Regression Analysis and Inversion Analysis of Surrounding Rock Parameters Based on the Monitoring Data of *Song* Pan Tunnel

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ABSTRACT

Taking the Song pan tunnel high ground stress large deformation area as the engineering background, the regression analysis and curve fitting of the arch crow settlement and horizontal convergence displacement data of the surrounding rock are carried out on the typical section of the tunnel, and then the final deformation of surrounding rocks are predicted. Based on the convergence observation data of the construction site, utilizing the golden section method combined with the MIDAS/GTS numerical analysis software, the displacement back analysis of the surrounding rock characteristic parameters E and K is carried out and verified. The results show that, this method is feasible and reasonable when determining the parameters employed in the tunnel in high ground stress under complicated geological conditions, so it has a certain guiding significance to adjust and modify the design support parameters and ensure the tunnel safety. Meanwhile, the rock mechanics parameters obtained can also provide reference for other similar projects.

KEYWORDS: Surrounding rock parameters; Regression analysis; Displacement back analysis; Golden section method

INTRODUCTION

In the construction of the railway, tunnel excavation in high ground stress is prone to large deformation and failure due to the complex strata and high ground stress, since if the construction process is not properly controlled, it will lead to cracking of supporting structure, instability and even collapse of surrounding rock, which cause drastically damage to the project[1]. At present, engineering analogy method is universally adopted in the tunnel design, which is based on the geological survey data to determine the level of surrounding rock, and then the construction method and supporting form of the tunnel are determined according to the previous engineering experience. When confronting with such deep buried tunnel with complex geological conditions, it is difficult to obtain the detailed exploration message by limited geological borehole data, great discrepancies would showed between the surrounding rock parameters obtained from the laboratory test and practical situation, thus easily leading to the insufficient or surplus design. In order to ensure the safety of construction and achieve the goal of optimal design, it is necessary to carry out the inversion analysis of the characteristic parameters of surrounding rock based on the displacement and deformation data of the tunnel. Vast studies have already done by Domestic scholars with respect to back analysis of surrounding rock parameters, Hao Zhe[2] analyzed the mechanical parameters of surrounding rock by difference method, orthogonal design and artificial neural network method; Liu Qiang[3] established the method of parameters back analysis of tunnel surrounding rock based on the system identification theory; Huang Kan[4] obtained optimal solution of the parameters of surrounding rock using the simulation prediction function of neural network combined with the analysis of genetic algorithm inversion. Although these inversion methods can be used to accurately calculate the characteristic parameters of surrounding rock, the algorithm is relatively complex, and the value in practical application is limited. Selecting a convenient and practical inversion algorithm, so that the results can be accepted by the project, and without losing the engineering precision requirements become a trend.

The golden section method based on mathematical algorithm has the characteristics of simple principle, convenient and reliable, which combine with the inversion theory can effectively obtain surrounding rock parameter by the numerical analysis software.

In this paper, through the regression analysis of in-situ monitoring data and the golden section method adopted for surrounding rock parameters inversion analysis, aimed to predict and control the deformation of the surrounding rock accurate and effectively, and then carry out the information and dynamic construction effectively, to achieve maximum benefits.

ENGINEERING SITUATION

Cheng lan railway is a significant transport hub that connect Chengdu city, Sichuan province with Lanzhou city, Gansu province. The total length of this railway is 457.59 Km, of which the tunnel length accounted for 72.6%, and the average length of each tunnel is more than 10Km.Song pan tunnel is one segment of Cheng lan high speed rail, and is located in the east of Song Pan county. It begins at D3K239+630 and end at D3K247+678. It belongs to long-large-deep tunnel with total length 8084m and maximum depth 400m. This tunnel is located at the footwall of Min Jiang fault zone, where the lithology is simple. Affected by the Min Jiang fault, the secondary small faults and folds developed well due to serious interlayer extrusion and diverse bedding occurrences. New Austrian method is used for the tunnel construction, while it is difficult for construction risk management since it's a virtually high risk tunnel. Song pan tunnel section D3K244 + 250 ~ D3K244 + 280, study conducted in this paper are at the depth of 400m in vertical, where the rocks are weak and broken, thus the structure condition is extremely complex in such high ground stress, high seismic intensity and high geological disaster risk area. The surrounding rock is carbonaceous slate with low strength and the corresponding saturated uniaxial compressive strength is 5~15MPa.

REGRESSION ANALYSIS OF MONITORING DATA

The field monitoring measurement

The new Austrian method can control the mechanical dynamics and the stability of the surrounding rock and supporting structure in the construction process by field monitoring measurement, the arrangement and analysis of the measured data can provide guidance for the following construction and realize the information and dynamic construction[5]. The selected object is at D3K244+269 section of Song Pan tunnel, in which the surrounding rock is identified as level Ⅳ, and the hole measured point (line) is arranged as shown in Figure 1, measured D3K244+269 section of the arch crown settlement (measured by measuring point A) and horizontal clearance convergence displacement (measured by measuring line BC) and the corresponding displacement time curves were obtained as shown in figure 2 and figure 3.

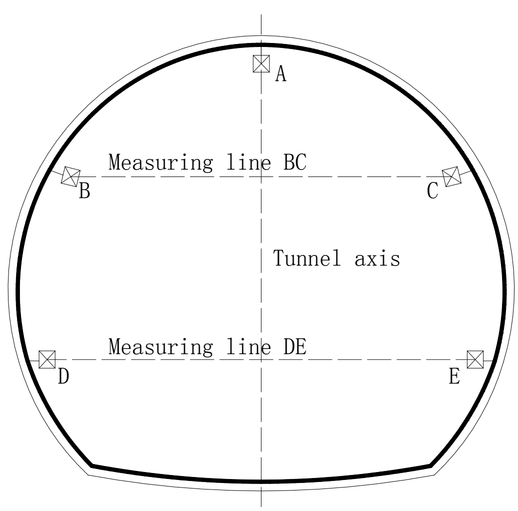

Figure 1: the arrangement of measuring line

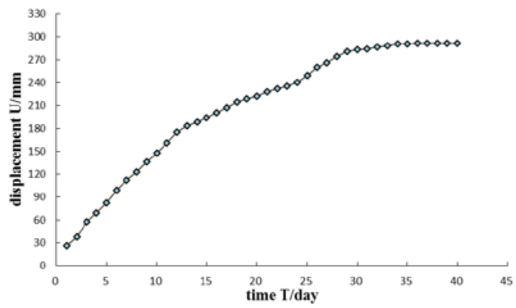

Figure 2: displacement time curve of vault settlement

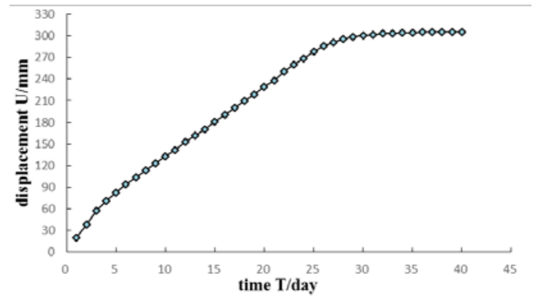

Figure 3: displacement time curve of horizontal clearance convergence

Regression analysis of measurement data

The regression analysis is carried out by three kinds of functions which are commonly used in engineering practice: the exponential function ($U = Ae^{-B/t}$), the hyperbolic function ($U = t/(A+Bt)$), and the logarithmic function($U = A\lg(1+t)+B$).The calculation results of the regression function of arch crown settlement and horizontal convergence displacement are shown in Table 1 .The curve fitting is then obtained by the regression function and then the fitting curves of crown settlement and horizontal convergence are compared with the measured curves, as shown in Figure 4 and 5.When the regression function is determined, the correlation coefficient R2 which can reflect the linear correlation coefficient of the fitting data and the measured data is compared and selected, and the function of the highest correlation coefficient is taken as the final regression function. Through the approach mentioned above, the arch crown settlement regression function can be obtained ($U = 241.528\lg(1+t) - 91.339$), and also the horizontal clearance convergence displacement regression function can be obtained ($U = t/(0.0475+0.0021t)$) .

Table 1: the results of regression analysis of vault settlement and horizontal convergence

Function types	Regression function of arch crown settlement	R^2	Regression function of horizontal clearance convergence	R^2
Exponential function	$U = 253.807e^{-3.035/t}$	0.791	$U = 261.0e^{-3.281/t}$	0.786
Hyperbolic function	$U = t/(0.0393+0.00268t$	0.978	$U = t/(0.0475+0.0021t)$	0.998
Logarithmic function	$U = 241.528\lg(1+t) - 91.$	0.980	$U = 267.73\lg(1+t) - 118.81$	0.952

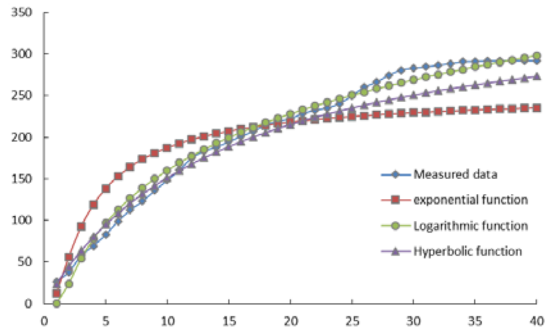

Figure 4: comparison of measured curve and regression function curve of arch crown

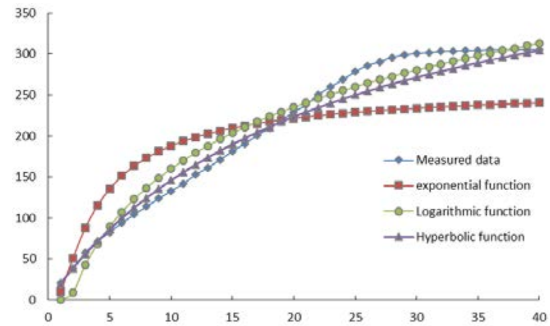

Figure 5: comparison of the measured curve and the regression function curve of horizontal convergence

It can be seen from the convergence deformation curve that, the deformation process of surrounding rock follows the deformation law of the significant deformation period, the slow deformation period and the basic stable period. And taking 60 days of deformation as the final deformation of surrounding rock[6] through the maximum correlation coefficient regression function. According to the calculation of the regression function, the final deformation of horizontal clearance convergence and arch crown settlement can be obtained, which are 345mm and 339mm,the final deformation of the surrounding rock has a certain reference value for the adjustment of the support parameters and the determination of the deformation of surrounding rock, it can also provide the basis for the subsequent error judgment of surrounding rock parameters inversion and result verification.

BACK ANALYSIS OF DISPLACEMENTS

Displacement back-analysis of underground engineering refers to inversion of the initial parameters of medium materials[7] by adopting the given engineering dielectric material model based on the displacement disturbance of the structure and the medium caused by the engineering construction. In order to evaluate the accuracy of the inversion parameters, the objective function of displacement back analysis is established: the displacement of program calculation is denoted as $X = X(\gamma, c, \varphi, E, \mu, K)$, $\gamma, c, \varphi, E, \mu, K$ are the main parameters of numerical analysis software, the displacement of each measuring point is recorded as $\overline{X} = \left(\overline{x_1}, \overline{x_2}, \cdots, \overline{x_i}, \cdots, \overline{x_n}\right)^T$, $i = 1, 2, \cdots, n$ for each monitoring point number, the difference between the calculated displacement and the measurement displacement can represent the accuracy of the inversion parameters. In the formula $R = \sum_{i=1}^{n} (\overline{x}_i - x_i)^2$, R is the objective function; $\overline{x}_i, x_i$ are the monitoring displacement and the calculated displacement value of the measuring point i respectively. The objective of the inversion analysis is to make the objective function value to be minimized by the parameter analysis.

The principle of golden section method

The golden section method is based on the interval of the independent variables of the known minimum function, and then the independent variable interval is divided, and the extreme value is approximated by narrowing the interval until the precision meets the requirement. The advantage of this method is no need the derivation of the function can obtain the minimum value.

Such as the interval for unimodal functions (a, b), then the interval for the golden section. Set the segmentation point c and d, there are:

$$\begin{cases} c = (1-\alpha)a + \alpha b \\ d = \alpha a + (1-\alpha)b \end{cases} \quad (1)$$

In formula (1), $\alpha = 0.618$ is the gold partition coefficient, the segmentation point c and d as shown in Figure 6. From the position relation of the graph, we can know that d<c, compare the function value $f(c)$ and $f(d)$ of c and d: if $f(c) < f(d)$, then take (d, b) as the independent variable of the new computing interval (a_1,b_1); if $f(c) > f(d)$, then take (a, c) as the independent variables of the new computing interval (a_1,b_1), by continually reducing the interval of the independent variable until the function value of the interval (a_n, b_n) satisfy the error requirement. $\alpha = 0.618$ is the partition coefficient of independent variable interval, from the formula (1) we can know that one of the segmentation points of the new district (a1, b_1) is one of the segmentation points on the upper one. Thus this segmentation method can be rapid and effective in narrowing the interval, and the computational efficiency is greatly improved.

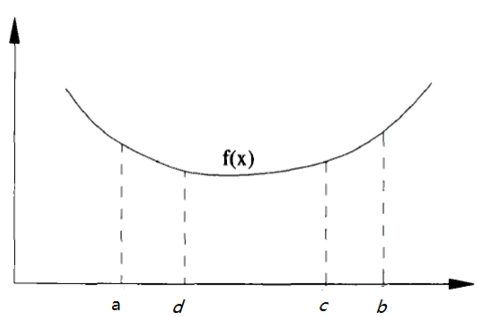

Figure 6: Schematic diagram of the golden section method

For finding the extreme value of multiple variables, the method of coordinate alternation can be adopted. There is only one variable for each partition, and the value of the other variable is replaced by the boundary value of the interval, after calculating the new area of the variable, the other parameters are also determined by the same method, by constantly narrowing the range of parameters until the objective function meets the accuracy requirements, we can stop and then get the minimum value and the independent variable range.

Selection of inversion parameters

Inversion parameters generally select the deformation properties parameters of surrounding rock, including unit weight(γ), cohesion (c), internal friction angle(φ), elastic modulus(E) and Poisson's ratio(μ). Among them,γ,c and φ can get accurate results through laboratory tests, while the rock Poisson's ratioμhas little changes, and the influence on surrounding rock deformation is relatively weak, the elastic modulus E of surrounding rock is affected by various factors such as joint and crack, and the data obtained from the experiment is quite different from the actual situation[8], so the elastic modulus E is selected as one of the inversion parameters.

The selection of initial ground stress field will directly affect the correctness of the construction design under the complicated geological conditions, however, it is difficult to measure the initial ground stress in the field, and also hard to carry out detailed field measurement. The gravity stress field can be calculated by the buried depth and the heavy weight of the rock mass, and the horizontal stress can be obtained by the gravity stress and the lateral pressure coefficient K (the ratio of the horizontal compressive stress and the vertical compressive stress). But the lateral pressure coefficient K is related to the surrounding rock, and there is no unified theory about how to value the side pressure coefficient at present[9], so the lateral pressure coefficient K is also selected as one of the inversion parameters.

The establishment of the model

Based on the measured data of D3K244+269 section of Song pan tunnel, the parameter E and K are selected as the inversion calculation, and the deformation parameters of surrounding rock are retrieved through the whole process of MIDAS/GTS simulation tunnel dynamic construction.

Given the buried depth of this section is 400m, in order to save computing resources, the upper part of the soil is properly simplified, partially covered by the external load input[10-11].Surrounding rock is broken carbonaceous slate, which surrounding rock grade is IV, its material parameters are cited from the railway tunnel design specification, relevant literature[12-14] and engineering experience, the selected range of values of E and K for inversion parameters are: elastic modulus E=1~5GPa, lateral pressure coefficient K=0.75~1.5. It is assumed that the surrounding rock of the tunnel is a homogeneous and continuous elastic-plastic medium, which is subject to the constitutive relation of the Mohr-Coulomb. The section of the tunnel is three-centered circular, the span of the model is 90m, the height is 96m, and the length of the tunnel axis is 48m, tunnel construction method with short steps, every excavation footage for 2.4m. The model is divided into 28120 elements and 29392 nodes by the mixed grid. Supporting parameters of tunnel surrounding rock and other structure parameters are shown in table 2. Figure 7 is the overall model of the tunnel, figure 8 is the tunnel anchor and lining model. The boundary conditions of the calculation model are set as follows: the displacement of the bottom surface constraint Z direction, the displacement of the front and back surface bound X direction, the displacement of the left and right constraint Y direction. The top surface of the model is considered as free surface.

Table 2: Table of parameters for tunnel model

Support materials	Unit weight γ (kN/m^3)	Elastic modulus E (GPa)	Poisson ratio μ	Cohesion c (MPa)	Internal friction angle φ (^{0})
Surrounding rock	18.5	1~5	0.3	0.15	28
Sprayed concrete	22	15	0.3	2	50
secondary lining	23	31.5	0.2	5	60
Steel arch centering	78.5	206	0.3	—	—
bolt	78.5	200	0.3	—	—

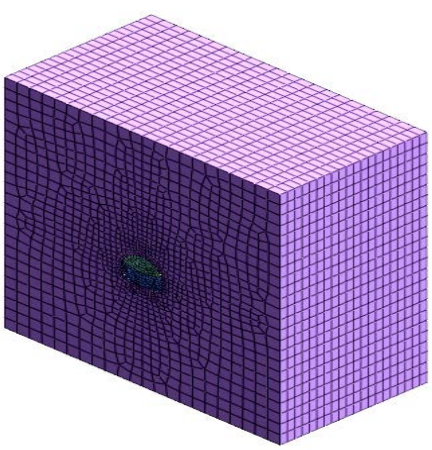

Figure 7: overall model

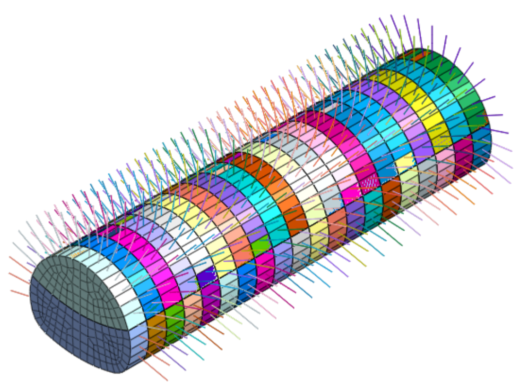

Figure 8: anchor and lining model

BACK ANALYSIS CALCULATION AND ANALYSIS

According to the principle of the golden section method, the tunnel's deformation function is assumed to be the $g(x)=g(E,K)$, the measured deformation of the tunnel surrounding rock is *G*, and construct a function $f(x)=|g(x)-G|$. The purpose of the inversion analysis is to obtain the optimal solution of E and K when the minimum value of $f(x)$ is obtained.

In the process of back analysis, the final deformation of the horizontal clearance convergence and arch crown settlement predicted by the regression analysis is used as the criterion of error estimation, in order to reflect the actual working condition of the surrounding rock more comprehensively, both horizontal displacement and arch crown settlement are used to judge the error. And then using the golden section coordinate alternation method to conduct the back analysis, which process is as follows:

The inversion calculation of *E* and *K* is carried out by using coordinate alternation method, first fixed *K*=0.75, and then the interval of *E* for the golden section, *c*=3.472,*d*=2.528,horizontal clearance converge and vault displacements are $g(3.472)_{horizon}$=70mm,$g(3.472)_{vault}$=176mm,the relative error is $\sigma_{horizon}$=(345-70)/345=80%,σ_{vault}=(339-176)/339=48%; $g(2.528)_{horizon}$=82mm,$g(2.528)_{vault}$=232mm, the relative error is $\sigma_{horizon}$=(345-82)/345=76%,σ_{vault}=(339-232)/339=32%.Thus available, $f(2.528)_{horizon} < f(3.472)_{horizon}$, $f(2.528)_{vault} < f(3.472)_{vault}$, we can select interval (1, 3.472) as the new interval of *E*.

Then fixed *E*=3.472, and the interval of *K* for the golden section, *c*=1.214,*d*=1.037, horizontal clearance converge and vault displacement $g(1.214)_{horizon}$=218mm,$g(1.214)_{vault}$=127mm, the relative error is $\sigma_{horizon}$=(345-218)/345=37%,σ_{vault}=(339-127)/339=63% ; $g(1.037)_{horizon}$=156mm,$g(1.037)_{vault}$=146mm,the relative error is $\sigma_{horizon}$=(345-156)/345=55%,σ_{vault}=(339-146)/339=57%.Compare with the function value of each point, we can know that $f(1.214)_{horizon} < f(1.037)_{horizon}$, the new interval of *K* should be (1.037, 1.5), $f(1.214)_{vault} > f(1.037)_{vault}$, and the new interval of *K* is (0.75, 1.214), in order to meet the requirements of the deformation of the horizontal clearance displacement and the vault displacement, the common interval (1.037, 1.214) of the two sections is selected as the new interval of *K*. Through the coordinate alternation method of the golden section method, the interval range is continuously reduced, and the error of the numerical calculation result and the measured result can be controlled in a reasonable range. Currently about inversion calculation error control is rare, and with no clear standard. According to the research of Dong Cheng long[15] combined with practical engineering, accuracy control of back analysis in the range of 5% - 10% can basically meet the engineering needs, therefore the range is applied as an error criterion for this paper, the final optimization result and calculation process are shown in Table 3. D3K244+269 section of the surrounding rock elastic modulus *E* and lateral pressure coefficient *K* the final inversion results are 1.361GPa and 1.105, respectively.

Table 3: back analysis results of surrounding rock parameters *E* and *K*

times	*E* value range (GPa)	*K* value range	Calculated *E* value (GPa)	Calculated *K* value	Calculated displacement (mm)		Displacement of regression prediction (mm)		Relative error (%)	
					horizontal	vault	horizontal	vault	horizontal	vault
1	(1, 5)	(0.75, 1.5)	1	0.75	124	523	345	339	64%	54%
			5	0.75	59	130	345	339	83%	62%
2	(1, 5)	(0.7, 1.5)	2.528	0.75	82	232	345	339	76%	32%
			3.472	0.75	70	176	345	339	80%	48%
3	(1, 3.472)	(0.75, 1.5)	3.472	1.214	218	127	345	339	37%	63%
			3.472	1.037	156	146	345	339	55%	57%
4	(1, 3.472)	(1.037, 1.214)	1.944	1.037	234	236	345	339	32%	30%
			2.528	1.037	194	189	345	339	44%	44%
5	(1, 2.528)	(1.037, 1.214)	1.584	1.037	270	280	345	339	22%	18%
6	(1, 1.944)	(1.037, 1.214)	1.944	1.105	274	221	345	339	21%	35%
			1.944	1.146	298	213	345	339	14%	37%
7	(1, 1.944)	(1.105, 1.146)	1.361	1.105	354	320	345	339	3%	6%

Back analysis results verification

In order to examine the accuracy and reliability of the inversion results of surrounding rock parameters, select one of the section(D3K244+257 section) to conduct the verification, since it is in the same construction section close to the D3K244+269 and their surrounding rock parameters can be considered the same[16]. Results of the inversion calculation (elastic modulus *E*=1.361GPa, side pressure coefficient *K*=1.105) as a substitute the known variables MIDAS / GTS calculation, figure 9 and figure 10 are the final results of arch crown settlement and clearance convergence displacement, the nephogram shows that the final calculation results of horizontal clearance convergence and arch crown settlement are 326mm and 313mm, respectively. Compared with the predicted final deformation of the horizontal convergence and vault settlement, which were 312mm and 304mm, respectively, using regression analysis method at D3K244+257 section, the error between the calculated and measured results is only 4.5% and 3%, definitely meets with the requirement of engineering error control. It can be seen that the surrounding rock parameters obtained by the inversion analysis based on measured data can accurately reflect the real deformation of the surrounding rock, this method has certain reference significance to the engineering that the parameters of surrounding rock of soft rock tunnel in high ground stress are not easy to be obtained accurately.

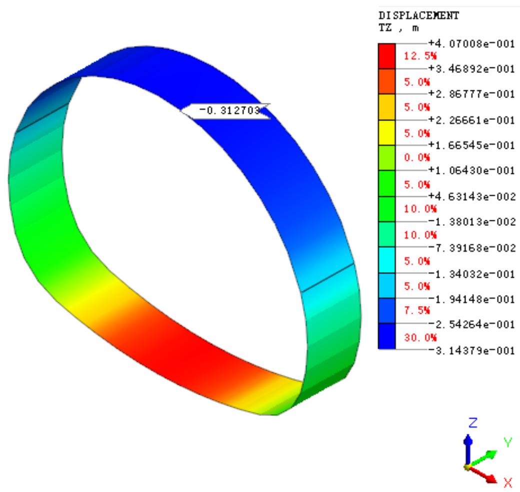

Figure 9: final displacement of arch crown settlement

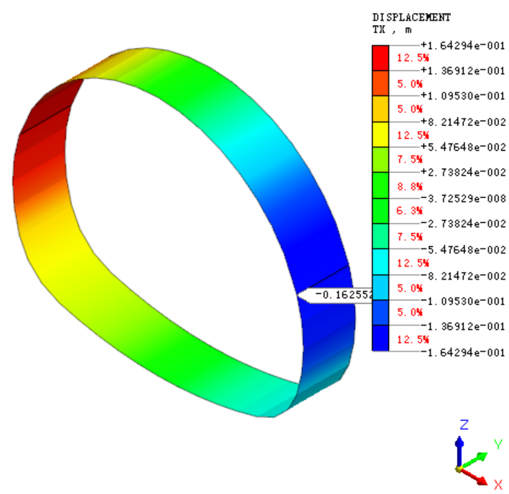

Figure 10: final displacement of horizontal clearance convergence

CONCLUSIONS

(1) Through the regression analysis and curve fitting of the measured displacement data, the final deformation of horizontal clearance convergence and arch crown settlement is predicted, which can provide valuable reference for tunnel's dynamic design and information construction. It is also proved that the mathematical regression analysis which based on the least square method is feasible to predict the ultimate deformation of soft rock tunnel with complicated geological conditions.

(2) On the base of in-situ monitoring data of D3K244+269 section, using MIDAS/GTS as the inversion calculation tool, combined with the golden section method, the elastic modulus *E* and the lateral pressure coefficient *K* of the surrounding rock of the tunnel were analyzed, the results of the analysis error can satisfy the demands of practical engineering using precision, the reliability of the inversion results is verified by the D3K244+257 section, it has great application value in the scientific design and control of the tunnel construction quality of the high stress soft rock tunnel with complex geological conditions.

(3) Based on the combination of field monitoring measurement, numerical simulation and displacement back analysis, the reasonable value of the physical and mechanical parameters of the surrounding rock can be obtained. This comprehensive method can effectively simulate and

predict the deformation and mechanical behavior of surrounding rock during the excavation process.

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