq. (11) and Eq. (12) with respect to \(z_b\) yield

\[
\begin{align*}
E_b' &= F E_a - G H_b' - G H_b \\
0 &= Q E_a + P H_b' + P H_b'
\end{align*}
\]  

(13)  

(14)

The governing equation at \(z_b\) can be written as

\[
\begin{align*}
E_b &= T_1 E_a + T_{12} H_b \\
H_b &= T_2 E_a + T_{22} H_b
\end{align*}
\]  

(15)  

(16)

Using Eq. (11) - Eq. (16) gives

\[
\begin{align*}
(F' - T_{11} F - G T_{12} F) E_a + (-G' - T_{12} - G T_{22} + T_{11} G + G T_{22}) H_b &= 0 \\
(Q' + P T_{22} F) E_a + (-P T_{22} G + P T_{22}) H_b &= 0
\end{align*}
\]  

(17)  

(18)

Noting that \(E_a\) and \(H_a\) are mutually independent yields the following equations

\[
\begin{align*}
F' &= T_{11} F + G T_{12} F \\
P' &= P T_{22} G - P T_{22} \\
G' &= -T_{12} - G T_{22} + T_{11} G + G T_{22} \\
Q' &= -P T_{22} F
\end{align*}
\]  

(19)

Combination of adjacent intervals

Consider two contiguous intervals \([z_a, z_b]\) and \([z_b, z_c]\), and applying Eq. (11) and Eq. (12) to them leads to
The intervals \([z_a, z_z]\) and \([z_b, z_c]\) can be combined into a large interval \([z_a, z_c]\) with

\[
E_c = F_1 E_a - G_1 H_c
\]

\[
H_a = Q_1 E_a + P_1 H_c
\]

Solving \(E_b\) and \(H_b\) by Eq. (20) and Eq. (23) gives

\[
E_b = (1 + G_2) j^i F_1 E_a - (G_2 j^i + Q_2) j^i P_1 H_c
\]

\[
H_b = (Q_2 j^i + G_2) j^i F_1 E_a - (1 + Q_2 G_2) j^i P_1 H_c
\]

Substituting Eq. (26) and Eq. (27) in Eq. (21) and Eq. (24) yields

\[
E_c = F_1 (1 + G_2 j^i) j^i F_1 E_a - [G_2 + F_1 (G_2 j^i + Q_2) j^i P_1] H_c
\]

\[
H_a = [Q_2 + P_1 (Q_2 j^i + G_2) j^i F] E_a + P_1 (1 + Q_2 G_2) j^i P_1 H_c
\]

Comparing Eq. (22) and Eq. (23) with Eq. (28) and Eq. (29) gives

\[
F_c = F_1 (1 + G_2) j^i F_1
\]

\[
G_c = G_2 + F_1 (G_2 j^i + Q_2) j^i P_1
\]

\[
Q_c = Q_2 + P_1 (Q_2 j^i + G_2) j^i F
\]

\[
P_c = P_1 (1 + Q_2 G_2) j^i P_1
\]

**Calculation of interval variables**

Eq. (30) has shown how two adjacent interval variables operate, but now no interval variable has been given, and only the system variables \(T_{11}, T_{22}, T_{12}\) and \(T_{21}\) are known. Therefore, it is now necessary to generate a set of interval variables from \(T_{11}, T_{22}, T_{12}\) and \(T_{21}\).

Assume the thickness of the \(i^{th}\) layer is \(d_i = z_i - z_{i-1}\). Firstly, it is divided into \(2^M\) (\(M\) is 6) sub-layers with uniform thickness \(d_i^{\text{sub}} = d_i / 2^M\). Then each sub-layer is further divided into \(2^N\) (\(N\) is 20) mini-layers with equal thickness \(d_i^{\text{sub}} / 2^N\). Because the layer thickness \(t\) here is extremely small, interval variable \(F, G, P\) and \(Q\) of mini-layer can be obtained by Taylor series expansion

\[
Q(t) = q_1 t + q_2 t^2 + q_3 t^3 + q_4 t^4
\]

\[
G(t) = g_1 t + g_2 t^2 + g_3 t^3 + g_4 t^4
\]

\[
F(t) = f_1 t + f_2 t^2 + f_3 t^3 + f_4 t^4
\]

\[
P(t) = p_1 t + p_2 t^2 + p_3 t^3 + p_4 t^4
\]

\[
F(t) = 1 + F'(t)
\]

\[
P(t) = 1 + P'(t)
\]

where \(q_i, g_i, f_i\) and \(y_i\) \((i = 1, 2, 3, 4)\) are complex constant.
Substituting Eq. 31 into Eq. 19 and comparing coefficients of various powers of $t$ give the following equations

\[
\begin{align*}
q_1 &= -T_{21}, g_1 = -T_{21}, j_1 = T_{11}, y_1 = -T_{22}, \\
q_2 &= -(y_1 T_{21} + T_{21} j_1)/2, g_2 = (T_{21} g_1 - g_1 T_{22})/2, \\
j_2 &= (T_{11} j_1 + g_1 T_{21})/2, y_2 = (T_{21} g_1 - y_1 T_{22})/2, \\
q_3 &= -(y_2 T_{21} + T_{21} j_2 + y_1 T_{21} j_1)/3, \\
g_3 &= (T_{11} g_2 - g_2 T_{22} + g_1 T_{21} g_1)/3, \\
j_3 &= (T_{11} j_2 + g_2 T_{21} + g_1 T_{21} j_1)/3, \\
y_3 &= (T_{21} g_3 + y_1 T_{21} g_1 - y_2 T_{22})/3, \\
q_4 &= -(y_3 T_{21} + T_{21} j_3 + y_2 T_{21} j_2 + y_1 T_{21} j_1)/4, \\
g_4 &= (T_{11} g_4 - g_4 T_{22} + g_2 T_{21} g_1 + g_1 T_{21} g_2 + g_1 T_{21} g_2)/4, \\
j_4 &= (T_{11} j_4 + g_3 T_{21} + g_2 T_{21} j_1 + g_1 T_{21} j_2 + g_1 T_{21} j_2)/4, \\
y_4 &= (T_{21} g_4 + y_1 T_{21} g_1 + y_2 T_{21} g_2 - y_3 T_{22})/4
\end{align*}
\]

(32)

Note that $F(t) = 1 + F'(t)$ and $P(t) = 1 + P'(t)$, in which $F$ and $P$ are very small because $t$ is extremely small. Therefore, it is important that $F$ and $P$ must be computed and stored independently so as to avoid losing effective digits. Hence Eq. (30) needs to be rewritten as

\[
\begin{align*}
F' &= (F'_1 - G_1 Q_1 - G_1 Q'_1 - G_1 Q'_P + F'_1 - G_1 Q_1 + G_1 Q'_P)/2 + P'_1 (G_1 Q_1 + G_1 Q'_P) + P'_P (G_1 Q_1 + G_1 Q'_P), \\
P'_c &= (P'_c - G_1 Q_1 - G_1 Q'_1 - G_1 Q'_P + P'_c - G_1 Q_1 + G_1 Q'_P)/2 + P'_P (G_1 Q_1 + G_1 Q'_P), \\
G_c &= G_c + P'_c (G_1 Q_1 + G_1 Q'_P) + P'_P (G_1 Q_1 + G_1 Q'_P), \\
Q_c &= Q_c + P'_c (G_1 Q_1 + G_1 Q'_P) + P'_P (G_1 Q_1 + G_1 Q'_P)
\end{align*}
\]

(33)

Once mini-layer interval variables $F_c, G_c, P_c$ and $Q_c$ have been obtained. Combined interval variables $F_c, G_c, P_c$ and $Q_c$ can be calculated by Eq. (33). As all intervals have equal thickness, $P = P_c, F = F_c$, and so on. Each pass of such interval combination reduces the number of intervals by a half. When all $N(=20)$ passes of such combinations have been finished, $F_{sub}, G_{sub}, P_{sub}$ and $Q_{sub}$ for sub-layer can be obtained. Then interval variables $F, G, P, Q, g_i$ of $i$th layer can be calculated by interval variables of sub-layers similarly. It should be noted that the combination formulas for the interval matrices are given by Eq. (30) because the layer thickness is not very small now. The combination of the interval matrices of the n layers into global interval variables $F_{total}, G_{total}, P_{total}$, and $Q_{total}$ is similar to the above by Eq. (30).
Source excitation

The sources are taken as input of electric and magnetic field components at the surface \((z = z_0)\) of \(n\)-layer structure (see Figure 1), and then the following boundary conditions are applied

\[
\begin{align*}
E(k, \omega, z_0^+) + E_u(k, \omega) &= E(k, \omega, z_0^-) \\
H(k, \omega, z_0^+) + H_u(k, \omega) &= H(k, \omega, z_0^-)
\end{align*}
\]

in which \(E_u(k, \omega)\) and \(H_u(k, \omega)\) are the input values of the electric and magnetic field in frequency-wavenumber domain. \(z_0^+\) and \(z_0^-\) represent the upper and lower faces of the interface \(z = z_0\), respectively.

Upper boundary condition

The wave motion equations should also satisfy the radiative conditions in semi-infinite space. Now assume that the state equation in the upper semi-infinite space, i.e. the 0th layer, is

\[
\dot{\mathbf{u}}_u = \mathbf{T}_u \cdot \mathbf{v}_u = \mathbf{M}_u \Lambda_u \mathbf{M}_u^{-1} \mathbf{v}_u
\]

where subscript \(u\) represents the medium of upper semi-infinite space, \(\Lambda_u = \text{diag}(\lambda_{u1}, \lambda_{u2})\) is the eigenvalue matrix of \(\mathbf{T}_u\), \(\mathbf{M}_u = (\mathbf{a}_{u1}, \mathbf{a}_{u2})\) is the corresponding eigenvector matrix, \(\mathbf{a}_u\) \((i=1, 2)\) is eigenvector corresponding to the eigenvalue \(\lambda_u\). Note that these eigenvalues and eigenvectors should be arranged such that the first corresponds to the upward waves while the second corresponds to the downward waves.

Let \(\mathbf{b}_u(z) = \mathbf{M}_u^{-1} \mathbf{v}_u(z)\) and Eq. (34) can be expressed as

\[
\dot{\mathbf{b}}_u = \mathbf{A}_u \mathbf{b}_u
\]

Based on first-order ordinary differential equation theory, the solution of Eq. (35) is

\[
\mathbf{b}_u(z) = \exp[\mathbf{A}_u(z - z_0)] \cdot \mathbf{b}_u(z_0) \quad z > z_0
\]

in which \(\mathbf{b}_u(z_0)\) is a 2×1 vector, the first element represents the upwards travelling waves, and the second element represents the downward travelling waves. The radiation condition requires that no
downward traveling waves exist in upper semi-infinite space, i.e. the second element of $b_1(z_0)$ must be equal to 0, therefore

$$v_0^+ = M_0 \begin{bmatrix} b_{11} \\ 0 \end{bmatrix} = \begin{bmatrix} a_{n11} & a_{n21} \\ a_{n12} & a_{n22} \end{bmatrix} \begin{bmatrix} b_{11} \\ 0 \end{bmatrix}$$ (37)

where the subscript 0 represents values at $z = z_0$, $b_{11}$ is the first element in $b_1(z_0)$. From Eq. (37) we have

$$E_0^+ = a_{n11} \cdot b_{11}, \quad H_0^+ = a_{n12} \cdot b_{11}$$ (38)

$$H_0^+ = a_{n21} \cdot a_{n11}^{-1} \cdot E_0^+, \quad R_0 = a_{n12} \cdot a_{n11}^{-1}$$ (39)

Eq. (39) is the radiative condition in upper semi-infinite space.

**Lower boundary condition**

Similarly, the solution of the state equation in lower semi-infinite space, i.e. the (n+1)th layer, can also be expressed as

$$b_1(z) = \exp[-\Lambda_b \int z_0^z \] \cdot b_1(z_n) \quad z < z_n$$ (40)

where subscript d represents lower semi-infinite space media. The radiation condition requires that no upper traveling waves exist in lower semi-infinite space, i.e. the first element of $b_1(z_n)$ must be zero. Therefore

$$v_0^- = \begin{bmatrix} a_{d11} & a_{d21} \\ a_{d12} & a_{d22} \end{bmatrix} \begin{bmatrix} 0 \\ b_{d2} \end{bmatrix}$$ (41)

$$E_n^- = a_{d21} \cdot b_{d2}, \quad H_n^- = a_{d22} \cdot b_{d2}$$ (42)

$$H_n^- = a_{d22} \cdot a_{d21}^{-1} \cdot E_n^-, \quad R_d = a_{d22} \cdot a_{d21}^{-1}$$ (43)

Eq. (43) is the radiation boundary conditions in lower semi-infinite space.

**Continuous conditions at the interfaces**

Assuming that the electric and magnetic fields are continuous at the interface gives

$$E(z = z_i^+) = E(z = z_i^-), (i = 1, 2, \cdots, n)$$ (44)

$$H(z = z_i^+) = H(z = z_i^-), (i = 1, 2, \cdots, n)$$ (45)
THE SIMULATION PROCESS OF GPR WAVE PROPAGATION IN LAYERED STRUCTURE BY PIM

Simulation process of GPR wave propagation in layered structure by PIM can be summarized as the following steps:

(1) Read the incident wave in time-space domain;
(2) Transform the incident wave in time-space domain into frequency-wavenumber domain;
(3) Define $i^{th}$ layer system matrix $T_i$ corresponding each angular frequency $\omega$, wave-number $k$, and dielectric parameters of the $i^{th}$ layer using Eq. (32);
(4) Solve interval variables of the $i^{th}$ layer. Firstly, mini-layer interval variables $F_i, G_i, P_i$ and $Q_i$ are calculated using Eq. (31); secondly, the sub-layer interval variables $F_{sub}, G_{sub}, P_{sub}$ and $Q_{sub}$ are obtained after $N(=20)$ passes of combinations by Eq. (33); finally, the $i^{th}$ layer interval variables $F_i, G_i, P_i$ and $Q_i$ are given using Eq.(30) after $M(=6)$ passes of combinations;
(5) Repeat steps (3) and (4), and the interval variables of each layer in n-layer structure are obtained;
(6) Solve the whole interval variables $F_{total}, G_{total}, P_{total}$ and $Q_{total}$ of the n-layer structure by Eq.(30)

$$E_n = F_{total} \cdot E_n - G_{total} \cdot H_n$$

$$H_0 = Q_{total} \cdot E_n + P_{total} \cdot H_n$$

(7) Substitute the whole interval variables $F_{total}, G_{total}, P_{total}, Q_{total}$, boundary conditions Eq. (39) and Eq. (43) and excitation source Eq.(44) and Eq. (45) into governing equation (10)

$$E_n = F_{total} \cdot E_n - G_{total} \cdot R_d \cdot E_n + F_{total} \cdot E_n$$

$$R_d \cdot E_0 = Q_{total} \cdot E_n + P_{total} \cdot R_d \cdot E_n + Q_{total} \cdot E_n - H_n$$

(8) Solve the reflected signal $E_n^*$ from Eq.(48) and Eq.(49)

$$E_n^* = \left( R_n - Q_d \cdot P_r \cdot R_d \cdot (1+G_r \cdot R_d) \right)^{-1} \cdot P_r \cdot R_d \cdot (1+G_r \cdot R_d)^{-1} \cdot F \cdot E_n + Q_d \cdot E_n - H_n$$

(9) Transform the reflected signal into time-space domain.
If the media are inhomogeneous, i.e. dielectric parameters of media varies with change of depth, the layer should be subdivided into several sub-layers, and each sub-layer is set different dielectric parameters. Solving process is similar to the above.

**NUMERICAL EXAMPLES**

**Example 1: four-layer semi-rigid pavement structure**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Material</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface</td>
<td>Asphalt concrete, 20cm</td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>Cement stabilized gravel, 20cm</td>
<td></td>
</tr>
<tr>
<td>Subbase</td>
<td>Limestone soil, z=35cm</td>
<td></td>
</tr>
<tr>
<td>Subgrade</td>
<td>Soil, lower semi-infinite space</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 2:** Diagram of the semi-rigid pavement structure

The designed geometry profile of four-layer semi-rigid pavement structure is shown in Figure 2. The test is developed by Pulse-V GPR operated at 1GHz center frequency. Because the air-coupled antenna is adopted, incident wave can be regarded as approximately uniform plane wave, i.e. the strength of electric and magnetic field is not related to wave-number k. Hence the matrix $T$ of Eq. (10) can be simplified as $T_{11} = T_{12} = 0$, $T_{12} = \sigma + i\omega \mu$ and $T_{21} = \sigma + i\omega \epsilon$. The input values of electric and magnetic field in Eq. (44) and Eq. (45) can be simplified to $E_n(\omega)$ and $H_n(\omega)$. The GPR incident signal (shown in Figure 3) can be recovered by placing a sufficiently large metal plate under the antenna.
Figure 3: Incident wave

<table>
<thead>
<tr>
<th>Media</th>
<th>Relative Permittivity</th>
<th>Conductivity (S/m)</th>
<th>Permeability (F/m)</th>
<th>Magnetic Permeability (Ω/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1</td>
<td>0</td>
<td>$\mu_0$</td>
<td>0</td>
</tr>
<tr>
<td>Surface</td>
<td>6</td>
<td>0.02</td>
<td>$\mu_0$</td>
<td>0</td>
</tr>
<tr>
<td>Base</td>
<td>13</td>
<td>0.02</td>
<td>$\mu_0$</td>
<td>0</td>
</tr>
<tr>
<td>Subbase</td>
<td>13</td>
<td>0.05</td>
<td>$\mu_0$</td>
<td>0</td>
</tr>
<tr>
<td>Subgrade</td>
<td>15</td>
<td>0.05</td>
<td>$\mu_0$</td>
<td>0</td>
</tr>
</tbody>
</table>

$\mu_0$ is vacuum permeability.

Dielectric parameters choices of four-layer semi-rigid pavement structure are collected in Table 1. Figure 4 and Figure 5 give comparison of the measured signal and simulated waveform by PIM and FDTD, respectively. As can be observed in Figure 4 and Figure 5, the results of two methods all fit well with the measured waveform, but computing time needed to PIM and FDTD are 0.4173s and 0.5895s, respectively. The PIM can save 40% CPU time compared with FDTD method.
Example two: Inhomogeneous layered media model

As shown in Figure 4, homogeneous media model can fit well with the measured wave in many aspects, such as wave amplitude, latency, but the fitting is not perfect. In this example, the surface of four-layer semi-rigid pavement structure is considered as inhomogeneous media, i.e. dielectric constants change with the depth. The surface layer is further divided into ten sub-layers with equal thickness, and each sub-layer is set different dielectric constants (shown in Tab. 2), the other parameters remain unchanged. Figure 6 shows the comparison diagram of the analog waveform and the measured waveform.
Table 2: sub-layer dielectric constant setting

<table>
<thead>
<tr>
<th>Sub-layer Number</th>
<th>Relative Permittivity</th>
<th>Sub-layer Number</th>
<th>Relative Permittivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z₁</td>
<td>6.7</td>
<td>Z₂</td>
<td>5.1</td>
</tr>
<tr>
<td>Z₃</td>
<td>5.8</td>
<td>Z₄</td>
<td>6.4</td>
</tr>
<tr>
<td>Z₅</td>
<td>5.6</td>
<td>Z₆</td>
<td>6.5</td>
</tr>
<tr>
<td>Z₇</td>
<td>5.6</td>
<td>Z₈</td>
<td>6.6</td>
</tr>
<tr>
<td>Z₉</td>
<td>7.3</td>
<td>Z₁₀</td>
<td>6.8</td>
</tr>
</tbody>
</table>

Figure 6: Comparison between the reflected waveform simulated by inhomogeneous model and that measured by GPR in the field.

As shown in Figure 6, excellent agreement is achieved between waveform simulated by inhomogeneous model and the measured one.

CONCLUSION

PIM with two-point boundary value conditions is presented to simulate GPR wave propagating in layered structure in this paper. Compared with measured GPR echo signal, simulated results show that the peak amplitude and time delay of simulated waveform calculated by homogeneous model fits well with the measured signal, but some other aspects are not very well; the simulated waveform calculated by inhomogeneous model is in an excellent agreement with measured one. In addition, the proposed method can save CPU time 40% compared with the FDTD method. Therefore, the proposed numerical model in this paper can improve simulation precision effectively.
REFERENCES


