A Damage Constitutive Model for A Rock Considering the Microcrack Deformation and Propagation

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ABSTRACT

Aiming at the shortcoming that the contribution of the microcrack deformation and propagation to the rock total deformation, the mixed propagation criterion of the microcrack and the influence of the damage on the number of the activated microcracks are not well considered at the same time in the existing rock damage constitutive model, the uniaxial compressive damage constitutive model for a rock is proposed basing on the microcrack propagation and deformation mechanism. Firstly, the stress-strain relationship for a rock under uniaxial compression is set up according to the crack sliding model and energy balance principle. It is also assumed that the distribution of the microcrack obeys weibull model, and the mixed fracture propagation length of the wing crack is solved with the strain energy density criterion and iteration method. Also the damage evolution variable for a rock is obtained with the wing crack propagation length. Finally the validation of the proposed model is verified with the numerical calculation and model test. The following results can be obtained. (1) The uniaxial compressive climax strength and strain both decrease with the increase in the microcrack length and the decrease in the microcrack friction coefficient. (2) Through the comparison between the calculation result and the test one of the rock under the uniaxial compression in the relevant reference, it shows that the theoretical calculation result obtained from the proposed model fits very well with the test one, which shows that the proposed damage model for a rock is valid.

KEYWORDS: microcrack; a sliding crack model; strain energy density; uniaxial compression; a damage constitutive model

INTRODUCTION

The existence of the microcracks and their propagation and evolution is assumed to be the main mechanical mechanism leading to the deterioration of the rock mechanical property [1-5]. Although the continuum mechanics based on the macroscopic damage mechanics has already made much progress in the study of the rock damage constitutive model, it does not involve the mesoscopic mechanism of the rock failure such as the closure and propagation effect of the microcracks and their interaction, so that it cannot reflect the intrinsic mechanism of the rock failure [6-8]. However, the progress and application of the meso-mechanics provide a powerful tool in the study of rock damage theory and therefore the corresponding mesoscopic damage constitutive model for a rock is proposed [1-2, 9]. Assuming the microcracks in the rock obeyed the random distribution, Zhou et al [2] proposed the uniaxial compressive damage constitutive model by considering the initiation and propagation of the microcracks. But he did not consider the effect of the deformation caused by the microcrack propagation and frictional slippage on the rock total deformation. Based on the crack sliding model, Liu et al [9] established the one-dimensional dynamic damage constitutive model for a rock under dynamic loading. But it also has two shortcomings. First, the propagation of crack type Ⅱ will probably occur at the microcrack tip under compression, but only the propagation of crack type Ⅰ is considered in this model. Secondly, the number of the micocracks to be activated and then to propagate will dramatically change with the increase in rock damage which is also not considered in this study. These two factors above will both lead to the error between the calculation result and the actual condition.

Therefore, on basis of the existing studies [1-2, 9], aiming at the problems stated above, this paper will set up a new uniaxial compressive damage model for a rock based on the microcrack deformation and propagation, and then its validity is tested with the test result.

THE CRACK SLIDING MODEL

W. F. Brace et al [11] firstly proposed the crack sliding model, and thereafter much progress has been made [12-13]. The propagation path of a single microcrack under compression is shown in Fig.1. Under compression, the microcrack firstly closed and then the shear stress on the microcrack face will make it have the trend to slide. But because of the closure of the microcrack, the friction will resist the slippage, but when the shear stress exceeds the corresponding friction, the rock block will slide along the microcrack. When the stress intensity factors at the microcrack tips P and P' satisfy the microcrack propagation criterion, the wing cracks Q and Q' will occur and the angle between their propagation direction and the maximum compressive stress is ϕ. With the propagation of the microcrack, the tensile crack will be almost parallel to the maximum compressive stress, shown in Fig.2. The formulation and propagation of lots of microcracks will finally lead to the occurrence of the macroscopic crack, and causes the rock to axial split failure.

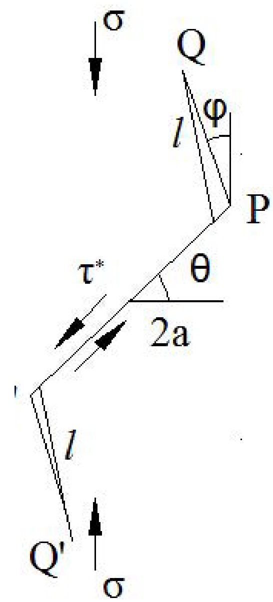


Figure 1: The sliding model for a microcrack

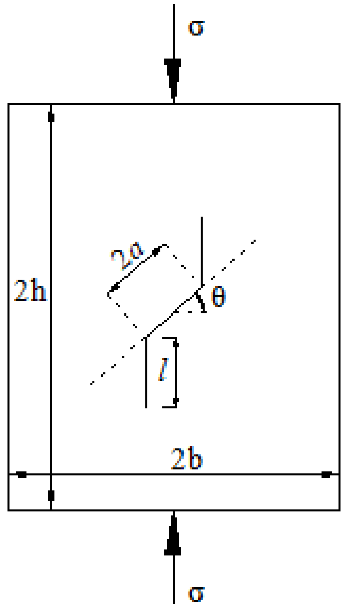


Figure 2: A unit cell model for a sliding crack under uniaxial compression

Under uniaxial compression, the resolved normal and shear stresses σ_θ and τ_θ on the microcrack face are respectively:

$$\sigma_\theta = \sigma\cos^2\theta \tag{1}$$

$$\tau_\theta = \sigma\sin\theta\cos\theta \tag{2}$$

where θ is the primary crack inclination angle.

Assume the friction angle on the microcrack face is φ, and then its coefficient is $f = \tan\varphi$. Then under uniaxial compression, the shear stress on the microcrack face will cause the rock mass to slide along the microcrack face. In turn, the normal stress on the microcrack face will produce the friction force to resist this slippage. So the slide force along the microcrack face τ^* must be larger than or equal to 0MPa. Therefore, it can be obtained from Eqs. (1) and (2):

$$\tau^* = \begin{cases} 0 & \tan\theta < \tan\varphi \\ \tau_\theta - f\sigma_\theta & \tan\theta \geq \tan\varphi \end{cases} \tag{3}$$

Under uniaxial compression, the stress intensity factor of the microcrack shown in Fig.2 is [13]:

$$K_\mathrm{I} = 2a\tau^*\cos\theta \Big/ \sqrt{\pi\left(l+l^*\right)},\ K_\mathrm{II} = -2a\tau^*\sin\theta \Big/ \sqrt{\pi\left(l+l^*\right)} \tag{4}$$

where l^*=0.27a.

STRESS-STRAIN RELATIONSHIP

According to the energy balance principle, the work done by the external load equals to the elastic strain energy of the system and consumed energy by friction:

$$W_1 = U_e + W_f \tag{5}$$

where U_e is the released elastic strain energy because of the microcrack propagation; W_f the consumed energy by friction; W_1 the work done by the external load.

Under the stress shown in Fig.2, the strain of the microcrack along and perpendicular to the compressive load is assumed to be $\Delta\varepsilon_1$ and $\Delta\varepsilon_2$ respectively. Then the work done by σ is:

$$W_1 = 4bh\sigma\Delta\varepsilon_1 \tag{6}$$

where $4bh$ is the element area.

According to elastic theory, there is:

$$\begin{bmatrix} \Delta\varepsilon_1 \\ \Delta\varepsilon_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ 0 \end{bmatrix} \tag{7}$$

where S_{ij} is the component of the flexibility tensor caused by one single microcrack.

According to the symmetry, $S_{12}=S_{21}$, and we can obtain by substituting Eq.(7) into Eq.(6):

$$W_1 = 4bhS_{11}\sigma^2 \tag{8}$$

According to fracture mechanics, the released elastic strain energy is because of the microcrack propagation (for a planar stress problem) :

$$U_e(l) = 2\int_0^l \left[\left(K_{\mathrm{I}}^2 + K_{\Pi}^2\right)/E\right] dl \tag{9}$$

Substituting Eq.(4) into Eq.(9), we can obtain:

$$U_e(l) = \frac{8a^2}{\pi E}\left\{\left(\tau^*\right)^2 \ln\left(1+\frac{l}{l^*}\right)\right\} \tag{10}$$

Assume the slip distance of the microcrack along its face is δ, the consumed energy because of the friction:

$$W_f = 2a\delta \cdot f\sigma_\theta \tag{11}$$

If we ignore the interval of the microcracks, δ can be expressed as [6]:

$$\delta = \frac{4\sqrt{2}a\tau^*}{E}\sqrt{\frac{l+l^{**}}{l+l^*}} \tag{12}$$

where $l^{**} = 0.083a$.

Substituting Eq.(12) into Eq.(11), we can obtain:

$$W_f = \frac{4\sqrt{2}a^2 f}{E}\left[2\tau^*\sigma\cos^2\theta\sqrt{\frac{l+l^{**}}{l+l^*}}\right] \tag{13}$$

From Eqs.(5), (10) and (13), we can obtain: $S_{11}=\dfrac{A_1B_1+A_2B_2}{E}$, where, $A_1=\dfrac{4a^2}{\pi bh}\left(\sin\theta-f\cos\theta\right)^2\cos^2\theta$, $A_2=\dfrac{2\sqrt{2}a^2f}{bh}\left(\sin\theta-\mu\cos\theta\right)\cos^3\theta$, $B_1=\ln\left(1+\dfrac{l}{l^*}\right)$ and $B_2=\sqrt{\dfrac{l+l^{**}}{l+l^*}}$.

Rock contains lots of microcracks, and for simplification, assume the microcracks have the same length, evenly distribute and parallel to each other. Assume the microcrack density is *N*, and ignore the interaction among the microcracks, the nonlinear strain caused by the microcrack propagation is:

$$\varepsilon^d = \mathrm{N}\begin{bmatrix}\Delta\varepsilon_1\\ \Delta\varepsilon_2\end{bmatrix} \tag{14}$$

The total strain can be divided into the elastic strain ε^e and the strain ε^d caused by the microcrack propagation, namely:

$$\varepsilon=\varepsilon^e+\varepsilon^d \tag{15}$$

According to Hooke's law, there is:

$$\varepsilon^e = C:\sigma \tag{16}$$

where, *C* is the flexibility tensor, and for a planar stress problem, $\mathrm{C}=\dfrac{1}{E}\begin{bmatrix}1 & -\nu\\ -\nu & 1\end{bmatrix}$.

From Eqs. (14)~(16), the total strain is:

$$\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\end{bmatrix}=\begin{bmatrix}\varepsilon_1^e+\varepsilon_2^d\\ \varepsilon_2^e+\varepsilon_2^d\end{bmatrix}=\frac{1}{E}\begin{bmatrix}1 & -\nu\\ -\nu & 1\end{bmatrix}+N\begin{bmatrix}S_{11} & S_{12}\\ S_{21} & S_{22}\end{bmatrix}\begin{bmatrix}\sigma\\ 0\end{bmatrix} \tag{17}$$

where, *E*, ν are the elastic modulus and Poisson's ratio.

The above equations are suitable for the planar stress problem, while for the planar strain problem, *E* and ν in above equations should be replaced by $E/(1-\nu^2)$ and $\nu/(1-\nu)$.

Then under uniaxial compression, the stress-strain relation of the rock in Fig.2 can be obtained from Eq.(17):

$$\varepsilon=\sigma/E+NS_{11}\sigma \tag{18}$$

Substituting S_{11} into Eq.(17), we can obtain:

$$\sigma=\frac{E}{\left[1+NA_1\ln\left(1+\dfrac{l}{l^*}\right)+NA_2\sqrt{\dfrac{l+l^{**}}{l+l^*}}\right]}\varepsilon \tag{19}$$

If the rock damage variable is introduced to describe the deterioration of the rock elastic modulus caused by the propagation of the microcracks, we can obtain:

$$\bar{E}=\mathrm{E}\left(1-D\right) \tag{20}$$

where,

$$D=\frac{NA_1\ln\left(1+\frac{l}{l^*}\right)+NA_2\sqrt{\frac{l+l^{**}}{l+l^*}}}{1+NA_1\ln\left(1+\frac{l}{l^*}\right)+NA_2\sqrt{\frac{l+l^{**}}{l+l^*}}} \tag{21}$$

PROPAGATION AND NUCLEATION OF THE MICROCRACK

Propagation criterion of the microcrack

Ashby et al. [13] assumed that the propagation of the wing crack is mainly caused by the tensile stress, and then they adopted the type Ⅰ stress intensity factor to solve the length of the wing crack. Thereafter, Huang et al. [14-17] also adopted this method to establish the corresponding damage constitutive model for a rock. However, the studies [14] showed that the propagation of the wing cracks is not only the type Ⅰ, and often accompanied with type Ⅱ, which is a mixed propagation. Therefore, the effective strain energy density criterion [14] is adopted here to calculate the wing crack length. It is assumed that the microcracks begin to propagate when the effective strain energy density at wing crack tip is larger than the minimum strain energy density S_c.

The effective strain energy density at wing crack tip can be expressed as follows [18]:

$$S=a_{11}K_{\mathrm{I}}^2+2a_{12}K_{\mathrm{I}}K_{\mathrm{II}}+a_{22}K_{\mathrm{II}}^2 \tag{22}$$

where, $a_{11}=\frac{1+\nu}{8\pi E}\left[(3-4\nu-\cos\theta_3)(1+\cos\theta_3)\right]$, $a_{12}=\frac{1+\nu}{8\pi E}(2\sin\theta_3)\left[\cos\theta_3-(1-2\nu)\right]$, $a_{22}=\frac{1+\nu}{8\pi E}\left[4(1-\cos\theta_3)(1-\nu)+(1+\cos\theta_3)(3\cos\theta_3-1)\right]$, and $\theta_3=0$, charactering the strain energy density at the extended direction of wing crack.

The minimum strain energy density S_c can be expressed as [18]:

$$\mathrm{S}_c=\frac{(1+\nu)(1-2\nu)}{2\pi E}K_{\mathrm{I}c}^2 \tag{23}$$

where $K_{\mathrm{I}c}$ is the rock fracture toughness.

The wing crack length l_t at time t is obtained as follows:

$$\begin{cases} l_t=0, & \mathrm{if}(\mathrm{S})_t\le S_c \\ increase\quad l_t, & \mathrm{until}(\mathrm{S})_t=S_c,\mathrm{if}(\mathrm{S})_t>S_c \end{cases} \tag{24}$$

Nucleation of the microcracks

Grady et al. [19] proposed a microcrack damage evolution model in studying on the oil shale blasting, and they assumed the density of the microcracks obeyed Weibull distribution:

$$N_0=k\varepsilon^m \tag{25}$$

where, N_0 is the number of the activated microcracks per volume under the strain ε . k and m are the parameters to describe the material mechanical parameters.

At the time step n, D_n denotes the fraction of the damage microcracks, and the fraction of the undamaged microcracks is $(1-D_n)$. Therefore the number N of the microcracks actually activated can be expressed as [19]:

$$N=\left(1-D_n\right)N_0=\left(1-D_n\right)k\varepsilon^m \tag{26}$$

ONE-DIMENSIONAL DAMAGE CONSTITUTIVE MODEL FOR A ROCK UNDER COMPRESSION

Because there are lots of random microcracks in the rock, these microcracks will be activated and propagate under compression. Then the stress-releasing area and accumulative damage is formed. When the damage accumulates to some degree, the rock will finally fail because of the deterioration of its strength and stiffness.

One-dimensional damage constitutive model for a rock under compression can be expressed as:

$$\sigma=\mathrm{E}\left(1-D\right)\varepsilon \tag{27}$$

where, D is the damage which can be solved with Eq.(12).

The calculation procedure of the model can be summarized in the following steps:

① The initial stress should be assumed to be zero, and the stress increment $\Delta\sigma$ is given. Then the stress intensity factor at the crack tips is calculated with Eq.(4), and whether the microcrack could propagate should be determined according to Eqs.(21)~(24). If it does not meet with the microcrack propagation criterion, N and l should all be zero. Then the strain increment $\Delta\varepsilon$ corresponding to the stress increment $\Delta\sigma$ can be solved with Eq.(27). Thereafter the stress gradually increases until the microcrack begin to propagate, and assume the time moment, stress and strain are t_n, σ_n and ε_n respectively. This stage is the linear one at which the microcrack does not begin to propagate.

② Assume the strain increment is $\Delta\varepsilon_{n+1}$ at time moment t_{n+1} and D_n=0, and the number N_n of activated microcracks can be calculated with Eq.(21). Let the propagation length of the microcracks l_n =0, then the corresponding damage increment ΔD_{n+1} can be calculated with Eq.(21), and the corresponding stress increment $\Delta\sigma_{n+1}$ can be calculated with Eq.(27).

③ Substituting the damage increment ΔD_{n+1} at time moment t_{n+1} into Eq.(26) to calculate the number ΔN_{n+1} of the activated microcracks, and then substituting it into Eq.(21) to calculate the damage incremental ΔD_{n+1} corresponding to the propagation length increment Δl_{n+1} of the microcrack.

The stress-strain relationship of the rock under damage can be obtained by solving ② and ③ in loop iteration. Then combining with that of the rock under the elastic stage, the whole stress-strain curve of the rock under uniaxial compression can be obtained.

VERIFICATION OF THE PROPOSED MODEL

The test done by Zhou et al. [2] is adopted here to verify the proposed model. The rock is the sandstone from Xiangjiaba hydraulic power station. Its elastic modulus, Poisson's ratio, density and fracture toughness are 34GPa, 0.3, 2600kg/m^3 and 1.1MPa·m$^{1/2}$, respectively. The sample is a cylinder with 100mm in height and 50mm in diameter, and its test curve under uniaxial compression is shown in Fig.3. The parameters in the proposed can be adopted as follows: k=5e22m^{-3}, m=7.1, 2a=2e-4m, μ=0.26 and θ=45º. The theoretical curve is shown in Fig.3. It can be seen that: (1) The theoretical curve fits very well with the test one, especially at the elastic stage. Their climax strengthes are 131.03MPa and 137.68MPa respectively which shows that the error is little. After the climax strength, these two curves are almost the same. (2) According to the proposed model, the rock elastic limit stress is 79.3MPa, 60.5% of its climax strength 131.03MPa, thereafter the damage occurs. Many studies [20] show that the rock elastic limit stress is about 30%~70% that of its climax strength, which agrees with the existing studies.

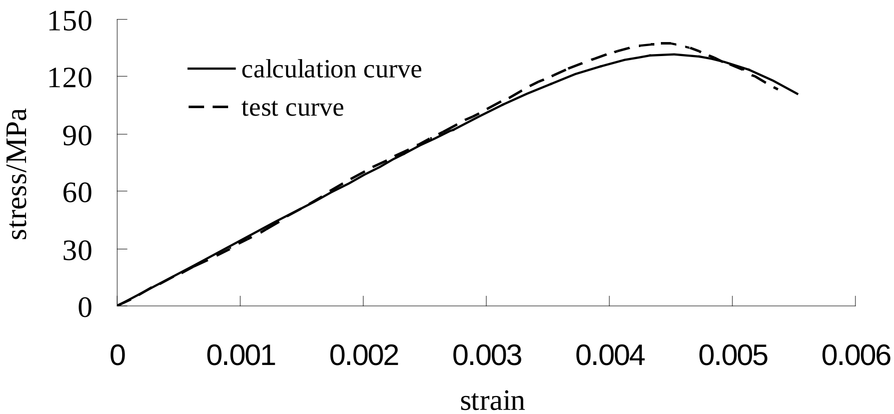


Figure 3: Comparison of experimental and simulation data for sandstone under uniaxial compression

CALCULATION EXAMPLES

The mechanical behavior of the rock under uniaxial compression

Here the rock elastic modulus, Poisson's ratio, toughness and density are assumed to be 40GPa, 0.2, 0.4MPa·m$^{1/2}$ and 2500kg/m^3 respectively, and the calculation model is shown as Fig.2. The sample is a cylinder with 100mm in height and 50mm in diameter, and it is calculated according to the planar stress theory. The microcrack dip angle, length and friction coefficient are 45°, 60 μm and 0.7 respectively. The Weibull parameters of the microcracks are as follows: k=4e23/m^3 and m=6.5. So, the curves of the stress-strain and strain-damage are shown in Fig.4.

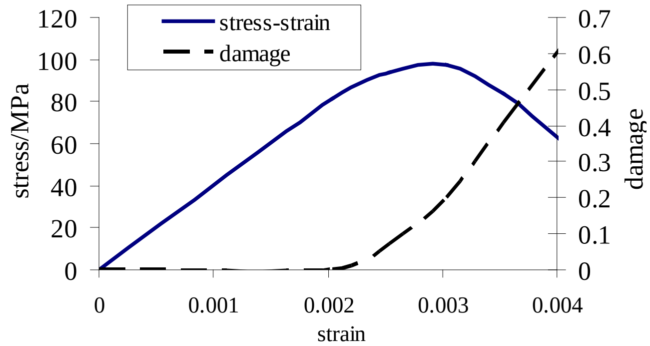
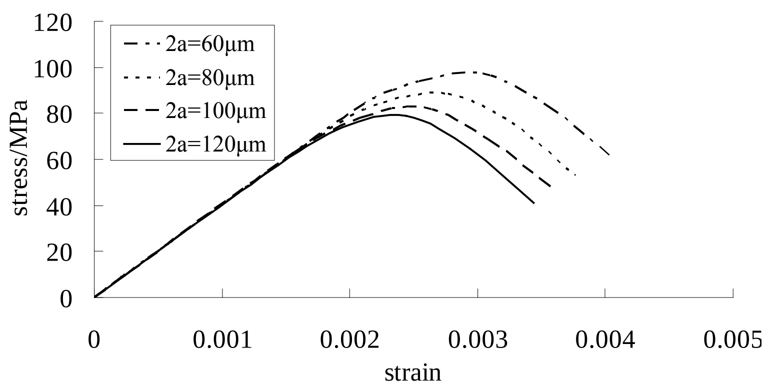


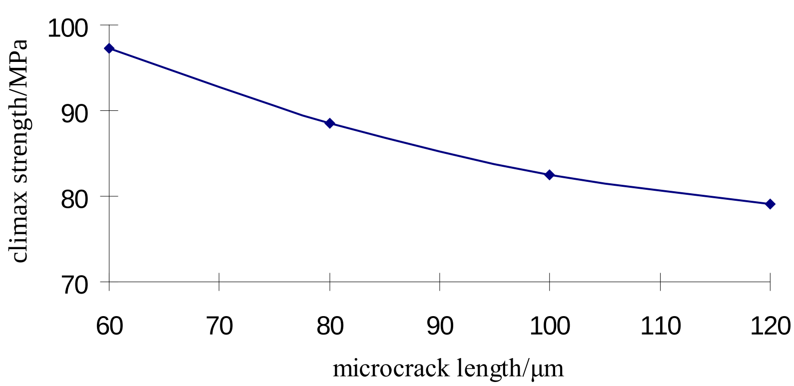
Figure 4: Curves of the axial stress and damage vs. axial strain under uniaxial compression

The effect of the microcrack length

Here the microcrack length 2*a* is assumed to be 60, 80, 100 and 120μm respectively, and the other parameters are stated above. Then it can be seen from Fig.5 that the rock uniaxial compressive strength linearly decreases and its climax strain gradually decreases with the increase in the microcrack length. Because under the same external load, the stress intensity factor at the crack tip will increase with the microcrack length. When the fracture toughness is the same, accordingly the energy needed to make the microcrack propagate will reduce. Therefore, the rock climax strength and damage will gradually decrease and increase respectively with the increase in the microcrack length.

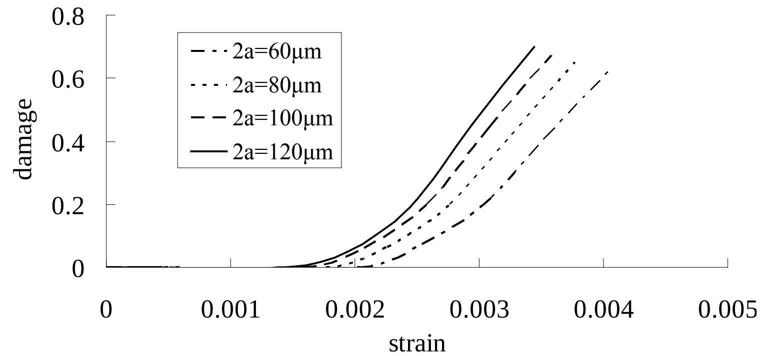


(a) the stress-strain curves of the sample with microcrack length



(b) the sample climax strength vs. the microcrack length

Figure 5: *Continues on the next page*

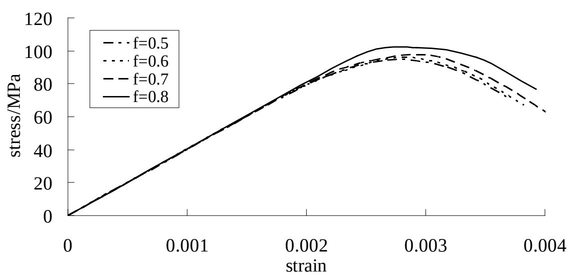


(c) the sample damage evolution vs. the microcrack length

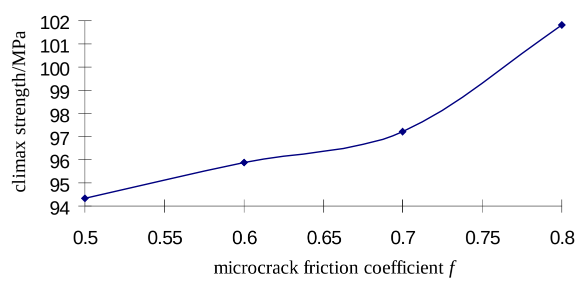
Figure 5: The mechanical property of the samples with different microcrack length

The effect of the microcrack friction coefficient

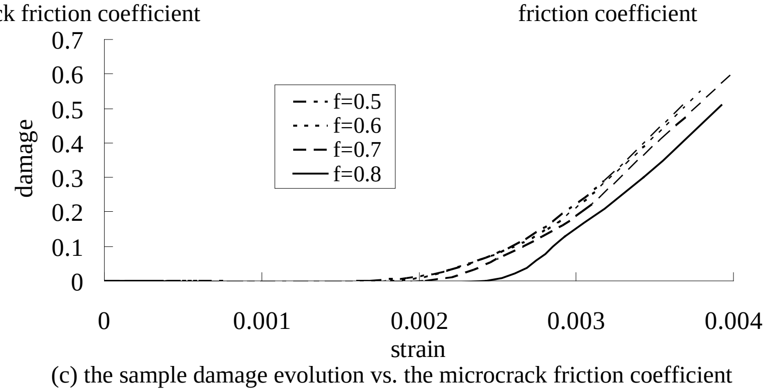
Here the microcrack friction coefficient *f* are assumed to be 0.5, 0.6, 0.7 and 0.8 respectively, and the other parameters are stated above. Then it can be seen from Fig.6 that the rock uniaxial compressive strength linearly increases with the increase in the microcrack friction coefficient, but it increase extent is different. When the microcrack friction coefficient is 0.5~0.7, the increase of the sample's climax strength is little. However, when the microcrack friction coefficient is 0.7~0.8, the sample's climax strength increases rapidly. This is because it is more difficult to make the rock produce more deformation and fail with the increase in the microcrack friction coefficient, which finally leads to the increase in the sample's climax strength.



(a) the stress-strain curves of the sample with different microcrack friction coefficient



(b) the sample climax strength vs. the microcrack friction coefficient



(c) the sample damage evolution vs. the microcrack friction coefficient

Figure 6: The mechanical property of the samples with different micorcrack friction coefficient

CONCLUSIONS

(1) A new damage constitutive model for a rock is proposed by comprehensively considering the contribution of the microcrack deformation and propagation to the rock total deformation, the mixed propagation criterion of the microcracks and the effect of the damage on the number of the activated microcracks. It can perfectly reflect the fact that the failure of the rock is caused by the deformation and propagation of lots of microcracks in the rock.

(2) The validity of the proposed model is verified with the test result, and the effect of the microcrack length and friction coefficient on the rock uniaxial compressive strength and climax strain is discussed. All these work proves that the proposed model is reasonable.

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