Correlation of Dynamic and Static Elastic Parameters of Rock

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ABSTRACT
Differences exist in dynamic and static elastic parameters of the same rock. It has important significance for fracturing in reservoirs to obtain the transformation functions between dynamic and static elastic parameters, and experiments were done under reservoir conditions. Rock mechanical properties of tight sandstones under reservoir conditions (including formation temperature and pressure) in A Gas field were obtained. It demonstrated that the static elastic parameters (the static Young's modulus and the static Poisson's ratio) are different from the dynamic elastic parameters (the dynamic Young's modulus and the dynamic Poisson's ratio) in the same core samples under the same experimental conditions, when the temperature or pressure changes. Analyses on the mechanism of this phenomenon were done, and the transformation functions between the dynamic and static elastic parameters under different temperature or pressure were obtained. The parameters under reservoir conditions were useful in tight sandstones in A Gas field, and they are foundation for the development of oil layer fracturing program.

KEYWORDS: rock mechanics, Young's modulus, Poisson's ratio, static/dynamic elastic parameters, high pressure high temperature

INTRODUCTION
The natural productivity of the tight sandstones reservoir of A Gas field is low because of the dense lithology, and fracturing is necessary when the Gas field is developed. Mechanical parameters of reservoir rock are the basis of fracturing design. The static method and dynamic method are used to
measure rock elastic mechanics parameters. The static elastic mechanics parameters are obtained by measuring the deformation of rocks when pressure is offered to the rocks, and the dynamic elastic mechanics parameters are got by acoustic velocity transformation in the rocks. The static elastic mechanics parameters are traditionally used in practical engineering. However, the static elastic mechanics parameters are obtained by experiment in the laboratory, which is expensive, difficult, and time-consuming, especially under reservoir conditions (including formation temperature and pressure). A large number of core experiment data is required to accurately describe the mechanics characteristics of the reservoir. For the mechanics characteristics under reservoir conditions can be obtained continuous with depth by the dynamic method (including well logging method and seismic prospecting method), the dynamic method is better to obtain the mechanics characteristics of the reservoir\textsuperscript{[1-3]}.

![Experimental device](image)

**Figure 1:** Experimental device

Zisma\textsuperscript{[4]} (1933) summarized that differences existed in dynamic and static elastic parameters. After that, researchers\textsuperscript{[5-7]} have done lots of work on the relationship between dynamic and static elastic parameters (including Young's modulus and Poisson's ratio), and the results demonstrated that static Young's modulus has a good correlation with dynamic Young's modulus, and dynamic Young's modulus is one to ten times larger than static Young's modulus, but there is no obverse relationship between dynamic and static Poisson's ratio. The conversion factor of dynamic and static Young's modulus varies in different areas. In order to establish suitable transformation model between
dynamic and static elastic parameters for a reservoir, experiments on the relationship between
dynamic and static elastic parameters are necessary to do under reservoir conditions.

EXPERIMENTS FOR DYNAMIC AND STATIC ELASTIC
PARAMETERS UNDER RESERVOIR CONDITIONS

In this study, core samples from tight sandstone reservoir in A Gas field have been used for the
experimental measurements. The core samples had the length ranging from 4.50 to 5.00 cm, the
average diameter is 2.54 cm, the average porosity is 8.00%, and the average permeability is $0.44 \times 10^{-3}$
$\mu m^2$. In order to simulate the reservoir conditions, AutoLab 1500 from NER Company is used as
measuring instrument in the experiment, and the maximum confinement pressure and pore pressure is
68 MPa, while the maximum temperature is 150$^\circ$C. The maximum axial pressure provided by
AutoLab 1500 is 823 kN. Strain gauge or LVDT is used to measure stress and the two methods both
measure strain in two axial and one radial. A series of rock elastic parameters test were done to
measure the intensity of the core samples. A pair of p-wave transducer and two orthogonal shear
wave transducers are both composed of Acoustic sensors.

The center frequency is 1 MHz, and transducers can withstand some pressure, in order to do
experiments for dynamic and static elastic parameters under reservoir conditions.

Core sample M was used as an example to describe the progress of the experiments. Core sample
M was fully saturated with the NaCl solution of simulated formation water salinity first. Then the
core sample M was assembled into the core holder. Core sample M was heated to the corresponding
formation temperature, and then added pressure to core sample M to the corresponding formation
pressure. When the temperature and pressure were balanced, acoustic waveform was collected to
calculate the compression and shear wave velocity and dynamic elastic parameters. Increased the
axial compression, collected the two axial compression cycle of stress and strain, and calculated the
static elastic parameters. The above two measurement are independent, and they are both automatic
program controlled while pressure is uniform added.

RELATIONSHIP BETWEEN DYNAMIC AND STATIC
ELASTIC PARAMETERS

According to the elastic wave propagation theory$^{[8]}$, the dynamic elastic modulus $E_d$ and dynamic
Poisson’s ratio $\mu_d$ are obtained as follows:

\[
E_d = \left[ \rho v_s^2 (3v_p^2 - 4v_s^2) / (v_p^2 - v_s^2) \right] \times 10^{-6} \\
\mu_d = (v_p^2 - 2v_s^2) / [2(v_p^2 - v_s^2)]
\]
where \( \rho \) is the density of the core sample, g/cm\(^3\), \( v_p \) and \( v_s \) are compressional and shear wave velocity respectively, m/s. take the average of wave velocity before and after the acquisition constant static mechanics as \( v_p \) and \( v_s \).

The static Young’s modulus \( E_s \) is the ratio of axial stress \( \sigma_1 \) and axial strain \( \varepsilon_1 \). The static Poisson’s ratio \( \mu_s \) is the opposite of the ratio of radial strain \( \varepsilon_2 \) and axial strain \( \varepsilon_1 \). In this experiment, \( E_s \) and \( \mu_s \) are the obtained by the slope of the stress and strain curves, shown in Fig. 2. Fig. 2a represent the relationship between axial stress \( \sigma_1 \) and axial strain \( \varepsilon_1 \), and the slope 12.529 is the static Young’s modulus of the core sample \( E_s \), GPa. Fig. 2b shows the relationship between the radial strain \( \varepsilon_2 \) and axial strain \( \varepsilon_1 \), and the opposite of the slope 12.529 is the static Poisson’s ratio of the core sample \( \mu_s \), fraction. Fig. 2b also shows that the radial deformation of the core sample is not fully recover after unloading stress, so the straight segment under the first compression cycle is used to calculate the static elastic parameters. The elastic parameters of core samples such as bulk modulus, shear modulus, Lame's constant can be calculated by Young’s modulus and Poisson’s ratio, so only Young’s modulus and Poisson’s ratio are analyzed.

(a) Relationship between axial stress and axial strain  (b) Relationship between the radial strain and axial strain

**Figure 2:** Calculation of static elastic parameters
Figure 3: Relationship between dynamic and static elastic parameters

Fig. 3 represents the relationship between dynamic and static elastic parameters. We can see from Fig. 3(a) that the static and dynamic Young’s modulus are linear correlation:

\[ E_s = 0.5640E_d - 3.4941, R^2 = 0.7246 \]  

Fig. 3(b) shows that the static and dynamic Poisson’s ratio of the core samples have no obvious correlation. The static Poisson’s ratios of the core samples have a wide distribution ranging from 0.05 to 0.31, while the dynamic Poisson’s ratios of the core samples are concentrated ranging from 0.21 to 0.35.

RELATIONSHIP BETWEEN DYNAMIC AND STATIC ELASTICITY PARAMETERS AND TEMPERATURE OR PRESSURE

The experiment to measure dynamic and static elasticity parameters under reservoir conditions (including high temperature and high pressure) is expensive, so research on the relationship between dynamic and static elasticity parameters and temperature and pressure is necessary, and the research reflects the differences of the mechanisms of dynamic and static elasticity parameters under reservoir conditions.

Three sets of strain gauges are pasted to the core sample (two groups of strain gauges are used to measure axial strain, and one group of strain gauges are used to measure radial strain). The dynamic and static elasticity parameters are obtained under 5 different temperature and 6 different confinement pressure. The strain measured by strain gauge has a high precision, the measure area is large, and it
has the characteristics of automatic temperature compensation. In consequence, the static elasticity parameters are more accurate measured.

(a) Relationship between dynamic Young’s modulus and pressure
(b) Relationship between dynamic Young’s modulus and temperature

Figure 4: Relationship between dynamic Young’s modulus and temperature or pressure

(a) Relationship between elasticity Young’s modulus and pressure
(b) Relationship between elasticity Young’s modulus and temperature

Figure 5: Relationship between elasticity Young’s modulus and temperature or pressure
(a) Relationship between dynamic Poisson's ratio and pressure
(b) Relationship between dynamic Poisson's ratio and temperature

**Figure 6:** Relationship between dynamic Poisson's ratio and temperature or pressure

(a) Relationship between elasticity Poisson's ratio and pressure
(b) Relationship between elasticity Poisson's ratio and temperature

**Figure 7:** Relationship between elasticity Poisson's ratio and temperature or pressure

Fig. 4 to Fig. 7 show the Relationship between dynamic and static elasticity parameters (including Young’s modulus and Poisson's ratio) and temperature or pressure, and the change rule are shown in Tab. 1.
The Table 1 shows the relationship between dynamic and static elasticity parameters and temperature or pressure.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>When pressure increases</th>
<th>When temperature increases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Young’s modulus</td>
<td>Logarithmic increases</td>
<td>Linear decreases</td>
</tr>
<tr>
<td>Static Young’s modulus</td>
<td>Linear increases</td>
<td>Linear increases</td>
</tr>
<tr>
<td>Dynamic Poisson's ratio</td>
<td>Exponential decreases</td>
<td>Slightly decreases</td>
</tr>
<tr>
<td>Static Poisson's ratio</td>
<td>Logarithmic increases</td>
<td>Greatly decreases</td>
</tr>
</tbody>
</table>

The Fig 4 to 7 and Tab. 1 show that when temperature or pressure increases, the changes of dynamic and static elasticity parameters are different, and in some cases the changes are on the contrary. The dynamic Young’s modulus decreases but the static Young’s modulus increases when the temperature increases. The dynamic Poisson's ratio decreases but the static Poisson's ratio increases, when the pressure increases.

**MECHANISM ANALYSIS OF THE DIFFERENCE OF DYNAMIC AND STATIC ELASTIC MECHANICS PARAMETERS**

For comparison, the dynamic and static elasticity parameters of a standard aluminum sample were measured at a room temperature when the pressure is 10 MPa, and the results are shown in Tab. 2.

**Table 2: The dynamic and static elasticity parameters of a standard aluminum sample**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Young’s modulus (GPa)</th>
<th>Poisson's ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>72.24</td>
<td>0.345</td>
</tr>
<tr>
<td>Static</td>
<td>72.56</td>
<td>0.340</td>
</tr>
<tr>
<td>Error</td>
<td>0.44%</td>
<td>1.45%</td>
</tr>
</tbody>
</table>

The core samples are multiphase composite medium. Micro cracks distribute in core samples, and there is fluid inside micro cracks, which is the internal reason for the difference between the dynamic and static elasticity parameters of a standard aluminum sample and of a core sample. The external cause of the difference between the dynamic and static elasticity parameters is the strain amplitude. When the strain changes greatly, the pore or fissure space will change, the strain will increase, and thus the static elasticity parameters will decrease. The strain caused by acoustic wave is slightly, so dynamic elasticity parameters are greater than static elasticity parameters.

Mechanism analysis of the difference of dynamic and static elastic Young’s modulus is as follows. In the process of dynamic test, the core sample will be in the “undrained” condition. If the confinement pressure increases, the core sample micro crack will closure, pore fluid will be compressed, the equivalent of pore fluid stiffness will increase, and it will provide additional Young’s
modulus. When the temperature increases, micro cracks increases, and Young’s modulus of the core sample decreases.

The static Young’s modulus is the ratio of axial stress and axial strain. When confining pressure increases, the core sample will become “longer”, the axial stress will increase, and thus static Young’s modulus will increase. When temperature increases, some minerals in the core sample will expand with heat, which can offset some of the compression deformation, and axial strain will decrease. As a result, Young’s modulus will increase, when temperature increases.

The above are the reasons for the difference and relationship between dynamic and static elasticity parameters.

CONCLUSIONS

The dynamic and static elastic mechanics parameters of 22 core samples were measured under the same condition. The compressional velocity, shear velocity, dynamic and static elastic mechanics parameters of core sample A were obtained when temperature or pressure changed. The following conclusions may be drawn from the present study:

(1) The dynamic Young’s modulus is greater than the static Young’s modulus under the same condition, and the static and dynamic Young's modulus are linear correlation. The dynamic Poisson's ratio is almost larger than the static Poisson's ratio, but there is no obvious relationship between the static and dynamic Poisson's ratio. The static Poisson's ratio distributes widely, while the dynamic Poisson's ratio is concentrated.

(2) The dynamic and static elastic mechanics parameters of the same core sample were measured under the reservoir conditions, but the relationships between dynamic and static elastic mechanics parameters are different, even opposite. When pressure increases, the dynamic Poisson's ratio will logarithmic increase, and the dynamic Young’s modulus will linear decrease when temperature increases. The static Young’s modulus will linear increase when pressure or temperature increases. The dynamic Poisson's ratio will exponential decrease when pressure increases, and it will slightly decrease when temperature increases. The static Poisson's ratio will logarithmic increase when pressure increases, and it will greatly decrease when temperature increases.

REFERENCES


