Major Effects of Control Parameters on Kinetic Characteristic Analysis in 3-D DDA

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ABSTRACT

Discontinuous Deformation Analysis (DDA) is a new numerical simulation method in geotechnical engineering to analyze discontinuum mechanics. In this paper, three typical examples of block motion including sliding, open-close iteration and rebound are carried out to investigate the effect of different spring stiffness, time step, elastic modulus and other factors on the kinetic characteristic of block in the three-dimensional (3-D) DDA. The results indicate the characteristics of several control parameters in the kinetic analysis. The size of time step and the spring stiffness play an important role to ensure the reasonable computational accuracy and kinetic behavior of blocks. And the value range of actual physical quantity-elastic modulus can be extended to investigate more materials except for rock masses in DDA. This study can provide a theoretical basis of the selection of DDA parameters for the application in dynamic engineering computation.

KEYWORDS: DDA; Kinetic characteristic; Elastic modulus; Spring stiffness; Time step

INTRODUCTION

Discontinuous Deformation Analysis (DDA)[1,2] is a numerical analysis method to study the discontinuous displacement and deformation of the block system. This method allows the simulation of not only translation, rotation and deformation of individual blocks, but also large-scale sliding and opening along discontinuities[3,4]. Compared with other discontinuum-based numerical methods, DDA benefits from the rigorous mathematical and physical theories, such as the classic variational principle[1], the general contact theory[5], the elegant simplex integration[6], the strictly convergent open-close iteration[1,7] and the unconditionally stable time integral scheme[8]. Significant developments have been achieved since DDA was first introduced as a two-dimensional (2-D) mode. With continuous modifications and improvements, 2-D DDA has been more efficient and suitable to cover practical engineering problems of landslide[9], rock fall[10], tunnel[11], masonry structures[12] and many others.

Because DDA uses penalty method to substitute actual force and use the open-close iteration method, some control parameters[13] (i.e. spring stiffness, time step, etc.) have great effect on the kinematic characteristics. The effects of many control parameters on the kinetic characteristics in 2-D DDA have been researched by many experts. Shi[2] and Jiang[14] suggested that the spring stiffness *p* is suitable to be 20～100 times of the elastic modulus, and as long as the spring stiffness is large enough, the computation results will not depend on the choice of spring stiffness. Amadei[15] found that the results are too dependent on the spring stiffness. Tsesarsky[16] concluded that unreasonable time step $\triangle t$ will affect the accuracy of the calculated convergence value, and excessive spring stiffness can also make the stiffness matrix seriously ill-conditioned, making errors in calculation. Through numerical examples, Liu[17] recommended the range of time step, step displacement and spring stiffness. Wu[18] initially identified the reasonable interval of time step and spring stiffness through free fall model and inclined sliding block model to study the impact of time step and the spring stiffness on the calculation results.

In recent years, various three dimensional numerical methods have been used widely in different research fields, for the simulation results of these numerical methods are in good agreement with real situation, such as the application of Finite Difference Method (FDM) for tunnel’s construction[19], Finite Element Method (FEA) for the slope stability[20], the Discrete Element Method (DEM) for soil, rock and concrete mechanics[21], and so on. But the research filed of DDA is still limited because of its difficulty in choosing the parameters. Therefore, by analyzing the major effects of control parameters on the kinetic characteristics, it is helpful to extend the practical applications for 3-D DDA.

With the development of the basic theory and contact mechanism for 3-D DDA[5,22-27], more and more applications in rock engineering adopt 3-D DDA to obtain more realistic results[7,28,29]. However, the effects of the control parameters on the computational results in 3-D DDA are still unclear. Normally, it is believed that the parameters used for the 2-D DDA can be applicable for the 3-D DDA. But, the values of the parameters in 3-D DDA are significantly different from that in 2-D DDA, resulting from the much more complicated contact theory in 3-D as compared to that in 2-D. For instance, the time step should be much smaller in 3-D than in 2-D. In addition, Shi’s recommendations for the spring stiffness in 2-D DDA might be not appropriate in 3-D[30]. However, the studies regarding on this topic in 3-D DDA are really rare to see currently. Therefore, it is in urgent need to investigate the effects of the control parameters on kinetic characteristics using 3-D DDA.

This paper will focus on three simple 3-D DDA models to study the effects of different control parameters on the kinetic characteristics of the block. Model 1 is an inclined sliding block model, which discusses the effect of the time step and the spring stiffness on the accuracy of sliding distance. Model 2 is a block open-close iteration model, which discusses the effect of different parameters on the open-close iteration process. Model 3 is a block rebound model, which discusses the effects of time step, spring stiffness on the computational accuracy and the actual agreement of elastic modulus during the rebound process.

Basic principles of 3D-DDA

Displacement and deformation of 3-D blocks

Different from the finite element method which chooses the node displacement as degree of freedom, DDA chooses the rigid body displacement and the strain of the block element as the basic unknown quantity, and obtains the large deformation and large displacement of the block system by accumulating small displacement and small deformation of the unit block at each time step. Suppose an arbitrary polygon block has constant stress, constant strain everywhere, then the motion and deformation of the block can be determined by the following 12 independent kinematic variables:

$$\mathbf{D}_i = [u_0, v_0, \omega_0, \alpha_0, \beta_0, \gamma_0, \varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}]^T \tag{1}$$

where $\mathbf{D}_i$ is the displacement variable of block i, u_0, v_0, ω_0 are the linear displacements , $\alpha_0, \beta_0, \gamma_0$ are the angular displacements, $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are the normal strain, and $\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ are the shear strain, then the displacement of any point (x, y, z) in the block can be expressed as:

$$[u\ v\ \omega]^{\mathrm{T}} = \mathrm{T_i}(x, y, z)\mathrm{D_i} \tag{2}$$

where

$$\mathbf{T}_i(x,y,z) = \begin{bmatrix} 1 & 0 & 0 & 0 & z-z_0 & y_0-y & x-x_0 & 0 & 0 & \frac{(y-y_0)}{2} & 0 & \frac{(z-z_0)}{2} \\ 0 & 1 & 0 & z_0-z & 0 & x-x_0 & 0 & y-y_0 & 0 & \frac{(x-x_0)}{2} & \frac{(z-z_0)}{2} & 0 \\ 0 & 0 & 1 & y-y_0 & x-x_0 & 0 & 0 & 0 & z-z_0 & 0 & \frac{(y-y_0)}{2} & \frac{(x-x_0)}{2} \end{bmatrix},$$

(x_0, y_0, z_0) is the centroid coordinates. $\mathbf{T}_i(x, y, z)$ is the displacement transformation matrix of the block and it can reflect the block's translation, rotation, normal strain, shear strain and the displacement condition under complex stress state.

General equilibrium equations

The block system is composed of several single blocks, by contacts between blocks and by displacement constraints on single blocks. Assuming there are n blocks, the equilibrium equations have the form:

$$\begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \mathbf{K}_{13} & \cdots & \mathbf{K}_{1n} \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \mathbf{K}_{23} & \cdots & \mathbf{K}_{2n} \\ \mathbf{K}_{31} & \mathbf{K}_{32} & \mathbf{K}_{33} & \cdots & \mathbf{K}_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{K}_{n1} & \mathbf{K}_{n2} & \mathbf{K}_{n3} & \cdots & \mathbf{K}_{nn} \end{bmatrix} \begin{Bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \\ \mathbf{D}_3 \\ \vdots \\ \mathbf{D}_n \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \mathbf{F}_3 \\ \vdots \\ \mathbf{F}_n \end{Bmatrix} \tag{3}$$

Because each block has twelve degrees of freedom, [$\mathbf{K}_{ij}$] is a 12×12 submatrix. [$\mathbf{K}_{ii}$] completely depends on the material properties and geometric dimensions of *Block i*, while [$\mathbf{K}_{ij}$] ($i \neq j$) depends on the contact condition of the *Blocks i* and *j*. Both {$\mathbf{D}_i$} and {$\mathbf{F}_i$}are 12×1 submatrix. {$\mathbf{D}_i$} represents the deformation variable of *Block i* and {$\mathbf{F}_i$} is the external loading that causes the block to move and generate deformation.

According to the variational principle and constant strain-displacement mode, these equilibrium equations are derived by minimizing the total potential energy Π_p done by the stresses and external forces:

$$\mathbf{K}_{ij} = \frac{\partial^2 \Pi_p}{\partial d_{ri} \cdot \partial d_{sj}} \qquad r, s = 1,2 \cdots, 12 \tag{4}$$

$$\mathbf{F}_i = -\frac{\partial \Pi_p}{\partial d_{ri}} \qquad r = 1,2 \cdots, 12 \tag{5}$$

Joint contact

Shi[1,2] used the penalty function method to realize the non-penetrated and non-tensile contact criteria between the interface of a block in the original DDA. The spring stiffness directly determines the contact condition. The open-close iteration[7] is the procedure of adding and removing stiff springs depending on the changes in contact state. A normal contact prevents interpenetration of objects, and a tangential constraint enforces sticking or slipping. The iteration is employed to obtain the locations and contact forces of until limited penetration or tension is found among all contacts within each time step. The point-to-face contact has three possible states: open, sliding and locked. Depending on the contact locations and the application conditions of the contact springs, the three types of contact states can be determined by using a joint contact model based on concentrated cohesion failure criteria (Table 1):

Table 1: Criteria for contact states

Contact state	Open	Sliding	Locked
Criteria	$d_N \geq 0$	$\mathbf{d_N} < 0$ $p_S\|\mathbf{d_S}\| > - p_N \mathbf{d_N} \tan\varphi + cA_c/m$	$\mathbf{d_N} < 0$ $p_S\|\mathbf{d_S}\| \leq -p_N \mathbf{d_N} \tan\varphi + cA_c/m$

where p_N and p_S are the stiffnesses of the normal and shear contact springs (Figure 1), $\mathbf{d_N}$ and $\mathbf{d_S}$ are the normal penetration distance and relative shear displacement vector, φ is the friction angle, c is the joint cohesion, A_c is the joint area, and m is the number of the dominant sub-contacts of the joint contact.

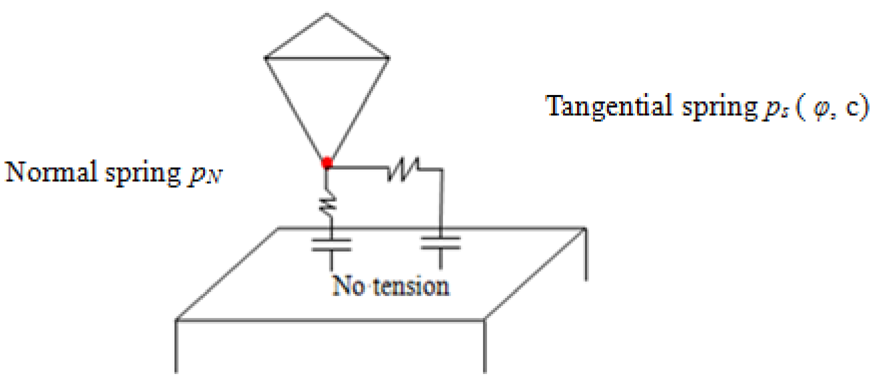

Figure 1: The constitutive model of contact in 3-D DDA

Time integration

The governing dynamic equation[8] for block system is:

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\}+[\mathbf{C}]\{\dot{\mathbf{u}}\}+[\mathbf{K}]\{\mathbf{u}\}=\{\mathbf{F}\} \quad (6)$$

where [**M**], [**C**], [**K**] is mass, damping, stiffness matrix, {**F**} is the time dependent applied force vector, and {**ü**}, {**u̇**}, {**u**} are acceleration, velocity, displacement vectors, respectively.

It is a difficult problem to investigate the stability of the nonlinear DDA system, for it includes the behavior of the open-close iteration. Hence, by considering the time step, change the differential equation to the difference equation. The computation of the next time step *n*+1 is based on the current time step *n*. Let $\{\mathbf{u}\}_n$ and $\{\mathbf{u}\}_{n+1}$ denote the approximation to the value of $\boldsymbol{u}(t)$ and $\boldsymbol{u}(t+\triangle t)$, respectively. The dynamic equation for the *n*+1 time step is:

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\}_{n+1}+[\mathbf{C}]\{\dot{\mathbf{u}}\}_{n+1}+[\mathbf{K}]\{\mathbf{u}\}_{n+1}=\{\mathbf{F}\}_{n+1} \tag{7}$$

The time integration scheme can be shown as the following equations:

$$\{\mathbf{u}\}_{n+1}=\{\mathbf{u}\}_n+\Delta t\{\dot{\mathbf{u}}\}_n+\frac{(\Delta t)^2}{2}\{\ddot{\mathbf{u}}\}_{n+1} \tag{8}$$

$$\{\dot{\mathrm{u}}\}_{n+1}=\{\dot{\mathrm{u}}\}_n+\Delta t\{\ddot{\mathrm{u}}\}_n \tag{9}$$

Inclined sliding block model

Because inclined sliding block model is the basis for the application of slope stability analysis of geotechnical engineering, and it is easy to calculate the analytical solution, many scholars have used this model to discuss the kinetic characteristics of the block with different time step Δ*t*, spring stiffness *p* and internal friction angle *φ*[17,18,31,32]. But most of these studies limited in 2-D field. So, this paper will study the effects of the time step and the spring stiffness on the accuracy of the sliding displacement in 3-D field.

As shown in Figure 2, a block is on an inclined plane with the angle of 30°, the block size 1m×1m×0.5m, density (ρ) 2000 kg/m3, elastic modulus (E) 3×108Pa, Poisson ratio (ν) 0.3. The internal friction angle *φ* is 15°. Study the kinetic characteristic of the block when it is sliding.

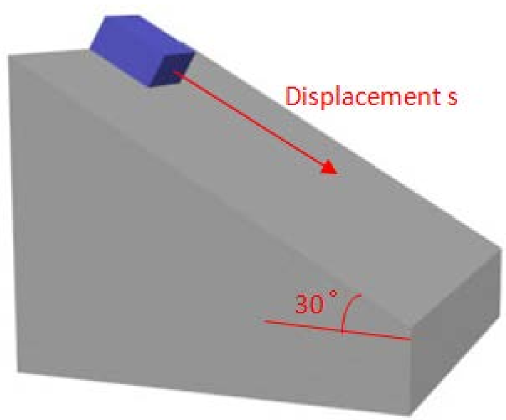

Figure 2: Schematic diagram of inclined sliding block model

When the sliding force is larger than the anti-sliding force, the block will slide. Because the cohesion *c* doesn't work after sliding, *c* can be ignored in the analysis. In the sliding process, the anti-sliding force is only provided by friction angle *φ*, so the displacement formula of the sliding block is:

$$s=\frac{1}{2}(\mathrm{g}\sin\alpha-\mathrm{g}\cos\alpha\tan\varphi)t^2 \tag{10}$$

where *s* is the displacement of the block, *g* is the acceleration of gravity, *α* is the slope angle.

The computational accuracy of displacement can be obtained by calculating the displacement relative error δ to compare with the analytical solution:

$$\delta = [s(\text{numerical solution}) - s(\text{analytical solution})] / s(\text{analytical solution}) \times 100\% \quad (11)$$

To study the effect of time step Δt on the computational accuracy of sliding displacement, keeping p = 3×10^9N/m, calculate the relative error of displacement respectively when Δt = 0.001s, 0.0005s, and 0.0002s (Figure 3). The relative error decreases with the increasing of total sliding distance. It is evident that when the time step smaller than 0.001s, the relative error is under 1%, which shows that the computational results are accurate enough. When the time step larger than 0.001s, the block will not normally slide mainly because of two reasons. One is that the block will penetrate too much into the plane. The other one is that the computational accuracy is not enough. So the above 3 different time steps are chosen. From Figure 3, it is useless to choose too small time step. Because small time step will decrease the computational efficiency. Too small time step may also cause too small displacement and too large diagonal elements in the matrix [$\mathbf{K}_{ii}$], which makes the results obviously distorted or non convergent. So the choice of time step is of great importance. Both too large and too small time step may cause large error. Among the above 3 time steps, 0.001s has enough computational accuracy and the highest computational efficiency, so it is recommended.

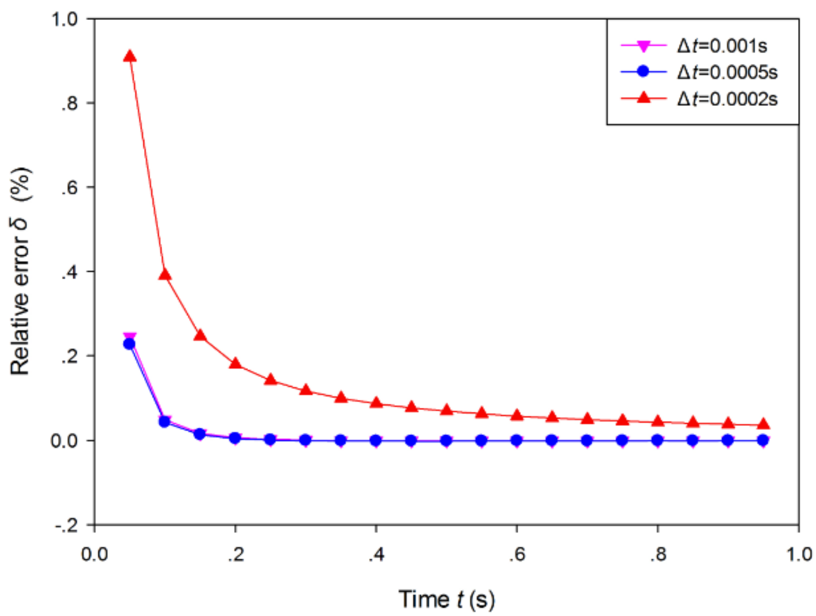

Figure 3: The relationship between time step Δt and relative error δ

To study the effect of spring stiffness p on the computational accuracy of sliding displacement, keep Δt =0.001s, and calculate the relative error of displacement when p= 3×10^8N/m, 3×10^9N/m, and 3×10^{10}N/m, respectively (Figure 4). It shows that when spring stiffness is large enough, the relative error is very small. The open-close iteration is used to enforce no penetration and no tension, so larger spring stiffness is better. But the spring stiffness can't be too large. From Eq. (3), if the spring stiffness is too large, [$\mathbf{K}_{ij}$] may be larger than [$\mathbf{K}_{ii}$], which makes the matrix computation not convergent. In addition, the displacement or deformation matrix $\{\mathbf{D}_i\}$ will be very small. When a small error in the $\{\mathbf{D}_i\}$ multiplies a very large spring stiffness, there may be a large error in load matrix $\{\mathbf{F}_i\}$, which can produce errors continuously after using this error in the iterative calculation. Certainly, the spring stiffness can't be too small. Otherwise, the block will directly penetrate into the slope. When the penetration depth is too large, there will be no normal contact force between the interface to keep the block on the slope. Although in this model, anti-sliding force is produced by the tangential spring force, the effect of tangential spring stiffness on the computational accuracy can't be analyzed correctly in this condition.

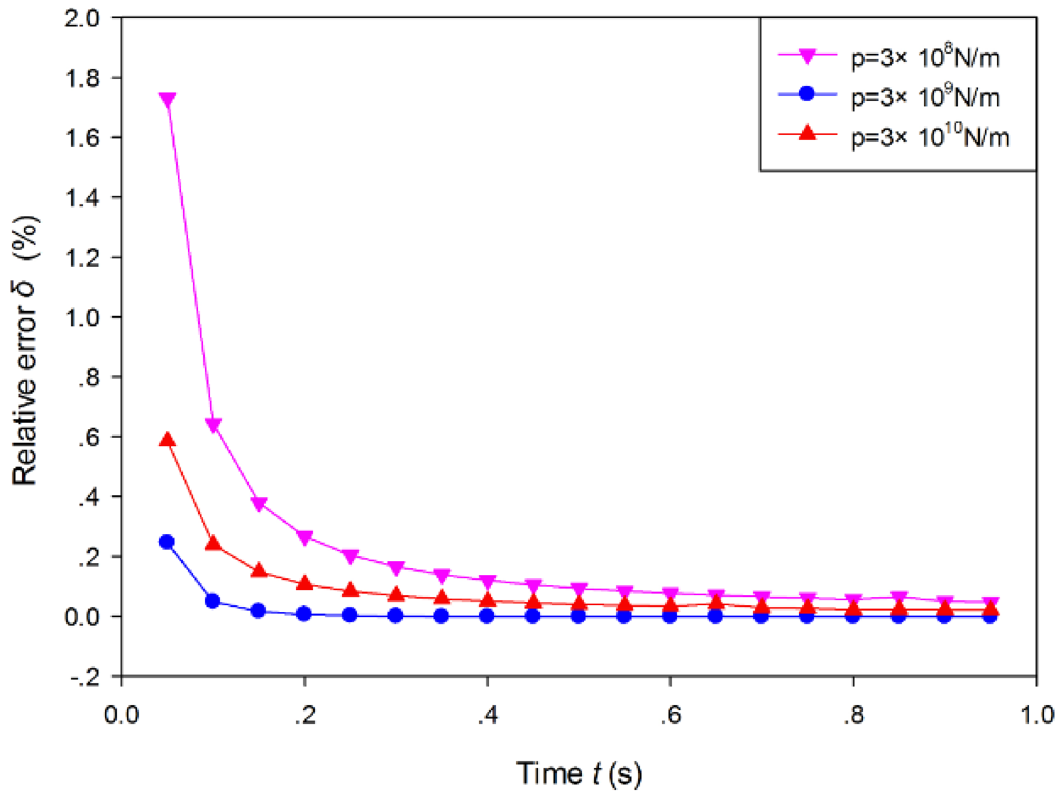

Figure 4: The relationship between spring stiffness *p* and relative error δ

Therefore, as long as the time step and the spring stiffness are reasonable, the numerical solutions are in good agreement with the analytical solution.

Block open-close iteration model

To quantitatively study the interaction relationship between the major control parameters of the contact open-close iteration process in the3-D DDA block system, a simplified model is established here to explain the process of the open-close iteration, which is actually the process of applying and removing the spring. As shown in Figure 5, there is a block on a fixed plane, the block size 2m×2m×2m, density (ρ) 2000 kg/m^3. To study the relationship between time step and the penetration depth, keeping the elastic modulus E = 3×10^{12}Pa and spring stiffness p = 3×10^{8}N/m, calculate the displacement d when the time step is 0.001s, 0.0005s and 0.0002s (Figure 6).

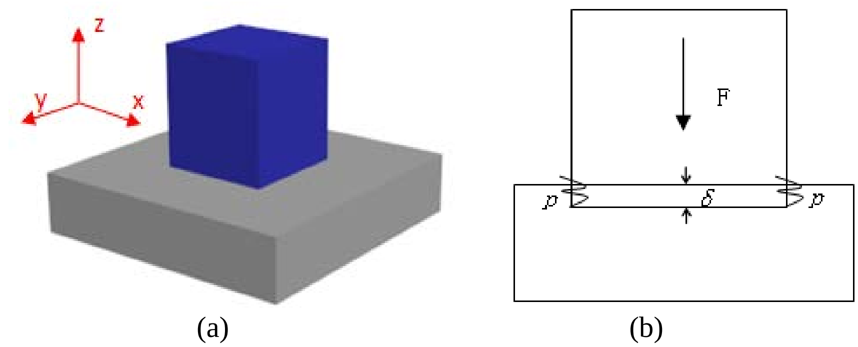

Figure 5: Schematic diagram of block open-close iteration model

In DDA, the contact interface needs to be ensured with no penetration and no tension no penetration. From Figure 6, under the conditions of different time step, the displacement *d* is much smaller than the smallest side length in the block system, which is only about 0.01% of the length of the block. So the penetration depth is smaller enough and almost can be ignored, which meets the requirement of no penetration. In addition, the displacement *d* is always less than 0, which also meets the requirement of no tension. In addition, in this model, it is better that the maximum penetration depth is small and the block system reach the relatively static state as soon as quickly. When time step is 0.001s, the convergence rate of the displacement is the fastest. And when the time step is less than 0.001s, the maximum penetration depth will become larger. Therefore, if the time step is too small, there is no good for the open-close iteration of the contact interface. The time step can be adjusted to speed up the open-close iteration convergence and ensure the accuracy of the solution. Hence, the time step 0.001s is recommended for it has enough computational accuracy and is better for the computational convergence rate.

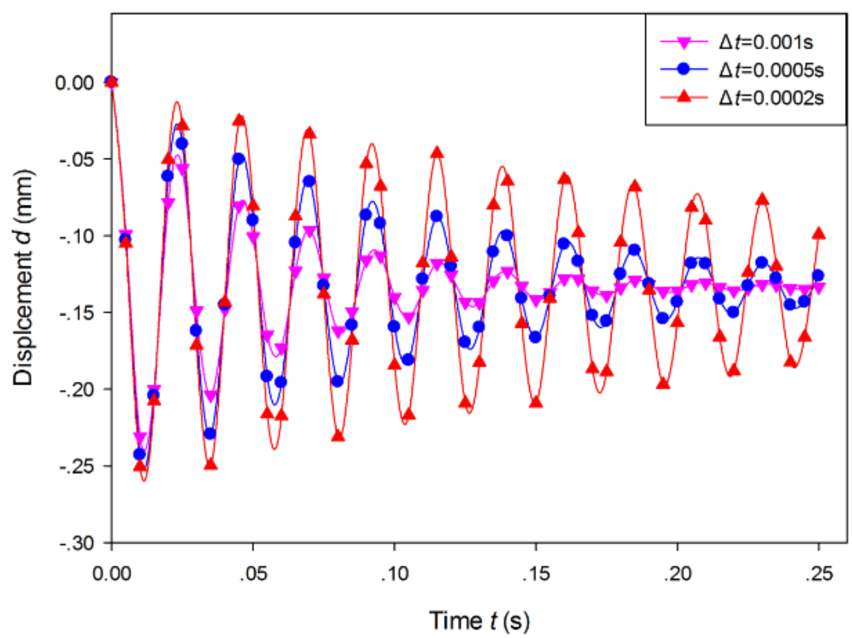

Figure 6: The displacement *d* with time

To study the effect of spring stiffness on the penetration depth, keep the time step Δt=0.001s and calculate the displacement *d* with different spring stiffness. It is easy to find that if the spring stiffness is larger, the displacement will be smaller at the same time point, the block will reach the balanced position more quickly. In principle, the spring stiffness can be adjusted according to the maximum penetration displacement. From Figure 7, the balanced position is just in inverse proportion to the spring stiffness:

	$$\frac{p}{p_0}\cdot\frac{d}{d_0}=1$$	(12)
	$$pd=p_0d_0=F$$	(13)

where p_0 is spring stiffness 3×10^8N/m, and d_0 is the block balanced position when p_0 is 3×10^8N/m. Eq. (12) and Eq. (13) mean that the force which balances the block gravity *m*g is the normal spring force $F = dp$ (Hooke's law, *d* is the deformation of spring or the penetration depth between the block and the plate).

From Figure 7, larger spring stiffness *p* is better for the requirement of no penetration. But *p* can't be too large, otherwise, the stiffness matrix $[\mathbf{K}_{ij}]$ in Eq. (3) will not be linearly dependent, which absolutely leads to the wrong computational result.

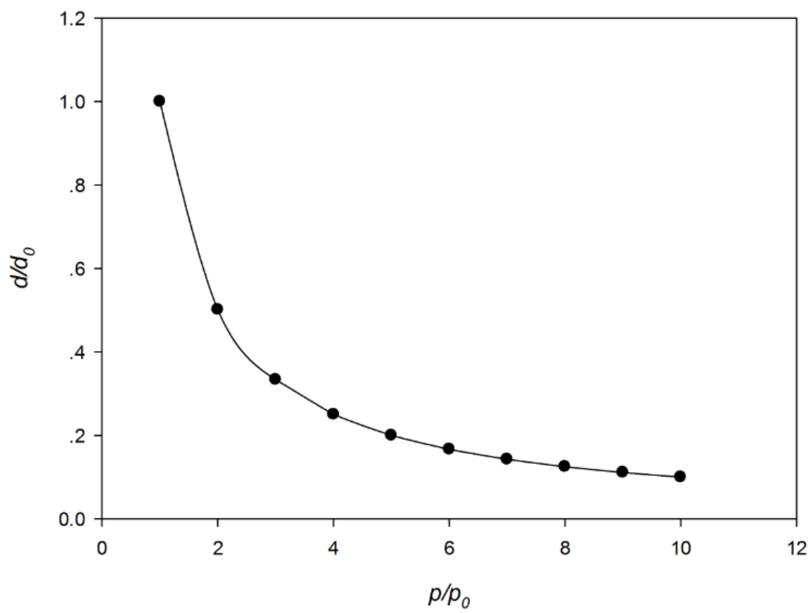

Figure 7: The relationship between spring stiffness *p* and balanced position *d*

Block rebound model

At present, there are few researches on the impact characteristics by using DDA. The block rebound model is a complex model which comprehensively considers various control parameters. As shown in Figure 8, there is a block over a fixed flat plate and then fall, the block size 2m×2m×2m, Poisson ratio (*v*) 0.15. The elastic modulus of the flat plate is very large, and the initial height is 3m. When the block impact the plate, there will also be a certain penetration depth *d* between the block and the plate (similar to the block open-close iteration model) , and then the block rebound up.

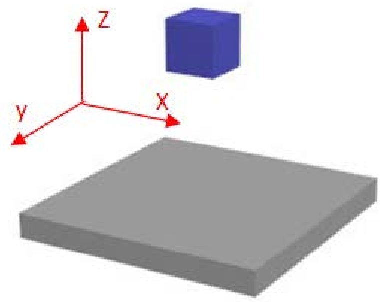

Figure 8: Schematic diagram of block rebound model

Every time the rebound height will decrease which means the energy losses at the impact moment. The total energy of the block system includes strain energy of block unit, initial stress potential energy, point load potential energy, line load potential energy, body load potential energy, bolt connection potential energy, inertia force potential energy, viscous force potential energy and so on[17]. In this model, joint parameters (friction, cohesion, tensile strength) are all equal to 0, so the loss of total potential energy is mainly because of the computational accuracy and the block deformation. Before each rebound, the gravitational potential energy and kinetic energy ($mgh+\frac{1}{2}mv^2$) remains unchanged. But after each rebound, the maximum height of block will be less than the previous one, which means the energy has been dissipated. The **dissipation coefficient** *m* is:

$$m = \left(1 - \frac{h_{n+1}}{h_n}\right) \times 100\% \tag{14}$$

where h_n is the maximum height after the n^{st} rebound, n = 0,1,2,3…, h_0 is the initial height 3m.

To study the relationship between time step and the **dissipation coefficient,** keep the block's density ρ = 2000kg/m^3, E =3×10^{8}Pa, p = 3×10^{8}N/m, and observe the energy dissipation condition when time step is 0.001s, 0.0005s and 0.0002s (Figure 9). It can be found that energy dissipates more seriously with the increasing of the time step. If the time step is too large, the block can't reach the real bottom of the penetration depth. Hence, smaller penetration displacement results to smaller spring force. Actually, the spring force will not provide large enough counter force to make the block rebound back to the initial height.

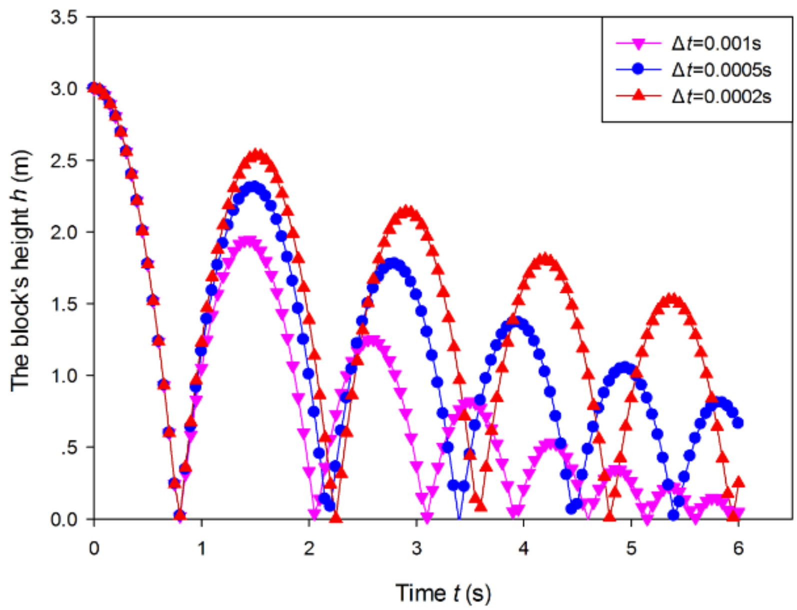

Figure 9: The relationship between the block's height *h* and time step Δ*t*

To study the relationship between the impact force and energy dissipation, observe the data in the same test, such as the above test when Δt = 0.001s. When the block quality doesn't change, the relationship between the maximum height before rebound and the dissipation coefficient can be concluded (Table 2). The data show that m is around 35%. The energy dissipation rate is stable in the same test. Different maximum height (or different impact velocity) almost has little effect on energy dissipation. Then change the block's density to 5000 kg/m3, 8000 kg/m3 and compare with the condition when the density is 2000 kg/m3. Observe the first penetration depth ratio δ/H (H is the minimum length in block system) and m (Table 3). It can be shown that impact quality has a great effect on energy dissipation. The heavier the block is, the larger the penetration depth is, the larger the maximum normal spring force is, and the block will rebound higher. Therefore, the impact force has a remarkable effect on the energy dissipation.

Table 2: The relationship between maximum height h_n and dissipation coefficient *m*

Rebound time *n*	Maximum height h_n (m) before rebound	Dissipation coefficient *m* (%) after rebound
1	3.00	35.3
2	1.94	34.4
3	1.27	35.2
4	0.82	34.8
5	0.54	35.4

Table 3: The relationship between impact quality and **dissipation coefficient** *m*

Density(kg/m^3)	*d/H*	*m*(%)
2000	0.0305	35.2
5000	0.0501	26.7
8000	0.0642	23.5

To study the effects of elastic modulus and spring stiffness on the **energy** dissipation, keeping the block's density ρ = 2000 kg/m^3 and time step Δt = 0.001s, calculate **dissipation coefficient** *m* with different elastic modulus *E* and spring stiffness *p* (Figure 10):

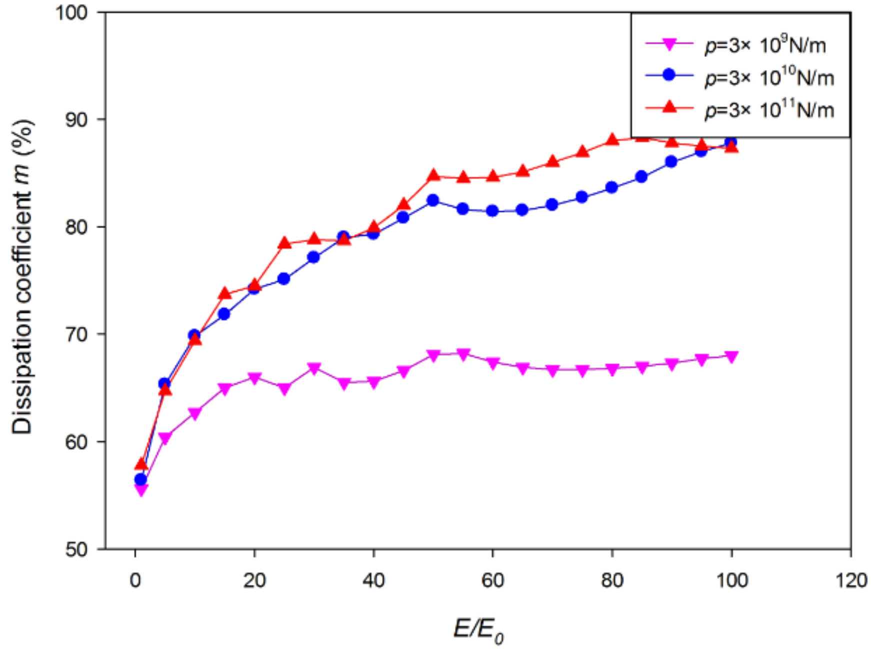

Figure 10: Dissipation coefficient *m* with different *E* and *p*

where E_0 is the elastic modulus 3×10^{7}Pa.

According to the trend of *m* in Figure 10, both elastic modulus and spring stiffness have effects on energy dissipation:

When the elastic modulus keeps unchanged**,** *m* increases with the increasing of spring stiffness. When the spring stiffness is large, the penetration depth is smaller. The time of the springs contact the block becomes shorter. According to the momentum conservation formula $Ft = mv$(F is the contact force, *t* is the duration of contact, *m* is the quality of the block, and *v* is the velocity of the block), the velocity will be not large enough to make the block rebound higher. Therefore, the block is more difficult to rebound with larger spring stiffness.

When the spring stiffness keeps unchanged, *m* increases with the increasing of the elastic modulus**,** which proves that DDA meets the physical laws. Suppose the elastic modulus of block and plane is very large, it belongs to completely inelastic collision. The block can't rebound and the deformation can't be restored after impact, so the energy loses most seriously. Suppose the elastic modulus of block and plane is very small, it belongs to elastic collision, and the deformation can be restored after impact, so there is no energy loss. The main function of elastic modulus is to decide whether the strain energy can transform back to the kinetic energy or the gravitational potential energy. Choosing different elastic modulus can decide different type of collision. Many scholars suggested that the spring stiffness should be 20*E* ~ 100*E*, which is based on the materials whose elastic modulus is similar to the rock. However, in some materials whose elastic modulus are obviously different from rock, the multiple relationship between elastic modulus and spring stiffness doesn't exist. One of the aims of this paper is to provide a theoretical basis to extend the scope of research materials other than the rock in DDA.

In conclusion, the energy dissipation in DDA is a very complicated process, for it depends on multiple parameters, such as the time step, elastic modulus, spring stiffness, impact force and so on.

The adjustment strategy discussion of the major control parameters

(1) The time step $\triangle t$ is of great importance: From the point of computational accuracy, small time step is better, because 3-D DDA uses time integration scheme. The time step should be small enough that the two order infinitesimal displacement $\frac{(\Delta t)^2}{2}\{\ddot{\mathbf{u}}\}_{n+1}$ in the Eq. (8) can be ignored, which makes the calculation more stable. From the point of computational convergence, time step should be large enough that there is enough mutual penetration depth between the block system to develop the contact force, which makes the block system convergence quickly. From the point of the formula computation, the time step can't be too small. Because small time step will decrease the computational efficiency. Furthermore, the displacement in each step will be too small and the spring stiffness will be too large, also resulting to the illness of Eq. (3).

In addition, time step is more sensible when the acceleration changes with time. In the sliding model and the open-close iteration model, the normal direction acceleration of block changes by small degrees, where time step 0.001s is recommended for enough computational accuracy and quick convergence speed.

While in the rebound model, the acceleration of block changes seriously with time. If the time step is not small enough, there will be large error of displacement in the same period of time for low computational accuracy. Time varying acceleration and low computational accuracy may make the 3-D DDA block system look like has energy dissipation. The energy dissipation is a very complicated process. In the real situation, it involves a variety of factors, such as air resistance, thermal energy consumption and so on. But now 3-D DDA can't simulate these factors. According to the previous models, the energy dissipation is related with the time step. Therefore, it is possible to adjust the value of time step to simulate similar kinetic results with these energy dissipation reasons in the future. If there is large energy dissipation in the real situation, the time step can be increased to reach the similar result.

(2) Because 3-D DDA uses penalty method, the spring stiffness p plays an important role to control the reasonable penetration depth and acceleration. From the point of contact, the spring stiffness can't be too small, otherwise penetration depth will be too large, resulting in no contact force in the DDA program, which is not meet the contact requirement of no tension and no penetration in 3-D DDA. If the contact force is vary large, the spring stiffness should be increased corresponding to reduce the penetration depth. From the point of computational validity, spring stiffness can't be too large. Otherwise, $[\mathbf{K}_{ij}]$ may be larger than $[\mathbf{K}_{ij}]$, which makes the matrix computation not convergent. And, the displacement or deformation matrix $\{\mathbf{D}_i\}$ will be very small. When a small error in the $\{\mathbf{D}_i\}$ multiplies a very large spring stiffness, there may be a large error in load matrix $\{\mathbf{F}_i\}$, which can produce errors continuously after using this error in the iterative computation. The range of spring stiffness should follow the above two value rules.

Furthermore, spring stiffness also has a great effect on the acceleration and velocity variation. In the rebound model, large spring stiffness can decrease the contact time or acceleration time, which results in less velocity variation. So it may also make the 3-D DDA block system look like has energy dissipation. If there is large energy dissipation in the real situation, the spring stiffness can be also increased to reach the similar result.

(3) The elastic modulus *E* is also one of the important parameters in 3-D DDA. It is the inherent properties of materials, and it affects the stress-strain relationship and the deformation recovery ability of the block. Especially, in the rebound model, it decides the collision type between the blocks and the inter-transformation between strain energy and kinetic energy or gravitational potential energy.

CONCLUSIONS

This paper uses 3-D DDA method to analyze the effects of different control parameters on the kinetic characteristic of block in the sliding model, open-close iteration model and rebound model. It can be concluded that:

(1) Both too large and too small time step may increase the error. And the effect of time step on the computational accuracy is more obvious when the acceleration changes with time. In common, time step 0.001s is recommended because of its enough computational accuracy and fast convergence speed.

(2) The spring stiffness *p* plays an important role to control the reasonable penetration depth, which has a great effect on the acceleration time and the velocity variation. And the spring stiffness *p* is not related with the elastic modulus *E*, its range should be adjusted according to the penetration depth and ensure the computational stability of stiffness matrix.

(3) The energy dissipation is related with the time step and the spring force. If there is large energy dissipation in the real situation, the time step and the spring stiffness can be increased properly to simulate the similar result.

(4) In 3-D DDA, the characteristic of elastic modulus *E* is in good agreement with the reality. The study material should not only limited in the rocklike material and it can be extended to other materials, such as rubber.

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