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Elastoplastic Analysis on Hole Expansion Issue of Pile Based on Improvement Unified Strength Criterion with Hyperbolic Envelope

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ABSTRACT

The improvement twin shear unified strength criterion is the nonlinear improved expression of the two-parameter twin shear unified criterion. Based on the improved strength criterion with the hyperbolic type failure envelope, according to the cavity expansion theory, elasto-plastic analysis on hole expansion issue of pile is performed. The elasto-plastic analysis solution of the stress field and displacement field of the soil around the pile are given. With the parameter b varying between 0 and 1, a series of solutions, which can take account of intermediate principal stress effect quantitatively, are obtained. The results show that the parameter b in unified strength criterion has effects on the radius of plastic zone, the magnitude of stresses and displacement. When the hyperbolic type failure envelope is used, the radius of plastic zone is smaller, and the radial displacement and the limit expansion pressure are larger than those obtained by using linear type failure envelope.

KEYWORDS: Unified strength criterion; Hyperbolic failure envelope; Hole expansion of pile; Theoretical analysis

INTRODUCTION

Pile has been widely used to support road embankments, bridge, industrial or residential building, and other structures[1][2]. Displacement piles driven are adverse to surrounding environment because of compacting induced the displacement [3]. The bad effect caused by driving pile is severe. In order to decrease the compacting effecting of jacked piles, the prebored hole will construct [4]. How to predict and reduce the compacting effect on surrounding region became an important subject. It has theoretical and practical value. In the process of driving pile, the theoretical research on the stress and displacement field of the soil around pile hole are complicated. A amount of theoretical and experimental research are carried out [5-9]. The Mohr-Coulomb criterion is used in the process of theoretical derivation in the past[10-13]. The Mohr-Coulomb strength criterion has been also widely used in geomechanics and other branches of applied mechanics. As is well known, the Mohr-Coulomb strength criteria do not consider the intermediate principal stress. The existence of the intermediate principal stress effect has now been well recognized as characteristics of the materials[14][15] The unified strength criterion proposed by YU[16]takes into consideration the effect of all the stresses in the characteristics line field, which can reflect the effects of the intermediate principle. The unified strength criterion extends the Mohr-Coulomb criterion and has been applied successfully to some problems [17-19]. However for failure envelope, the unified strength criterion proposed by YU has the linear type of the failure envelope as same as the Mohr-Coulomb strength criterion. When the failure envelope of the materials is nonlinear, the unified strength criterion proposed by YU maybe is not suitable. In this study, based on the improved strength criterion with the hyperbolic type failure envelope [20], the paper investigates the use of the cavity expansion theory [21] to predict the ground displacements. The parametric study is used to investigate the effect of intermediate principal stress. Comparisons are made with the results obtained by Unified strength criterion with the linear type failure envelope.

IMPROVEMENT OF UNIFIED STRENGTH CRITERION

The hyperbolic failure envelope can be written as[22]:

$$\frac{(\sigma + a + \sigma_t)^2}{a^2} - \frac{\tau^2}{b^2} = 1 \tag{1}$$

where $a = \dfrac{2\sigma_t^2}{\sigma_c - 3\sigma_t}$, $b = \sigma_t\sqrt{\dfrac{\sigma_t}{\sigma_c - \sigma_t}}$, σ_t and σ_c are the material's uniaxial tensile strength and uniaxial compressive strength, τ and σ are the shear stress and normal stress.

It can be expressed in terms of the major and minor principle stress σ_1 and σ_3 as follows

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right)^2 = \frac{\sigma_c - 3\sigma_t}{\sigma_c + \sigma_t}\left[\frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_t(\sigma_c - \sigma_t))}{\sigma_c - 3\sigma_t}\right]^2 - \frac{\sigma_t^3}{\sigma_c - 3\sigma_t} \tag{2}$$

Eq.(2) can be written as

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right)^2 = \frac{1 - 3\alpha}{1 + \alpha}\left[\frac{\sigma_{1.} + \sigma_3}{2} + \frac{\sigma_t(1-\alpha)}{1 - 3\alpha}\right]^2 - \frac{\alpha\sigma_t^2}{1 - 3\alpha} \tag{3}$$

where $\alpha = \sigma_t/\sigma_c$ is the material's tensile-compressive strength ratio.

Analysis of the Mohr circles proves that the original two-parameter double shear unified strength criterion has the linear type failure envelope. The improved strength criterion with the hyperbolic type failure envelope is given out in order to describe the envelope nonlinear property. Analyses of loci in the π plane and meridian lines show that the hyperbolic type failure envelopes have no influences on the shape of the loci in the π plane, but the shape of meridian lines is changed from straight lines to curved lines. The improvements of unified strength criterion with the hyperbolic type failure envelope are as follows [20]

where $\sigma_2 \le \dfrac{\alpha\sigma_1 + \sigma_3}{1 + \alpha}$

$$\left[\frac{(1+b)\sigma_1 - b\sigma_2 - \sigma_3}{2(1+b)}\right]^2 = \frac{1 - 3\alpha}{1 + \alpha}\left[\frac{(1+b)\sigma_1 + b\sigma_2 + \sigma_3}{2(1+b)} + \frac{\sigma_t(1-\alpha)}{1 - 3\alpha}\right]^2 - \frac{\alpha\sigma_t^2}{1 - 3\alpha} \tag{4a}$$

where $\sigma_2 \ge \dfrac{\alpha\sigma_1 + \sigma_3}{1 + \alpha}$

$$\left[\frac{\sigma_1+b\sigma_2-(1+b)\sigma_3}{2(1+b)}\right]^2=\frac{1-3\alpha}{1+\alpha}\left[\frac{\sigma_1+b\sigma_2+(1+b)\sigma_3}{2(1+b)}+\frac{\sigma_t(1-\alpha)}{1-3\alpha}\right]^2-\frac{\alpha\sigma_t^2}{1-3\alpha} \tag{4b}$$

where σ_1, σ_2 and σ_3 are the major, intermediate and minor principle stress. b is the intermediate principal stress effect parameter, $0 \le b \le 1$. when $b = 0$, the Eq.(4a) and Eq.(4b) are same with Eq.(3).

UNIFIED STRENGTH CRITERION

The unified strength criterion with the linear type failure envelope was developed based on a twin shear element and multiple slip mechanism. In terms of principle stresses, the unified strength criterion with the linear type failure envelope can be expressed as [16][23]

When $\sigma_2 \le \frac{\alpha\sigma_1+\sigma_3}{1+\alpha}$ $\qquad \alpha\sigma_1-\frac{b\sigma_2+\sigma_3}{1+b}=\sigma_t$ (5a)

When $\sigma_2 \ge \frac{\alpha\sigma_1+\sigma_3}{1+\alpha}$ $\qquad \frac{\alpha}{1+b}(\sigma_1+b\sigma_2)-\sigma_3=\sigma_t$ (5b)

It is a series of failure criteria. It can be applied to wide variety of materials. The scope of application of the specific form is reflected by the parameter b and α. The Mohr-Coulomb criterion is special case of the unified strength criterion with the linear type failure envelope with $b = 0$.

ELASTO-PLASTIC ANALYSIS

Based on the cylindrical cavity expansion theory, the process of pile driving of pile was supposed as the expansion process of cylindrical cavity expansion with initialized aperture and terminal aperture which are respectively equal to inner diameter R_0 and external diameter R_u. The soil around the pile was modeled as continual elastic-plastic relation and unified strength criterion with the hyperbolic type failure envelope or the linear type failure envelope, taking into consider the effect of the intermediate principal stress. The cylindrical cavity expansion is simplified as an axisymmetric problem. The zone is divided into two zones: the plastic zone ($R_0 \le r \le R_p$) and the elastic zone $R_p \le r$, where R_p is the radius of the elasto-plastic interface under internal pressure p, as shown in Figure 1.

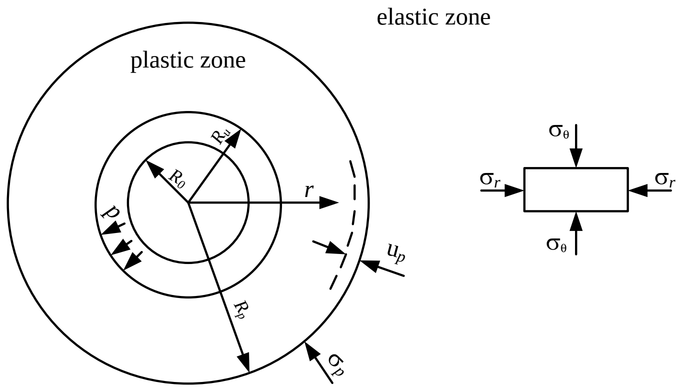

Figure 1: Soil element calculation model

The force balance equation as follows

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_\theta}{r}=0 \tag{6}$$

Geometric equation

$$\varepsilon_r=\frac{du_r}{dr}, \quad \varepsilon_\theta=\frac{u_r}{r} \tag{7}$$

where σ_r is radial stresses, σ_θ is tangential stresses, u_r is radial displacement, r is radius.

The strain components in the elastic region can be written as

$$\varepsilon_r=\frac{1-\mu^2}{E}(\sigma_r-\frac{1-\mu}{\mu}\sigma_\theta) \tag{8}$$

$$\varepsilon_\theta=\frac{1-\mu^2}{E}(\sigma_\theta-\frac{1-\mu}{\mu}\sigma_r) \tag{9}$$

where E is elastic modulus, μ is Poisson's ratio, ε_r is radial strain, ε_θ is tangential strain.

The boundary condition of this problem is $r=R_p$, $\sigma_r=\sigma_p$, σ_p is the stresses at interface between the elastic and plastic zones, R_p is also radius of plastic zone. σ_r, σ_θ, u_r in the elastic zone are expressed as

$$\sigma_r=\frac{\sigma_p R_p^2}{r^2} \tag{10}$$

$$\sigma_\theta=-\frac{\sigma_p R_p^2}{r^2}=-\sigma_r \tag{11}$$

$$u_r=\frac{1+\mu}{E}r\sigma_r \tag{12}$$

Analysis based on improvement of unified strength criterion

Under the axisymmetrical condition, the radial stresses σ_r, axial stress σ_z and tangential stresses σ_θ in the plastic zone will be principal stresses σ_1, σ_2 and σ_3 respectively. The soil around the pile was modeled as unified strength criterion with the hyperbolic type failure envelope. In the plane strain state, for simplicity, It is assumed that $\sigma_z = \sigma_2 = (\sigma_r + \sigma_\theta)/2$ [24]. Therefore, $\sigma_2 \geq \frac{\alpha\sigma_1 + \sigma_3}{1+\alpha}$, Eq.(4b) should be adopted. Eq. (4b) can be expressed as

$$\left[\frac{(2+b)(\sigma_r - \sigma_\theta)}{4(1+b)}\right]^2 = \frac{1-3\alpha}{1+\alpha}\left[\frac{(2+b)\sigma_r + (2+3b)\sigma_\theta}{4(1+b)} + \frac{\sigma_t(1-\alpha)}{1-3\alpha}\right]^2 - \frac{\alpha\sigma_t^2}{1-3\alpha} \tag{13}$$

We define $\frac{(2+b)\sigma_r + (2+3b)\sigma_\theta}{4(1+b)} = s$, Eq.(13) becomes

$$\frac{(2+b)(\sigma_r - \sigma_\theta)}{4(1+b)} = \left[\frac{1-3\alpha}{1+\alpha}s^2 + \frac{2(1-\alpha)}{1+\alpha}\sigma_t s + \frac{\sigma_t^2}{1+\alpha}\right]^{1/2} \tag{14}$$

We solve the Equation (14), we obtain the stresses in the plastic zone

$$\sigma_r = s + \frac{2+3b}{2+b}\left(a_1 s^2 + 2a_2 s + a_3\right)^{1/2} \tag{15}$$

$$\sigma_\theta = s - \left(a_1 s^2 + 2a_2 s + a_3\right)^{1/2} \tag{16}$$

where $a_1 = \frac{1-3\alpha}{1+\alpha}$, $a_2 = \frac{(1-\alpha)\sigma_t}{1+\alpha}$, $a_3 = \frac{\sigma_t^2}{1+\alpha}$

(1) The radius of plastic zone and the radial displacement

By using the balance condition of volume, the following system of equations can be obtained

$$\pi R_u^2 - \pi R_0^2 = \pi R_p^2 - \pi\left(R_p - u_p\right)^2 + \pi\left(R_p^2 - R_u^2\right)\Delta \tag{17}$$

where p_u is limit expansion pressure, R_0 is initialized aperture, R_u is terminal aperture, u_p is radial displacement at interface, Δ is of average volume strain of plastic zones.

By neglecting higher-order items of u_p, gives

$$1 - \frac{R_0^2}{R_u^2} = 2u_p\frac{R_p}{R_u^2} + \frac{R_p^2}{R_u^2}\Delta - \Delta \tag{18}$$

The boundary condition of this problem is $r = R_p$, $\sigma_r = \sigma_p$, σ_p is the stresses at interface between the elastic and plastic zones, R_p is radius of plastic zone. By using the improvement of unified strength criterion with the hyperbolic type failure envelope, Eq. (13) can be expressed as

$$\left[\frac{(2+b)(\sigma_p - \sigma_\theta)}{4(1+b)}\right]^2 = \frac{1-3\alpha}{1+\alpha}\left[\frac{(2+b)\sigma_p + (2+3b)\sigma_\theta}{4(1+b)} + \frac{\sigma_t(1-\alpha)}{1-3\alpha}\right]^2 - \frac{\alpha\sigma_t^2}{1-3\alpha} \tag{19}$$

By using the conditions is $r = R_p$, $\sigma_\theta = -\sigma_p$, gives as

$$\sigma_p = \frac{(1+b)\sigma_t}{2(1+b+\alpha+b\alpha+b^2\alpha)}\left\{\left[(1-\alpha)^2 b^2 + 4(1+b+\alpha+b\alpha+b^2\alpha)\right]^{1/2} - (1-\alpha)b\right\} \quad (20)$$

The radial displacement u_p at $r = R_p$ is calculated as follows

$$u_p = \frac{1+\mu}{E}R_p\sigma_p = \frac{1+\mu}{E}R_p m \quad 1)$$

Where $m = \frac{(1+b)\sigma_t}{2(1+b+\alpha+b\alpha+b^2\alpha)}\left\{\left[(1-\alpha)^2 b^2 + 4(1+b+\alpha+b\alpha+b^2\alpha)\right]^{1/2} - (1-\alpha)b\right\}$

By Eq.(18), Eq.(12) can be modified as follow

$$1-\frac{R_0^2}{R_u^2} = 2\frac{1+\mu}{E}\frac{R_p^2}{R_u^2}m + \frac{R_p^2}{R_u^2}\Delta - \Delta \quad (22)$$

The radius of plastic zone R_p gives as

$$R_p = R_u\sqrt{\frac{G\left(1+\Delta-\left(\frac{R_0}{R_u}\right)^2\right)}{m+G\Delta}} \quad (23)$$

where $G = \frac{E}{2(1+\mu)}$

The radial displacement at interface u_p gives as

$$u_p = \frac{1}{2G}R_u m\sqrt{\frac{G\left(1+\Delta-\left(\frac{R_0}{R_u}\right)^2\right)}{m+G\Delta}} \quad (24)$$

(2) The stress of plastic zone

Substitution of Eq. (15) and Eq. (16) into the equations of equilibrium Eq. (6), gives

$$\left[\frac{1}{(a_1s^2+2a_2s+a_3)^{1/2}} + \frac{(2+3b)(2a_1s+2a_2)}{2(2+b)(a_1s^2+2a_2s+a_3)}\right]ds + \frac{4(1+b)}{2+b}\frac{dr}{r} = 0 \quad (25)$$

By solving the Eq. (25), gives

$$r = Cf^{-\frac{2+3b}{8(1+b)}}\left[(a_1s+a_2)a^{-1/2} + f^{1/2}\right]^{-\frac{2+3b}{4(1+b)\sqrt{a_1}}} \quad (26)$$

where $f = a_1s^2 + 2a_2s + a_3$

By using the conditions is $r = R_p$, $\sigma_\theta = -\sigma_r$, $s = s_0$, Eqs.(15) and Eqs.(16), gives

$$s_0 + \frac{b}{2+b}\left(a_1 s_0^2 + 2a_2 s_0 + a_3\right)^{1/2} = 0 \quad (27)$$

From Eq. (27), s_0 can be derived as

$$s_0 = \frac{-a_2 b^2 - b\left[a_2^2 b^2 - a_3 a_1 b^2 + a_3(2+b)^2\right]^{1/2}}{a_1 b^2 - (2+b)^2} \quad (28)$$

Recalling the boundary conditions, C can be calculated, and gives

$$r = R_p\left(\frac{f}{f_0}\right)^{-\frac{2+3b}{8(1+b)}}\left[\frac{(a_1 s + a_2)a^{-1/2} + f^{1/2}}{(a_1 s_0 + a_2)a^{-1/2} + f_0^{1/2}}\right]^{-\frac{2+3b}{4(1+b)\sqrt{a_1}}} \quad (29)$$

where $f_0 = a_1 s_0^2 + 2a_2 s_0 + a_3$

Utilizing Eq. (15),(16) and (29), the relationship of the σ_r, σ_θ and r can be obtained.

Analysis based on unified strength criterion with the linear type failure envelope

*T*he soil around the pile was assumed to satisfy the unified strength criterion with the linear type failure envelope. By Eq.(5b), we can obtain

$$\frac{\alpha}{1+b}(\sigma_r + b\sigma_z) - \sigma_\theta = \sigma_t \quad (30)$$

By using the balance equation Eq. (6), $\sigma_r = p_u$ at $r = R_u$, σ_r, σ_θ can be given as follows

$$\sigma_r = (p_u + \frac{\sigma_t}{1-\alpha})(\frac{R_u}{r})^{\frac{2(1+b)(1-\alpha)}{2+2b-b\alpha}} - \frac{\sigma_t}{1-\alpha} \quad (31)$$

$$\sigma_\theta = \frac{2\alpha + b\alpha}{2+2b-b\alpha}(p_u + \frac{\sigma_t}{1-\alpha})(\frac{R_u}{r})^{\frac{2(1+b)(1-\alpha)}{2+2b-b\alpha}} - \frac{\sigma_t}{1-\alpha} \quad (32)$$

The radius of plastic zone R_p gives as

$$R_p = R_u\sqrt{\frac{G\left(1+\Delta - \left(\frac{R_0}{R_u}\right)^2\right)}{\frac{(1+b)\sigma_t}{1+b+\alpha} + G\Delta}} \quad (33)$$

The radial displacement u_p at $r = R_p$ is calculated as follows

$$u_p = \frac{1+\mu}{E}R_p\sigma_p = \frac{1+\mu}{E}R_p\frac{(1+b)\sigma_t}{1+b+\alpha} \quad (34)$$

The limit expansion pressure can be derived as

$$p_u = \left[\frac{(1+b)\sigma_t}{1+b+\alpha} + \frac{\sigma_t}{1-\alpha} \right] \left[\frac{G\left(1+\Delta-\left(\frac{R_0}{R_u}\right)^2\right)}{\frac{(1+b)\sigma_t}{1+b+\alpha}+G\Delta} \right]^{\frac{(1+b)(1-\alpha)}{2+2b-b\alpha}} - \frac{\sigma_t}{1-\alpha} \quad (35)$$

COMPARISONS AND PARAMETRIC ANALYSIS

In order to investigate the integrated influences of the intermediate principle stress, an example of rock mass is analyzed, the material's uniaxial tensile strength $\sigma_t = 25$ kPa, the uniaxial compressive strength $\sigma_c = 86$ kPa, average volume strain of plastic zones $\Delta = 0.02$, Poisson's ratio $\mu = 0.35$, elastic modulus $E = 5.0$ Mpa, inner diameter $R_0 = 0.3$ m and external diameter $R_u = 0.5$ m. Table 1 shows that the effects of the parameter *b* in the unified strength criterion with the hyperbolic type failure envelope and the linear type failure envelope on the radius of plastic zone. The parameter *b* can vary from 0 to 1. The different *b* values correspond to different intermediate principal stress effects. It is show that the parameter b in unified strength criterion has significant effects on the radius of plastic zone. The radius of plastic zone is decreased, because of the effect of intermediate principal stress.

Table 1: Results of the radius of plastic zone

b	0	0.25	0.50	0.75	1.0	
R_p	2.27	2.26	2.25	2.24	2.23	hyperbolic type
	2.34	2.31	2.29	2.28	2.27	linear type

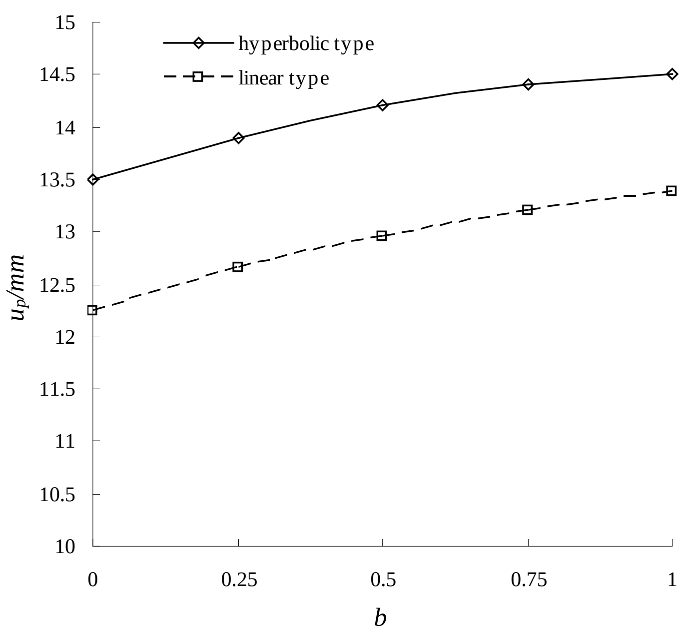

Figure 2: Influences of *b* on radial displacement of the plastic region boundary

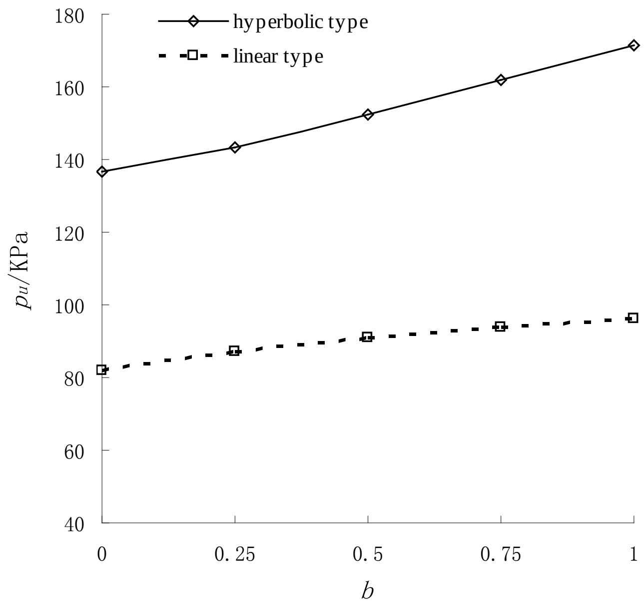

Figure 3: Influences of b on the limit expansion pressure p_u

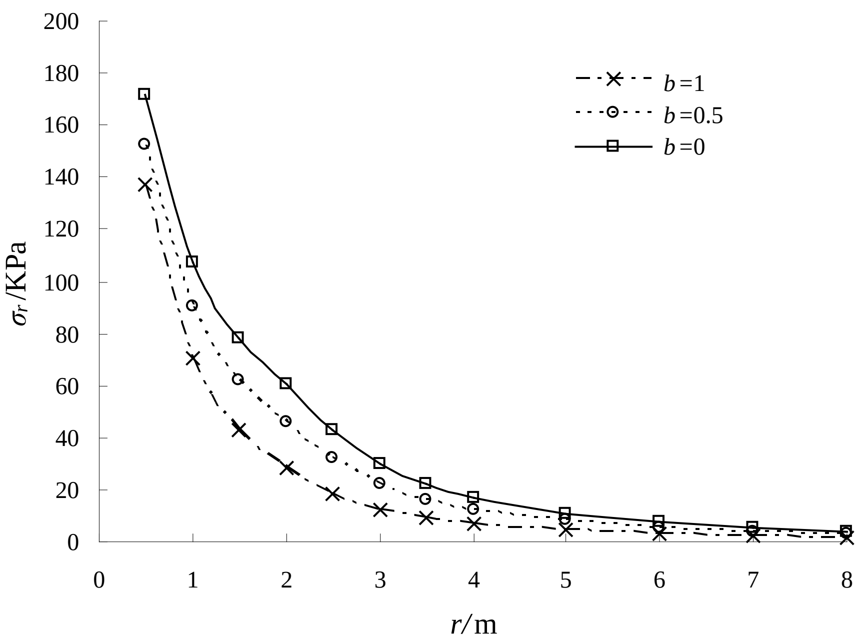

Figure 4: Influences of b on distribution of radial stress σ_r (hyperbolic type)

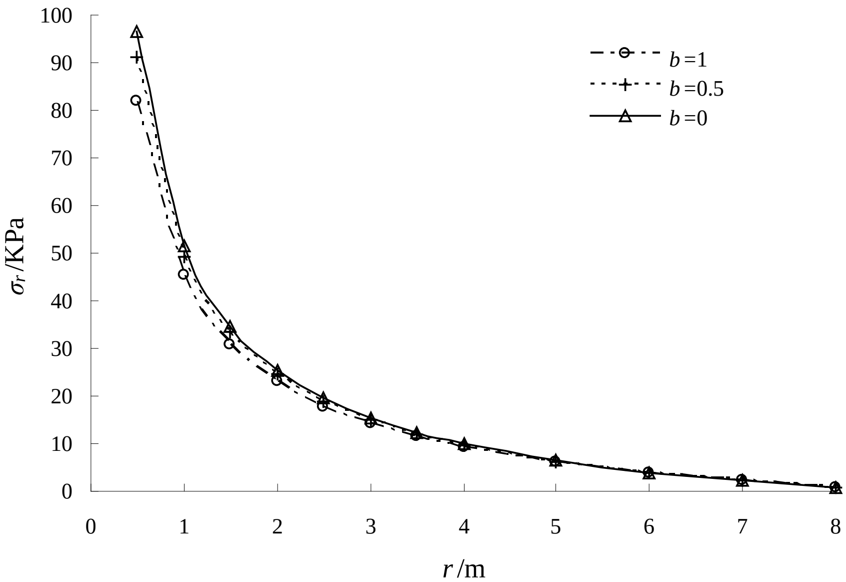

Figure 5: Influences of b on distribution of radial stress σ_r (linear type)

Figure 2 shows influences of *b* on radial displacement u_p of the plastic region boundary. It is show that radial displacement of the plastic region boundary increases with the increasing parameter b. The radial displacements of the plastic region boundary obtained by unified strength criterion with the hyperbolic type failure envelope are larger than results with the linear type failure envelope. Figure 3 indicates influences of b on the limit expansion pressure p_u. Unified strength criterion with the hyperbolic type failure envelope and the linear type failure envelope with $b=0$ give the lower limit expansion pressure. Both of them with $b=1$ give the upper limit expansion pressure. The limit expansion pressure obtained by unified strength criterion with the hyperbolic type failure envelope increase more quick with increasing the parameter *b* than result obtained by unified strength criterion with the linear type failure envelope. The distribution of radial stress σ_r are influenced by the parameter *b* , as shown in Figure 4 (hyperbolic type) and Figure 5 (linear type). Three basic criteria of the unified strength criterion with $b=0, b=0.5, b=1$ are applied to analysis of the radial stress σ_r. The improved unified strength criterion with $b=0$ gives the maximum result, the improved unified strength criterion with $b=1$ gives the minimum result and the improved unified strength criterion with $b=0.5$ give the median result. The results are similar to such which obtained with unified strength criterion with the linear type failure envelope. All results obtained by unified strength criterion with the linear type failure envelope with $b=0$ are results by using of Mohr-Coulomb and without the intermediate principle stress effect. Comparing with the results obtained by unified strength criterion with the linear type failure envelope, when the hyperbolic type failure envelope is used, the radius of plastic zone is smaller, the radial displacement and radial stress and the limit expansion pressure are larger.

CONCLUSIONS

To investigate the comprehensive influence of the intermediate principal stress, the example was used in an elastic plastic model analysis. The elasto-plastic analysis solution of the stress field and displacement field of the soil around the pile are given. The improved unified strength criterion with the hyperbolic type failure envelope is used in this paper. The effect of intermediate principal stress has been taken into account in the criterion. The solutions for different parameter *b* are given for comparison. The parameter *b* in unified strength criterion has effects on the radius of plastic zone, the magnitude of stresses and displacement. The improvement of unified strength criterion can be easily to analytical and numerical methods for the geotechnical engineering. The results obtained by unified strength criterion with the linear type failure envelope and the hyperbolic type failure envelope are similar. The prediction of the radius of plastic zone, the radial displacement, the limit expansion pressure and distribution of radial stress is influenced strongly by choosing of the yield criterion and strength parameter. Choosing the reasonable strength criterion in the research and design is very important. Above the theoretical results can be effectively applied to engineering problems in the field of theoretical analysis.

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