An Implementation Method of Shear Plane Fracture of Multi-Crack Rock Under Compression-Shear Loading

Zhi Wang
Associate Professor
School of Mechanics & Engineering Science, Zhengzhou University,
Zhengzhou 450001, China
Corresponding author, e-mail: zhiwang4960_cn@126.com

Jianhua Hou, Chaoya Wang, Fei Yang
School of Mechanics & Engineering Science, Zhengzhou University, Zhengzhou
450001, China

ABSTRACT

Taking the parallel multiple cracks rock as research object, a clamp to impose compression and shear loading was designed. Different lateral pressure can be adjusted by the clamp to achieve cracking angle's control and the shear plane fracture can be obtained. It can reveal the failure mechanism of Mode II fracture under combined loading. The Mode II stress intensity factor was calculated by finite element method and the Mode II fracture toughness calculation formula was derived. Numerical calculation and experimental results show that the tensile stress in the crack tip may be restrained when the lateral pressure imposed by clamp reaches a certain value. However, the lateral pressure must be restricted in a proper range. The specimen might be crushed by too large compression loading. The tested Mode II fracture toughness of sandstone is about 2-3 times to the Mode I fracture toughness. It is a effective way to achieve true Mode II fracture by applying compression and shear loading straightly.

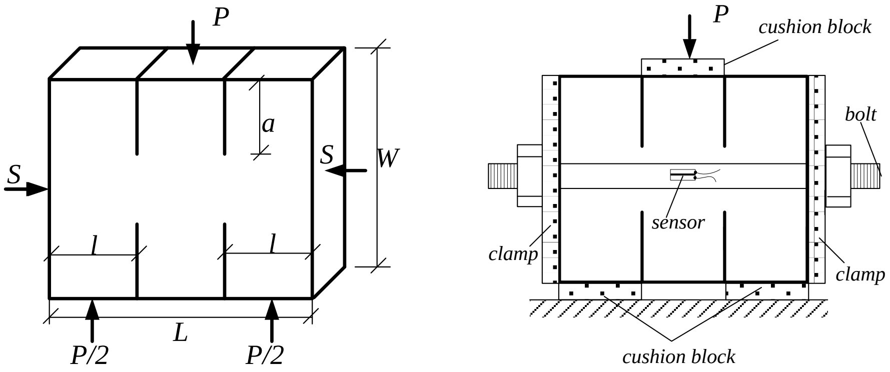
KEYWORDS: compression-shear loading, Mode II fracture, shear plane fracture, lateral pressure, rock

INTRODUCTION

The rock fracture mode of rock under direct shear loading is Mode I tensile fracture which was proved by many scholars. Wang and Sun[1-4] studied the fracture mechanism of rock under pure shear loading through direct shear experiment and plane four-point bending experiment of marble. It was suggested that under the pure shear load the maximum tensile stress was always reach its limit value before the maximum shear stress. Pure Mode II loading will lead to Mode I fracture. Whether the Mode II fracture can be obtained will depend on the ratio of the Mode II fracture toughness, K_{IIC}, and the Mode I fracture toughness, K_{IC}. Liu[5] tested the fracture toughness of rubber through plane four-point bending experiment and the K_{IIC} was 3 times to the K_{IC}. But the crack propagation was decided by the maximum circumferential tensile stress and it was obviously Mode I fracture. Petit[6] found that the crack did not propagate along the original crack surface under punching shear loading and the initial angle was about 70°which was Mode I fracture[7]. To obtain a ture Mode II fracture, Rao[8-9] introduced a shear box test which can be used to test the Mode II fracture toughness. And the test results show that the K_{IIC} was 2-3 times to the K_{IC}. The compression stress could restrain the tensile stress of the crack tip effectively. The size and shape of the samples of shear box experiment must be limited and the box have a larger size which is not suitable for high temperature, corrosion and other special environment. A convenient implementation clamp to impose compression and shear loading was designed to study the influence law of the shear and compression ratio on the fracture mode of collinear multi-crack rock..

EXPERIMENT SCHEME

The experimental materials is yellow sandstone and the mechanical parameters such as tensile strength ,σ_t, compressive strength ,σ_c, Elastic modulus, E, Poisson ratio,μ, Cohesion, C, internal friction angle, ϕ, Mode I fracture toughness, K_{IC}, are listed in Table 1. As shown in Figure 1, a device was designed to impose lateral pressure based on the punching shear experiment. The shear loading was imposed to the specimen through three cushion blocks and the lateral pressure was imposed by clamp with fastening bolt. The sensor was attached to the bolt so that the lateral pressure can be monitored in any time. The specimen was a rectangular body with a size of 75 mm×50 mm ×20 mm. Double notchs were prefabricated and the crack length, a, was about 12.5mm, the crack spacing, l, was about 24mm. The electric hydraulic servo testing machine was introduced and the loading rate was set as 5 mm/min. The peak value of the load, PM, and the sensor force value of bolt were recorded after the experiment.



(a)Sample size (b)Loading

Figure 1: Schematic diagram of loading

Table 1: Mechanical parameters

σ_t(MPa)	σ_c(MPa)	E(GPa)	μ	C	ϕ(°)	K_{IC} (MPa×m$^{1/2}$)
2.58	61	11	0.27	12MPa	35	0.951

EXPERIMENT SCHEME

There are no common formulas to calculate the Mode II stress intensity factor, K_{II}, for the four notched specimens under direct compressiong and shear loading. According to RAO's thesis [8], K_{II} can be obtained through finite element method. An eight-node hexahedron element was adopted for meshing and the mesh was refined around the crack tip (See Figure 2).

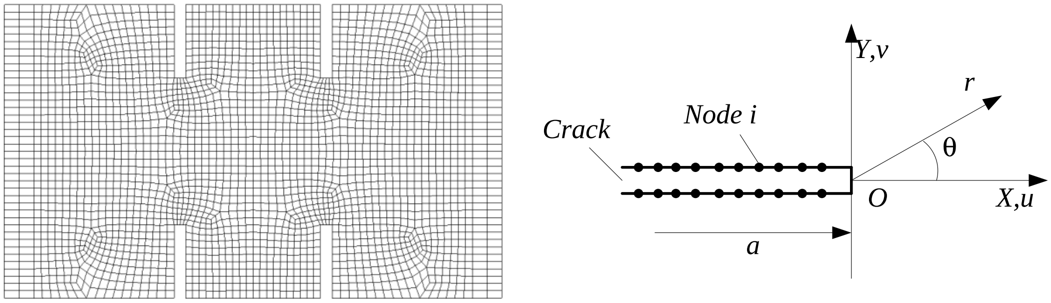


Figure 2: Finite element mesh and Coordinate system

As illustrated in Figure 2, the displacement u and v of the X and Y directions can be described as follows according to Irwin's the crack tip displacement equation[4]: An equation with 3-cell table method:

$$u = \frac{K_I}{4G}\sqrt{\frac{r}{2\pi}}\left[(2k-1)\cos\frac{\theta}{2} - \cos\frac{3\theta}{2}\right] + \frac{K_{II}}{4G}\sqrt{\frac{r}{2\pi}}\left[(2k+3)\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right] \qquad (1)$$

$$v = \frac{K_I}{4G}\sqrt{\frac{r}{2\pi}}\left[(2k+1)\sin\frac{\theta}{2} - \sin\frac{3\theta}{2}\right] + \frac{K_{II}}{4G}\sqrt{\frac{r}{2\pi}}\left[-(2k-3)\cos\frac{\theta}{2} + \cos\frac{3\theta}{2}\right] \qquad (2)$$

$k=(3-4\mu)/(1+\mu)$ for plane stress problem.
$k= 3-4\mu$ for plane strain problem.
$G = \frac{E}{1(1+\mu)}$.

When $\theta=\pm 180°$, formula (1) can be written as:

$$u = \frac{K_{II}}{4G}\sqrt{\frac{r}{2\pi}}\left[(2k+3)\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right];\ v = \frac{K_I}{4G}\sqrt{\frac{r}{2\pi}}\left[(2k+1)\sin\frac{\theta}{2} - \sin\frac{3\theta}{2}\right] \qquad (3)$$

Thus, the Mode II stress intensity factor, K_{II}, can be determined by the displacement u of the nodal point on the crack surface. The displacement u_i of node i in X direction is:

$$u_i^+ = \frac{K_{II}^i}{2G}\sqrt{\frac{r_i}{2\pi}}(k+1)\ ,\theta=+180°;\ u_i^- = -\frac{K_{II}^i}{2G}\sqrt{\frac{r_i}{2\pi}}(k+1)\ ,\theta=-180° \qquad (4)$$

Where the K_{II}^i is the Mode II stress intensity factor of node i. The relative displacement of node i in X direction can be determined as:

$$U_i = u_i^+ - u_i^- = \frac{K_{II}^i}{G}\sqrt{\frac{r_i}{2\pi}}(k+1) \tag{5}$$

$$K_{II}^i = \frac{G}{k+1}\sqrt{\frac{2\pi}{r_i}}U_i \tag{6}$$

When the node distance from the crack tip is far less than the crack length, ie, r << *a*, the stress intensity factors of crack tip can be obtained by the linear regression of K_{II}^i, ie, *r* = 0. The Mode II stress intensity factor is calculated as follows:

$$K_{II} = \tau\sqrt{\pi a}F(\frac{2a}{W}) = \frac{Q_{em}}{(W-2a)t}\sqrt{\pi a}F(\frac{2a}{W}) \tag{7}$$

where Q_{em} equivalent shear loading, *P* is shear loading, *S* is later compressiong loading, ϕ is internal friction angle and $Q_{em} = P/2 - S\tan\phi$.

The shape factor *F*can be obtained through formula (7):

$$F(\frac{2a}{W}) = \frac{K_{II}(W-2a)t}{Q_{em}\sqrt{\pi a}} \tag{8}$$

To determine the Mode II stress intensity factor, K_{II}, and shape factor, *F*(2*a*/*W*), took a rectangular specimens with prefabricated crack as FEM model. The size of the specimens were 75mm×50 mm×20 mm and the lengths of the crack were 10mm, 12.5mm, 15mm, 17.5mm respectively. The shear loading was determined as *P*=1*N*. The Mode II intensity factor calculation results at node *i* can be seen in Figure 3. The linear regression results of the Mode II stress intensity factors at crack tip and the shape factor obtained through formula (7) were listed in Table 2. The caculationg formula fo shape factor can be obtained by fitting data from Table 2(see Figure 4). The formula of shape factor, *F*(2*a*/*W*), can be described as:

$$F(2a/W) = -1.975(2a/W)^3 + 17.6(2a/W)^2 - 25.67(2a/W) + 12.08 \tag{9}$$

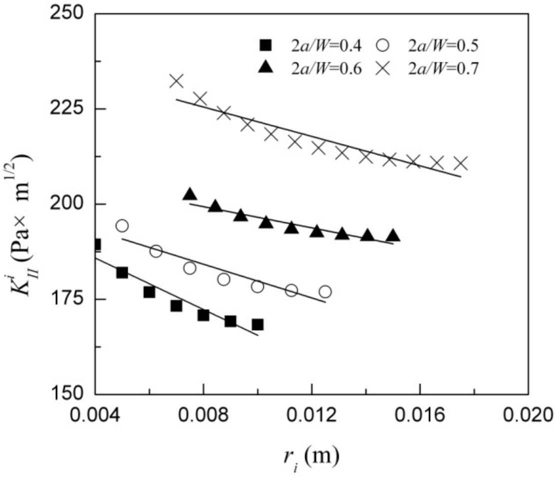


Figure 3: Node stress intensity factor variation with node position

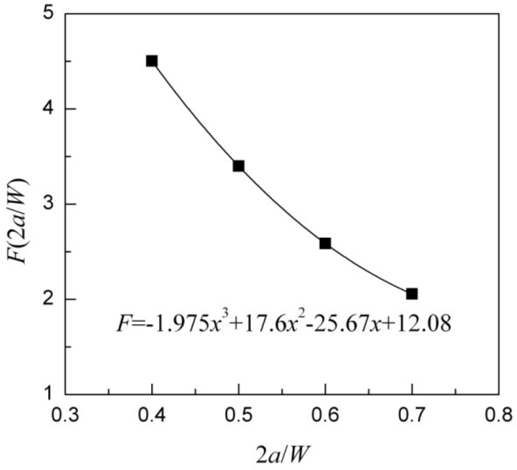


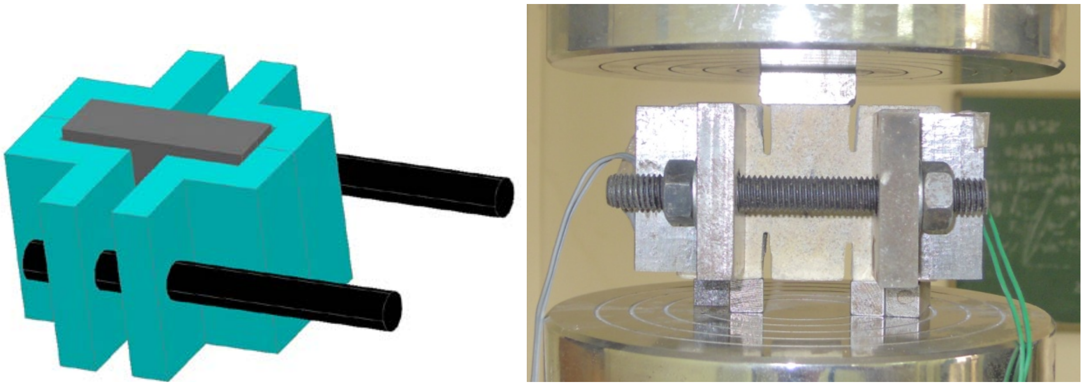
Figure 4: Shape factor variation with 2*a*/W

Table 2: Stress intensity factor and shape factor

S/P	*2a/W*	K_{II}(Pa×m$^{1/2}$)	*F(2a/W)*
0.5	0.4	199.393	4.503
0.5	0.5	201.911	3.399
0.5	0.6	210.462	2.587
0.5	0.7	240.936	2.056

RESULTS AND ANALYSIS

In order to study the influence law of fracture mode of muti-crack rock under compression-shear loading, we have designed a clamp to impose compression loading and achieved a ture shear fracture through sandstone experiment (see Figure 5). An initial lateral pressure was imposed to the specimen and then was the shear loading. The lateral pressure had a little increasing with the increased of the shear loading becasuse of the dilatancy effect of the rock. The initial fracture occured when the shear loading reached its peak value. The crack propagated through the original crack surface and a ture shear fracture was obtained (see Figure 6). The results of test were listed in Table 3. It can be seen that the Mode II fracture toughness of sandstone, K_{IIC}, was 2-3 times to the K_{IC}, which was well agree with Rao[8-9]. The peak value of the shear loading, *P*, increased with the increasing of later compression loading, *S*, while the K_{IIC} did not change significantly. According to the FEM calculation results, a stress concentration phenomenon could be seen from the stress distribution contours of the whole specimen (see Figure 7). The maximum □equivalent stressand minor principal stress occurred in crack tip and the tensile stress of the crack tip could be restrained by the clamp effectively.



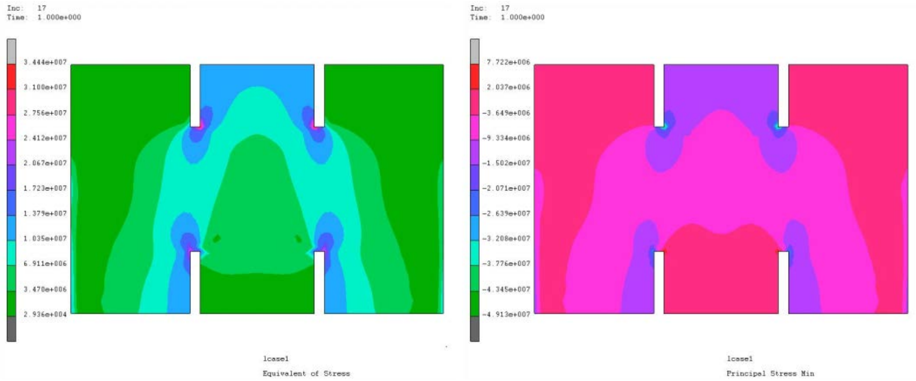
(a)design diagram (b)loading diagram

Figure 5: Clamp and equipment

Figure 6: Typical fracture trajectories

Table 3: Test Results

No.	S(N)	P(N)	S/P	KIIC (MPa×m1/2)	KIIC/KIC
1	1320	5151	0.26	2.224	2.338
2	1360	5921	0.22	2.705	2.844
3	2840	8135	0.32	2.800	2.944
4	3240	8177	0.40	2.451	2.577
5	3520	8266	0.43	2.247	2.363



(a) Equivalent stress (b) Minor principal stress

Figure 7: Stress distribution in crack tip

CONCLUSIONS

A clamp which can impose compression and shear loading succinctly was designed to study the fracture regularity of the parallel multiple cracks rock. The lateral compression loading can be adjusted to obtain shear plane fracture. The Mode II stress intensity factor was calculated by finite element method and the Mode II fracture toughness calculation formula was derived. Numerical calculation and experimental results show that the tensile stress in the cracp tip may be restrained when the lateral pressure imposed by clamp reaches a certain value. The Mode II fracture toughness of sandstone, K_{IIC}, was 2-3 times to the K_{IC}. The peak value of the shear loading, P, increased with the increasing of later compression loading, S, while the K_{IIC} did not change significantly. The maximum □equivalent stress and minor principal stress occurred in crack tip and the tensile stress of the crack tip could be restrained by the clamp effectively. It is a effective way to achieve true Mode II fracture by applying compression and shear loading straightly.

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