Stability Design Charts for Unsupported Wide Rectangular Tunnels

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ABSTRACT

This paper investigates the stability of a plane strain wide rectangular tunnel in cohesive soils. A strength reduction technique and the finite difference program *FLAC* are used to determine the factor of safety for unsupported wide rectangular tunnels. Results from the finite difference approach are compared with published rigorous upper and lower bound analysis. The comparison between these two methods finds very good agreement. Stability design charts for wide rectangular tunnels are presented for a wide range of practical scenarios using dimensionless ratios. The potential usefulness of this approach is demonstrated using a number of examples. This is considered a simpler way of analysing the stability of an unsupported tunnel.

KEYWORDS: Wide Rectangular Tunnel, Stability, Undrained Clay, Factor of Safety, FLAC, Shear Strength Method, Design Chart

INTRODUCTION

Nowadays underground tunnels have been constructed using TBM (Tunnel Boring Machine). The cross section of these tunnels is normally circular, and they are preferred in most designs due to geometric arching effects, which contribute significantly to tunnel stability. In comparison to circular tunnels, wide rectangular sections are more difficult to build. Besides, they do not provide the same level of soil arching, and therefore soil stability is of a major concern in wide rectangular tunnel design. The wide rectangular problem is similar to underground long wall excavations and it was rarely addressed in the past; with very limited research literature can be found in relation to this research.

Tunnels have had to be produced in various and increasingly difficult ground conditions from very soft to stiff clays with varying depths of excavation. During tunnel design and construction it is essential to know the soil stability. This is most often known by using the stability number (N) proposed by Broms and Bennermark (1967) who initiated a pilot study of the plastic flow of clay soil in vertical openings of retaining walls. Their work was then extended to the experimental study of a tunnel face supported by an internal air pressure in Mair (1979). The stability number was defined in equation 1.

$$N = \frac{\sigma_s - \sigma_t + \gamma(C + D/2)}{S_u} \tag{1}$$

Where the surface discharge load is given by σ_s and the internal tunnel pressure is given by σ_t. C is the tunnel cover and D is the tunnel height. S_u and γ represent the undrained shear strength and the unit weight of the soil respectively.

The stability number presented in equation 1 has been re-defined and approached by the upper and lower bound solutions (Davis et al. 1980; Sloan et al. 1988, 1989 and 1991). The problem was regarded as to find a critical pressure ratio $(\sigma_s - \sigma_t)/S_u$. It is possible to further simplify this critical pressure ratio by neglecting σ_s and σ_t to simulate an unsupported excavation in green-field conditions, which are ideally suitable for the purpose of preliminary designs. The problem can be reduced to a simpler factor of safety problem by assuming both of σ_s and σ_t equalling zero. This factor of safety is simply a function of the depth ratio C/D and the strength ratio $S_u/\gamma D$ for the undrained clay investigated in this paper. This approach is very similar to Taylor's design chart for slope stability analysis (Taylor, 1937).

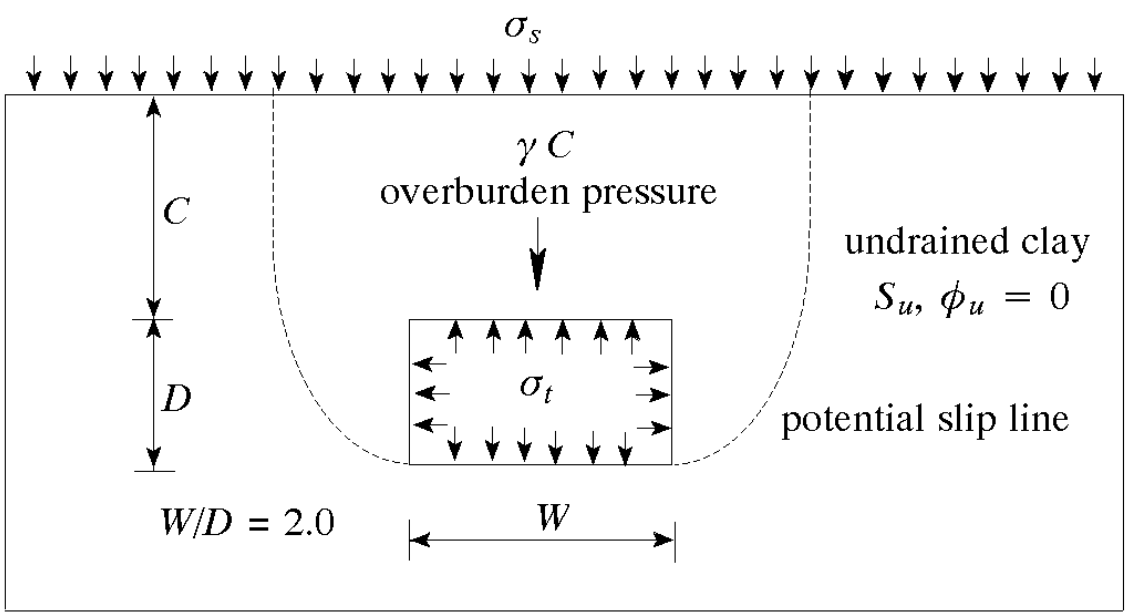

Figure 1: Problem definition of a wide rectangular tunnel (*W/D =2.0*)

Following Abbo et al. (2013) and Shiau et al. (2013 and 2014), this paper investigates the stability of wide rectangular tunnels in undrained clay. A strength reduction technique is used to determine the factor of safety in cohesive soils over a wide parametric range in green-field conditions. Results obtained from the strength reduction technique in *FLAC* are compared to those published using the finite element limit analysis *FELA*. A series of stability charts using the *FoS* approach is produced for practical applications.

PROBLEM DEFINITION AND MODELLING TECHNIQUE

Tunneling is a very complex three-dimensional problem in nature. Therefore it is often simplified to a two-dimensional one by assuming the transverse section as a very long tunnel. The idealized two-dimensional problem is defined in Figure 1.

The soil body is considered as undrained and modelled as a uniform Tresca material, which is the same as a Mohr-Coulomb material with zero soil friction angle (φ_u = 0), the non-zero undrained shear strength (S_u), and the saturated unit weight (γ). The dimension of the wide rectangular tunnel is ($W * D$) and the cover depth is C. The soil strength ratio (SR) is represented by $S_u/\gamma D$. The factor of safety (*FoS)* is used to represent the stability of the wide rectangular tunnel that is a function of the depth ratio (C/D) and strength ratio ($S_u/\gamma D$) as indicated in equation (2). The definition of a wide rectangular tunnel in this paper is for the width- height ratio W/D = 2.0. This ratio has been used to obtain all numerical results throughout this paper.

$$FoS = f\left(\frac{C}{D}, \frac{S_u}{\gamma D}\right) \tag{2}$$

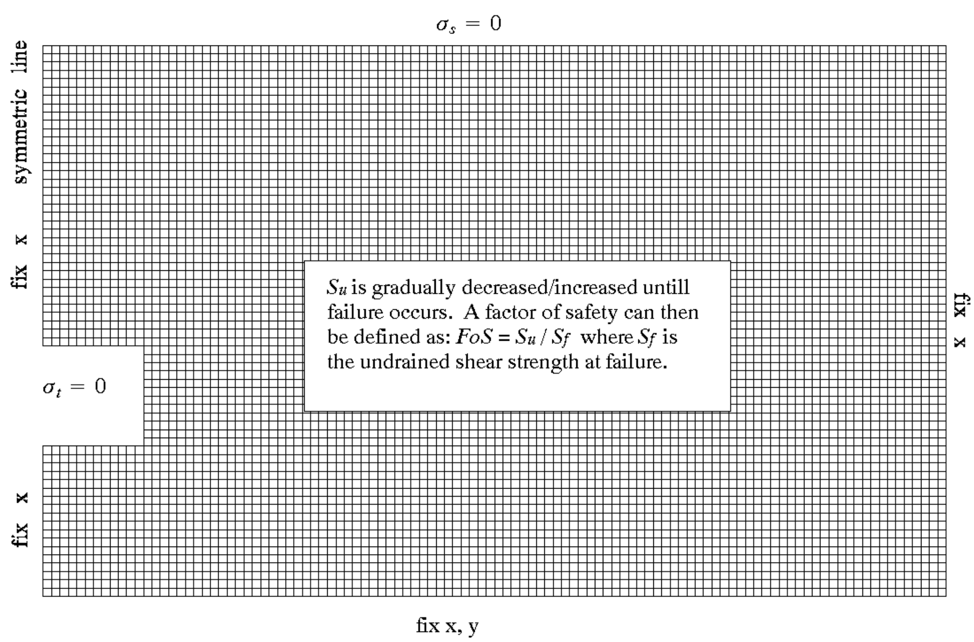

Figure 2: Typical half mesh and boundary conditions for a wide rectangular tunnel **(***W/D =2.0*)

A typical finite difference mesh of the problem in this paper is shown in Figure 2. The boundary conditions shown in the figure are important, they ensure that the entire soil mass is modelled accurately despite using a finite mesh. The soil domain size for each case was chosen to be large enough so that the failure zone of soil body is placed well with the domain. Note that the base and sides of the model are restrained in the *x* and *y* directions. For those nodes along the symmetrical line, only the x translation is restrained.

The shear strength reduction method is widely known and used for slope stability analysis by using finite element or finite difference methods, but this method has rarely been used for tunnel stability analysis. The factor of safety (*FoS*) being studied in this paper are computed by using explicit finite difference code via the software package *FLAC* and the built-in implementation of the strength reduction technique. Although the code is based on the explicit finite difference method, it is not very different from a nonlinear finite element program. The factor of safety is defined as a ratio of the strength necessary to maintain limiting equilibrium with the soil's available strength. If the material triggers the failure condition initially, then the cohesion and friction angle is increased until limiting equilibrium or failure state is reached. Once the actual and critical strength are known, they can then be used to calculate the factor of safety (*FoS*).

RESULTS AND DISCUSSION

Using the shear strength reduction method and the finite difference program *FLAC*, the values of factor of safety (*FoS)* were obtained for a range of parameters in undrained clay. This parametric study covered dimensionless parameters, including the depth ratio (*C/D*) and the strength ratio ($SR = S_u/\gamma D$). The parameters used in this paper are $S_u/\gamma D = 0.2 - 2$ and $C/D = 1 - 6$. This is to cover most of the realistic parameters and to ensure that the *FoS* design charts produced can be applicable to many different tunnel design and analysis problems.

Tables 1 and 2 present the numerical results obtained in this study. Graphical comparisons are also presented in Figures 3 and 4. In Figure 3, *FoS* increases linearly as the strength ratio $S_u/\gamma D$ increases, indicating that there exists a critical strength ratio $(SR)_c$ where the effective *FoS* is equal to one. This could be achieved by dividing the strength ratio ($SR = S_u/\gamma D$) by the *FoS* result for each case i.e. the critical strength ratio $(SR)_c = S_u/\gamma D(FoS)$. It should be noted that the rate of *FoS* increase is different for each *C/D* value. The gradient of the line is greater for smaller *C/D* values, indicating that by increasing soil strength for shallow tunnels will significantly improve tunnel stability. The benefit for deep tunnel is not that great when compared with shallow ones.

Figure 4 shows that the *FoS* decreases nonlinearly with increasing depth ratio *C/D* for all strength ratios defined as $S_u/\gamma D$. The strength ratio is normalised with respect to the γD, and the undrained shear strength (S_u) remains constant throughout the increasing depth ratios. When *C/D* increases and the undrained shear strength (S_u) remains constant, the results of *FoS* values decreases due to the increasing overburden pressure. This is different to the common belief that an increase to *C/D* always results in an increase to *FoS* due to the supporting arching effect.

Table 1: *FoS* results (*C/D* = 1, 2, and 3)

C/D	$S_u/\gamma D$	FLAC (Finite Difference) SSRM*
1	0.2	0.21
	0.4	0.41
	0.6	0.62
	0.8	0.82
	1.0	1.03
	1.3	1.34
	1.6	1.65

	2	2.06
2	0.2	0.20
	0.4	0.39
	0.6	0.59
	0.8	0.78
	1.0	0.98
	1.3	1.27
	1.6	1.56
	2	1.96
3	0.2	0.17
	0.4	0.35
	0.6	0.53
	0.8	0.70
	1.0	0.88
	1.3	1.14
	1.6	1.40
	2	1.75

** Shear Strength Reduction Method (SSRM)*

A simple observation can be made from Figure 1, where the active force (γC) is the weight of soil and the resisting force is given by the shear strength of the soil. Given two hypothetical tunnels in the same cohesive soil but at different depths, the tunnel with the smaller active force will have a higher factor of safety and therefore higher soil stability. This observation may not be true in a soil with internal friction angle due to the additional shear strength from the second term of the shear strength equation ($\sigma \tan\phi$) and the geometrical arching effects. In purely cohesive soils, the latter still occurs, but its effect is not enough to overcome that subsequent increase in active force.

Table 2: *FoS* results (*C/D* = 4, 5, and 6)

C/D	$S_u/\gamma D$	FLAC (Finite Difference) SSRM*
4	0.2	0.16
	0.4	0.32
	0.6	0.48
	0.8	0.64
	1.0	0.80
	1.3	1.04
	1.6	1.28
	2	1.60
	0.2	0.15
	0.4	0.29

5	0.6	0.44
	0.8	0.58
	1.0	0.72
	1.3	0.94
	1.6	1.16
	2	1.45
6	0.2	0.13
	0.4	0.27
	0.6	0.40
	0.8	0.53
	1.0	0.67
	1.3	0.87
	1.6	1.06
	2	1.33

* *Shear Strength Reduction Method (SSRM)*

Relying on one single numerical model is not prudent; result verification is required to improve the confidence in the obtained results. For this purpose, it is important to compare with rigorous upper bound and lower bound solutions. This is shown in Figure 5. The critical strength ratio $(SR)_c = S_u /\gamma D(FoS)$ is presented with various depth ratios *(C/D).* Note the finite difference results using shear strength reduction technique are in good agreement with the upper and lower bound solutions in Abbo et al. (2013).

Figures 6 and 7 show some typical plots of the shear strain rate for various values of *C/D* ($S_u/\gamma D$ = 1.0). This information of failure extent is important as it will assist practicing engineers in the decision making for monitoring ground movements. It was noted in this research that the strength ratios, $SR = S_u/\gamma D$, have no impact on the failure extent. This conclusion can be understood by observing the linear relationship between *FoS* and $S_u/\gamma D$ in Figure 3.

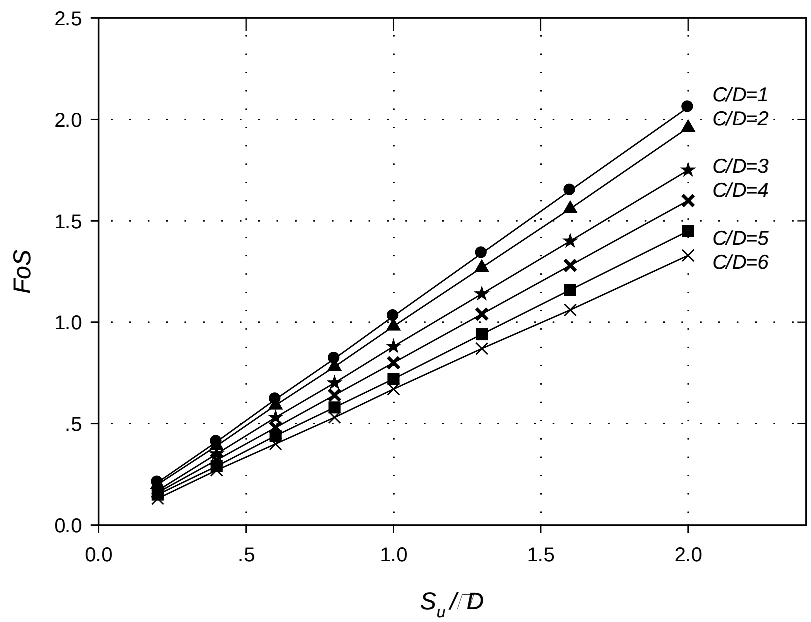

Figure 3: Comparison of *FoS* results with respect to $S_u/\gamma D$ for various values of *C/D*

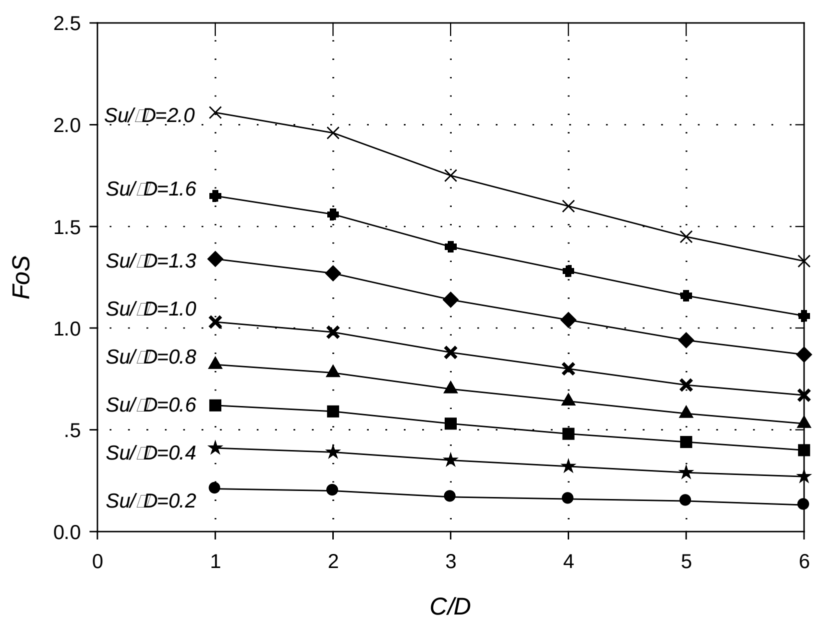

Figure 4: Comparison of *FoS* results with respect to *C/D* for various values of $S_u/\gamma D$

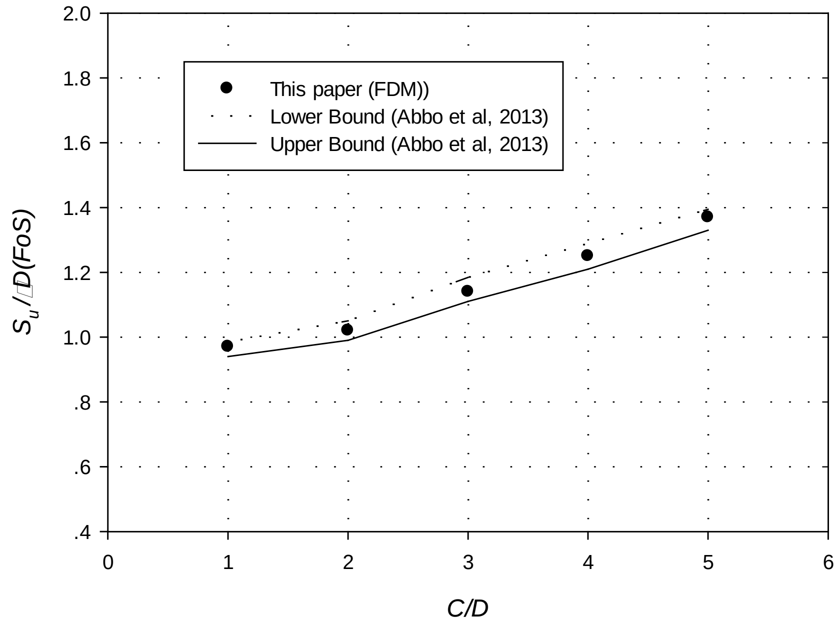

Figure 5: Comparison of critical strength ratio $(SR)_c = S_u/\gamma D(FoS)$

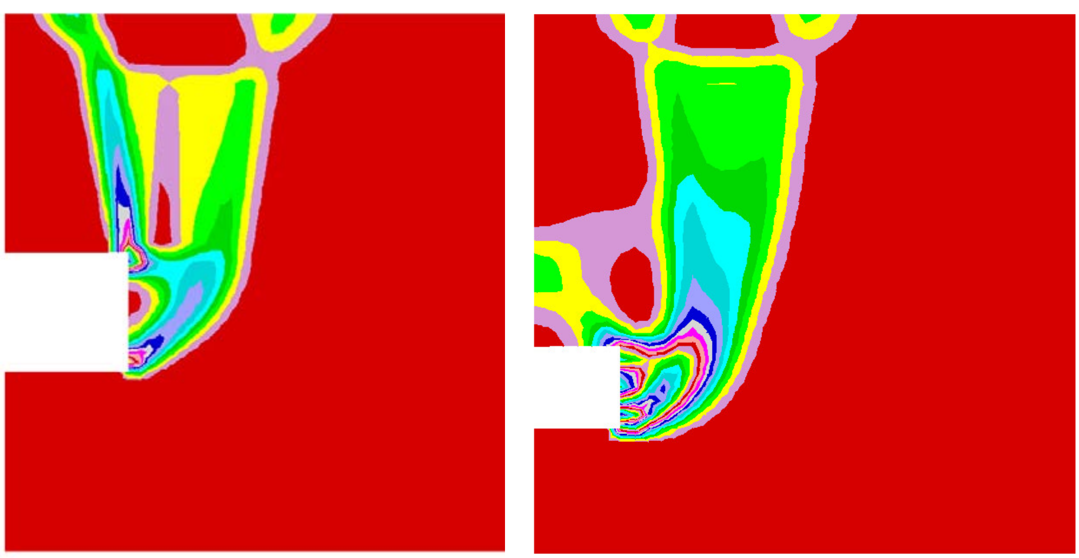

Figure 6: Plot of shear strain rate for *C/D* = 2.0 (left) and *C/D* = 4.0 (right)

Figure 7: Plot of shear strain rate for *C/D* = 6.0

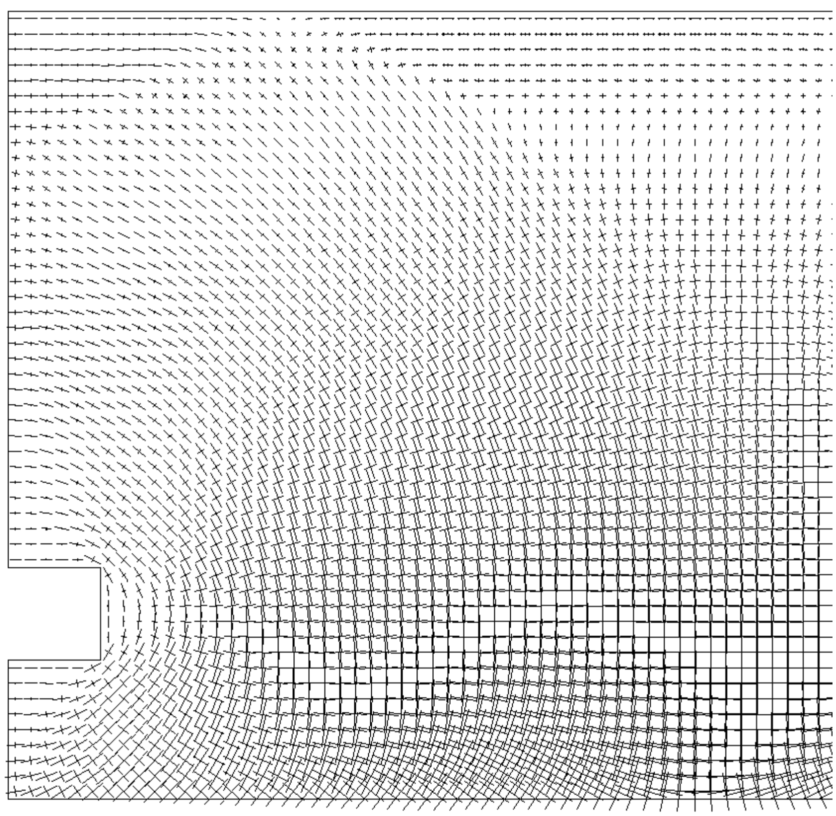

Figure 8: Principle stress tensor plot at collapse for *C/D*=6.0 and $S_u/\gamma D$ = 1.0

A typical principal stress tensor plot, which is normally used to study the potential effects of arching phenomenon is shown in Figure 8. This plot shows the directions of major and minor principal stresses, indicating weak soil arching throughout the soil body. As discussed, soils with an internal friction angle ($\phi \neq 0$) would have more potential for stability, with the internal frictional angle adding to the strength of the material by the soil arch due to the major principal stress rotation.

THE STABILITY CHART

The stability design chart is best demonstrated through a number of examples. Using the numerical results presented in Tables 1 and 2, a contour design chart for *FoS* has been produced in Figure 9 that can be used by tunnel engineers to relate the depth ratio (*C/D*), soil strength ratio ($S_u/\gamma D$) and factor of safety (*FoS*). Regression of the design chart gives the following relationship (equation 3) with $r^2 = 0.994$.

$$FoS = \left(\frac{S_u}{\gamma D}\right)\left\{0.0013\left(\frac{C}{D}\right) - 0.085\left(\frac{C}{D}\right) + 1.1238\right\} \quad (3)$$

Using the design chart (Figure 9) and equation (3), the following practical examples are illustrated for either analysis or design purposes.

Analysis of an existing unsupported wide rectangular tunnel

For an existing unsupported square tunnel without surcharge load (σ_s) and capacity to provide internal supporting pressure (σ_t), determine the factor of safety of the tunnel given the parameters S_u = 50 kPa, γ = 18 kN/m^3, C = 18 m, and D = 6 m.

1. Using C/D = 3.0, $S_u/\gamma D$ = 1.20, equation 3 gives a *FoS* of 1.06.

2. Using C/D = 3.0, $S_u/\gamma D$ = 1.20, Figure 9 gives an approximate *FoS* of 1.07 .

An actual computer analysis of this particular case gives a *FoS* of 1.05.

Design of an unsupported wide rectangular tunnel

The soil properties are known at the tunnel project site, and the dimension is specified. A target factor of safety is chosen, and the designers need to specify a maximum cover depth that will satisfy the target FoS. Parameters are given as: S_u = 172.8 kPa, γ = 18 kN/m^3, D = 6m, and the target *FoS* = 1.5

1. Using *FoS* = 1.5 and $S_u/\gamma D$ = 1.60, equation 3 gives a C value of 13.62 m (C/D = 2.27).

2. Using *FoS* =1.5 and $S_u/\gamma D$ = 1.60, Figure 9 gives an approximate C/D value of 2.3 and therefore C value of 13.80 m.

An actual computer analysis for this particular case (C value of 13.80m) gives a *FoS* of 1.51.

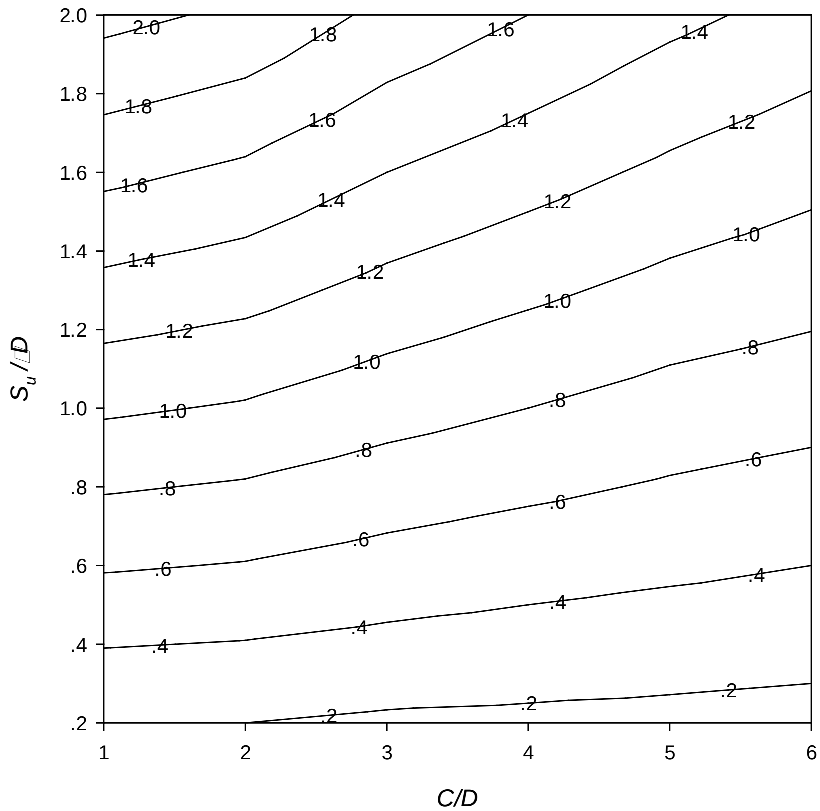

Figure 9: Stability chart for *FoS* with respect to *C/D* and $S_u/\gamma D$

CONCLUSION

Stability of plane strain wide rectangular tunnels was investigated in this paper. Numerical results of factor of safety (*FoS*) were computed by using a shear strength reduction method in *FLAC* and were compared with rigorous upper and lower bound limit analysis. The comparison of the results from both of these numerical approaches was very promising. Design charts and equation were then produced and two examples illustrated on how to use them.

It is concluded that the factor of safety approach to tunnel stability provides an alternative option for the designer and is a useful approach to provide direct information and understanding of tunnel stability. Further research is required on how this method can be applied to tunnels with a non-zero surcharge (σ_s) and internal pressure (σ_t).

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