A Calculation Model for the Analysis of Soil Arching Between Piles Based on *Ito* Plastic Theory

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ABSTRACT

Considering the influence of synergistic effect between friction soil arching and end-bearing soil arching, a calculation model of interaction between landslide and square stabilizing piles is established based on Ito. plastic theory. Firstly, the concept of load sharing ratio of friction soil arching between the piles is proposed under the given stress conditions. Then the calculating formula is derived by using the equilibrium conditions of soil arching system and stress boundary conditions. Secondly, the calculated process of piles spacing, considering the soil arching effect, is illustrated through a concrete engineering example. And a reasonable result is obtained. Then the relationships between piles spacing, friction angle and cohesion of the soil arching are analyzed. The result shows that, friction soil arching is the mainly bearing arching, the piles spacing would make a linear increase and the load sharing ratio of friction soil arching would reduce linearly along with the increment of soil cohesion; piles spacing would present an upward concave curve and the load sharing ratio of friction soil arching would increase linearly along with the increment of inner friction angle.

KEYWORDS: Ito plastic theory; stabilizing pile; soil arching; pile spacing

INTRODUCTION

Stabilizing piles are widely used around the world as means to recover collapsed slopes or secure slope stability (Song et al., 2012; Salem et al., 2012; Lai et al., 2015). These piles are usually embedded through the slip zone into slip bed so as to stabilize slopes by offering anti-slide force and anchor function from the stable stratum (Hu et al., 2005). With consideration of the complexity of the interaction of slopes and stabilizing piles, many scholars conducted research into the mechanism of load transfer between pile and soil and established the calculating models of pile spacing.

Soil arching is caused by the inhomogeneous displacement of the loose debris in slip body. The formation of soil arching changes the strain condition and leads to the rearrangement of strain. The stress on the back and front of the arching then loads on the sides of arching as well as the surrounding soil (Jia et al., 2003). In addition to material condition, the development of soil arching also requires the structure condition. The structure in contribution to the soil arching refers to the fixed arching foot for supporting the horizontal thrust and the arching ring formed by material with certain strength. In the arching plane, the stress in the soil is loading on the arching ring. The sliding thrust of landslide is delivered to the piles on both sides through the soil shown in Figure 1.

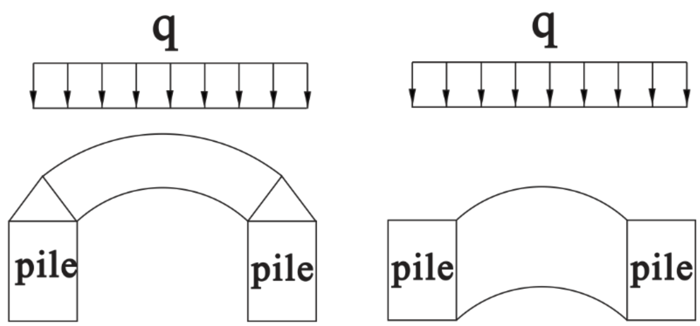

Figure 1: Sketch of the soil arching between piles

Currently, the theoretical study of soil arching focus on two aspects. On one aspect, the study of soil arching is based on the external force balance condition of the soil between piles. Pan (1980) established the upper limit equation of pile spacing on consideration of the friction resistance between piles while Wang (1997) proposed the lower limit equation. Wang (2001) studied the soil arching mechanical characteristics and he arching foot between piles were provided by the friction between piles and soil arching, which means the ignorance of the effect of the supporting arching. On the other aspect, the study of soil arching is based on the mechanical properties and strength condition of soil arching behind the piles. The arching foot is provided by the piles and a calculation model of the pile distance based on soil arching is established with only the consideration of the supporting arching behind piles (Chang, 1998; Zhou and Xiao, 2004).

On the whole, the idea on studying the soil arching lay particular emphasis on the friction between piles but ignored the effects of breadth of pile section. Or it would focus more on the supporting arching without consideration of height of pile section. When the stabilizing piles are enduring the sliding thrust, the interaction between soil and piles happens all round the pile section, indicating that on studying the transmission of load, both the effects of soil between piles and soil behind piles should be taken into consideration.

Here, both the friction arching between piles and the supporting arching behind piles are considered comprehensively to study the interaction of square pile and the slip body based on Ito plastic theory. A more comprehensive calculation equation is proposed on the consideration of soil arching.

ITO PLASTIC THEORY

Ito (1975; 1981) proposed a plastic theory based on plastic deformation of soil layers, which divided the soil arching into two parts (AA’EE’ and BB’EE’ shown in Figure 2).

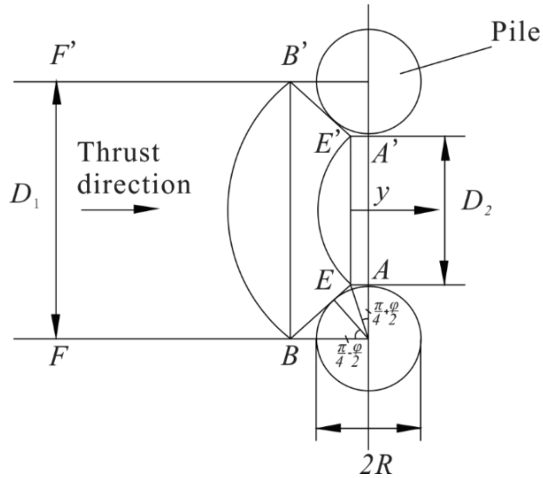

Figure 2: State of plastic deformation in theground just around piles

Equilibrium analysis is conducted aiming at the two deformation areas AA'EE' and BB'EE'. First, the interaction system of friction arching is analyzed. Based on the differential equations of horizontal principal stress and distance along with the assumed boundary condition of soil arching between piles, the relationship of horizontal stress and distance is obtained without consideration of the soil stress in front of the piles. Then, the supporting arching behind piles is analyzed based on the continuity of the boundary stress of the arching. The horizontal lateral force of unit-thickness layers of soil on piles is concluded and the relation formula of horizontal stress and distance is shown as follows:

$$p(z)=cD_1\left(\frac{D_1}{D_2}\right)^{G_1}\left[\frac{1}{N_\varphi\tan\varphi}\left\{\exp\left(\frac{D_1-D_2}{D_2}G_3\right)-2N_\varphi^{\frac{1}{2}}\tan\varphi-1\right\}+\frac{G_2}{G_1}\right]-c\left(D_1\frac{G_2}{G_1}-2D_2N_\varphi^{-\frac{1}{2}}\right) \\ +\frac{\gamma z}{N_\varphi}\left\{D_1\left(\frac{D_1}{D_2}\right)^{G_1}\exp\left(\frac{D_1-D_2}{D_2}G_3\right)-D_2\right\} \quad (1)$$

where D_1 and D_2 are the distance to the center of piles and the net distance between the piles.

In equation (1):

$G_1=N_\varphi^{\frac{1}{2}}\tan\varphi+N_\varphi^{-1}$, $G_2=2\tan\varphi+2N_\varphi^{\frac{1}{2}}+N_\varphi^{-\frac{1}{2}}$, $G_3=N_\varphi\tan\varphi\tan\left(\frac{\pi}{8}+\frac{\varphi}{4}\right)$ and

$N_\varphi=\tan^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)$

When saturated cohesive soil is experiencing the undrain shear failure, $\varphi=0$. The lateral force equation can further be simplified as follow:

$$p(z)=c\left\{D_1\left(3\log\frac{D_1}{D_2}+\frac{D_1-D_2}{D_2}\tan\frac{\pi}{8}\right)-2\left(D_1-D_2\right)\right\}+\gamma z\left(D_1-D_2\right) \quad (2)$$

The value of lateral force can be obtained by computing the integral for slip body depth with Equation (1) and (2). Then, the stress analysis could be carried out aiming at different deformation areas and studying the internal force of piles and the interaction mechanism between pile and soil.

THE CALCULATION MODEL OF SOIL ARCHING

Assumptions

The calculation model is based on three assumptions:

(1) Assume that the back of the soil arching between piles is under uniform distributed load and only consider the interaction of piles and unit-thickness soil layer. In this way, the complex three dimensional problem is simplified into a two dimensional one.

(2) Assume that in the transmission process of lateral force, there is no loss and most or all the horizontal stress turn into the lateral force on piles (Wang and Chen, 2001).

(3) Assume that the friction arching and supporting arching bear the load together and the friction arching sustains k times the total thrust load, this k is noted as the ratio of friction arching in the following analysis.

Analysis of soil arching

According to Ito plastic theory, when the soil arching is under critical state, the friction arching between piles and supporting arching behind piles undertake the landslide thrust together. In the interaction model between landslide and square piles, friction arching formed between the piles and soil behind piles is under the effect of thrust and end bearing arching formed then. The soil arching state around piles is shown in Figure 3. The AA’EE’ stands for friction arching while BB’EE’ represents the supporting arching.

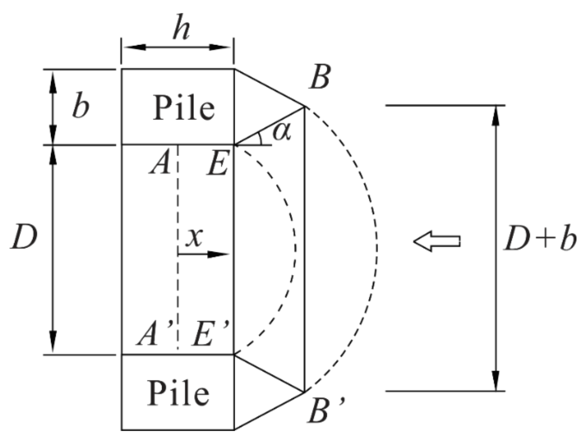

Figure 3: State of soil arching in the ground just around piles

On analysis of the friction arching between piles, according to the mechanic characteristics of friction arching, a coordinate system is established at the start of the friction arching AA’ and the AA’ is set as the zero of horizontal axis, as shown in Figure 4.

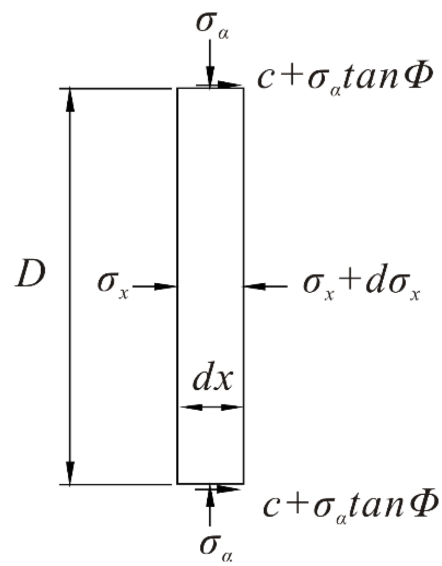

Figure 4: Small element of plastically deforming ground (AEE'A')

Considering the balance of horizontal force system, an Equation is drawn as follow:

$$D\cdot\sigma_x + 2(c+\sigma_\alpha \tan\phi)\cdot dx = D\cdot(\sigma_x + d\sigma_x) \quad (3)$$

where D stands for the distance between piles while σ_x represents horizontal principal stress and σ_α means the vertical lateral force of piles; c refers to the cohesive force of pile and soil while ϕ is the internal friction angle of pile and soil. For the ease of study, assume that c and ϕ equals to the cohesion of soil and the internal friction angle of soil, respectively.

According to assumption (2), an equation is concluded:

$$\sigma_\alpha = \sigma_x$$

Equation (3) can be simplified as follow:

$$2(c+\sigma_x \tan\varphi)\cdot dx = D\cdot d\sigma_x \quad (4)$$

According to Equation (4), a differential equation of horizontal stress and horizontal distance can be drawn:

$$\frac{d\sigma_x}{dx} - 2\frac{\sigma_x}{D}\tan\varphi = \frac{2c}{D} \quad (5)$$

and the solution is as follow:

$$\sigma_x = C_1\cdot e^{\frac{2\tan\varphi}{D}x} - \frac{c}{\tan\varphi} \quad (6)$$

According to boundary condition, when the shear strength of soil in passive area is low and the dip angle of slip surface is more than 5°, there is almost no passive pressure. However, when the shear strength is high and the dip angle is higher than 5°, the passive pressure exists. The allowance of the sliding deformation of soil arching compression is small, the passive soil stress that can be used as supporting force is smaller. Only when the soil arching is under the critical state, the passive soil stress can be taken full advantage of. Therefore, in the actual calculating, the passive stress can be ignored. That is to say, according to the coordinate that mentioned above:

$$\sigma_x\big|_{x=0} = 0$$

Take his Equation into Equation (6), we can conclude that:

$$C_1 = \frac{c}{\tan\varphi}$$

From which the relationship between horizontal stress and distance is displayed as follow:

$$\sigma_x = \frac{c}{\tan\varphi}\cdot(e^{\frac{2\tan\varphi}{D}\cdot x} - 1) \quad (7)$$

According to the boundary condition of friction arching, the friction arching is under uniformly distributed load $k\cdot p$ in unit-thickness soil, expressed in the Equation below:

$$\sigma_x\big|_{x=h/2} = k\cdot p$$

taken which into Equation (7) we can draw the formula:

$$\frac{c}{\tan\varphi}\cdot(e^{\frac{2\tan\varphi}{D}\cdot\frac{h}{2}} - 1) = kp \quad (8)$$

after simplification

$$D = \frac{h\tan\varphi}{\ln(\frac{kp\tan\varphi}{c}+1)} \quad (9)$$

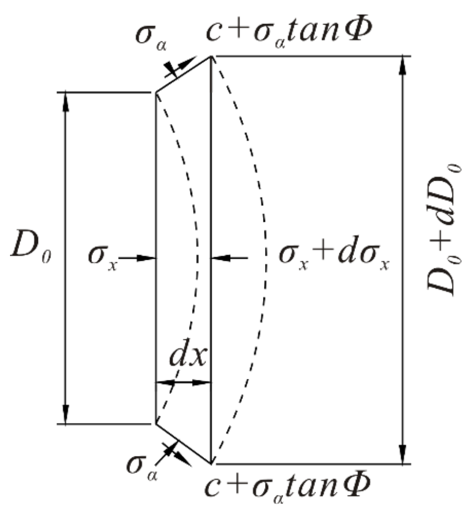

Figure 5: Small element of plastically deforming ground (EBB'E')

To analyze the stress of the supporting arching as shown in Figure 5, we can conclude the equation as follow in consideration of the balance of horizontal force system:

$$-D_0 d\sigma_x - \sigma_x dD_0 + 2dx(\sigma_\sigma\tan\alpha + \sigma_\sigma\tan\phi + c) = 0 \quad (10)$$

where D_0 refers to the distance of the supporting arching while σ_x stands for the horizontal principal stress and σ_α represents the lateral vertical stress of piles. c is the cohesive force between pile and soil and ϕ is the internal friction angle of pile and soil. For the ease of study, assume that c and ϕ equals to the cohesion of soil and the internal friction angle of soil, respectively.

According to assumption (2):

$$\sigma_\alpha = \sigma_x$$

After simplification, the Equation (10) can be transferred into the following formation:

$$-D_0 d\sigma_x - \sigma_x dD_0 + 2dx(\sigma_x \tan\alpha + \sigma_x \tan\varphi + c) = 0 \qquad (11)$$

According to the geometric relationship, an expression can be obtained:

$$D_0 = 2(x - \frac{h}{2})\tan\alpha + D \qquad (12)$$

where $D \le D_0 \le D + b$, $\frac{h}{2} \le x \le \frac{b}{2\tan\alpha} + \frac{h}{2}$.

The equation can be shown as follow after simplification:

$$(2x\tan\varphi - h\tan\varphi + D)d\sigma_x = 2(c + \sigma_x \tan\varphi)dx \qquad (13)$$

Integrate Equation (13) and the Equation (14), it can be concluded:

$$\ln(2\sigma_x \tan\varphi + 2c) = \frac{\tan\varphi \ln(2x\tan\alpha + D - h\tan\alpha)}{\tan\alpha} + C_1 \qquad (14)$$

where C_2 refers to a constant.

According to the boundary condition of the coordinate, $\sigma_x|_{x=h/2} = kp$, we can draw the equation as follow when put the boundary condition into Equation (14):

$$C_2 = \ln(2kp\tan\varphi + 2c) - \frac{\tan\varphi}{\tan\alpha}\ln D$$

After simplification, the relationship of horizontal principal stress and distance is shown as follow:

$$\sigma_x = \frac{(\frac{2x\tan\alpha - h\tan\varphi}{D} + 1)^{\frac{\tan\varphi}{\tan\alpha}} \cdot (kp\tan\varphi + c) - c}{\tan\varphi} \qquad (15)$$

On consideration of friction arching and the supporting arching, the expression of the principal stress along the horizontal thrust direction is shown as follows:

$$\begin{cases} \sigma_x = \frac{c}{\tan\varphi} \cdot (e^{\frac{2\tan\varphi}{D} \cdot x} - 1), 0 \le x \le \frac{h}{2} \\ \sigma_x = \frac{(\frac{2x\tan\alpha - h\tan\varphi}{D} + 1)^{\frac{\tan\varphi}{\tan\alpha}} \cdot (kp\tan\varphi + c) - c}{\tan\varphi}, \quad \frac{h}{2} \le x \le b / 2\tan\alpha \end{cases} \qquad (16)$$

According to the boundary condition of supporting arching, the uniformly distributed load p in unit-thickness soil acts on the back of piles can be expressed as follow:

$$\sigma_x\big|_{x=h/2+b/2\tan\alpha} = p$$

Put the expression into Equation (15) and after simplification, an equation can be drawn:

$$p\tan\varphi + c = (kp\tan\varphi + c)(\frac{b}{D}+1)^{\frac{\tan\varphi}{\tan\alpha}} \tag{17}$$

When it comes to a case study, the landslide thrust p, the strength parameter c and φ, the size of pile b and h are all constants. The only two unknowns are the ratio of friction arching k and the distance between piles D. Based on Equations (9) and (17), the two unknowns can be solved. When only the anti-sliding force of friction arching between piles is considered, the ratio of friction arching k =1. Therefore, based on Equation (9), the calculation formula of pile distance can be obtained as follow:

$$D = \frac{h\tan\varphi}{\ln(\frac{p\tan\varphi}{c}+1)} \tag{18}$$

CASE STUDY

Ziyang landslide is located at Ziyang County, Ankang city, Shaanxi province, China. The landslide develops at the north bank with a dip angle towards Han River of 20° (Zhao et al., 2010). The landslide is divided into two parts, the east part and the west part. The east and west parts obtain the width at the east to west direction of 90m and 100m, respectively and the length along the south to north direction is 100m and 130m, respectively. The slip body is formed with the debris materials composed of clay and silty clay with gravel and the slip bed is composed of phyllite. According to investigation information, a slip surface has developed inside of the upper slip body, which is composed of the loose material formed in Quanternary, while the lower slip body would slip along the bed rock. The cohesive force and internal friction angle of the slip surface on the bed rock are 130kPa and 15°, respectively.

According to residual thrust method, the landslide thrust near piles is 1076.6 kN/m. The pile section is of the length of 3m and width of 2m and the length of the pile sustaining the load is 10.5 m. Usually, the thrust would considered to be uniformly distributed along the pile, making the thrust force value that act on unit-thickness soil arching p =1076.6/10.5 =102.5 kPa.

Based on the equations above, the ratio of friction arching is 0.67 while the distance between piles is 6.04m. In the real case calculation, the distance would be 6 m for the safety and ease of calculation. The distance between the center of the piles is 8m.

The analysis of parameters affecting the pile spacing

It is noted that the difference distance between pile centers, marked as S, and the distance between piles, marked as D, is only a constant value of the width of piles. To avoid repeated analysis, only D, the distance between piles is taken into consideration for further discussion.

According to Equation (9), the distance between piles is related to cohesion and friction angle of soil, the load behind piles and the ratio of friction arching. The function is shown as follow:

$$D = f(h, p_0, c, \varphi, k) \tag{19}$$

For the stabilizing of landslides, when the optimal position for the stabilizing piles is chosen, the load behind piles, noted as p_0, is a constant in analysis, which make the Equation (19) into a simpler one:

$$D = f(h, c, \varphi, k) \tag{20}$$

From the expression, it indicates that the distance between piles is directly proportion to the width of piles and the curve would go through the origin point. Therefore, the relationship of distance between piles and width of piles is out of consideration. The ratio of friction arching correlates with shear strength parameters and the discussion would focus on the relationship between the distance between piles and the cohesion and friction angle of slip body.

(1) Relationship between D and c

From Figure 6 we can conclude that when the friction angle of slip body is 15°, the distance between piles increases with the increase of cohesion, and basically showing linear change. In the same time, the ratio of friction arching shows a linear relation with cohesion and decrease with the increment of the value of cohesion. This is in consistence with the real force condition of the soil arching. When the cohesion of soil between piles is strong, the pile is able to provide large friction, leading to the increment of the stabilization ability of the friction arching. When cohesion increase by 20kPa, the distance between piles would increase by 0.9m. The ratio of friction angle is always larger than 0.5, indicating that the friction arching is the main part in bearing the landslide thrust when considering the soil arching comprehensively.

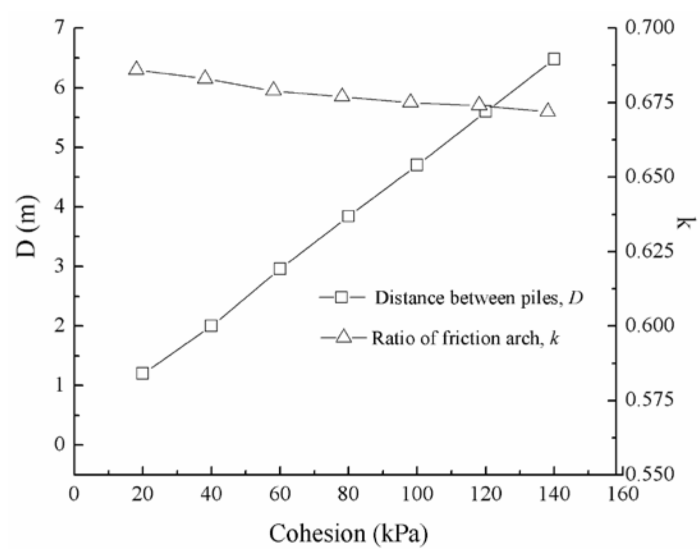

Figure 6: Relationship between *D* and *c*

(2) Relationship between D and φ

In Figure 7, the cohesion of the slip body is set to be 130kPa. The distance between piles would increase with the growth of internal friction angle and the increment rate would increase along. The ratio of friction arching increase linearly with the internal friction angle. The ratio of friction angle is always larger than 0.5, indicating that the friction arching is the main part in bearing the landslide thrust when considering the soil arching comprehensively.

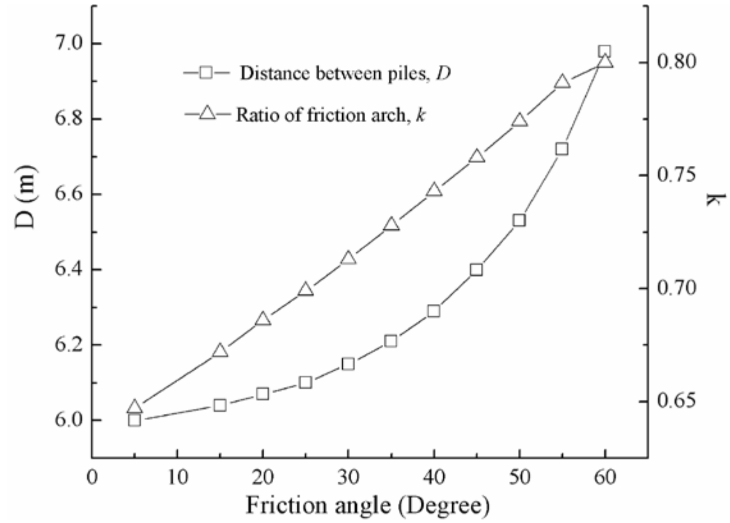

Figure 7: Relationship between *D* and φ

(3) Relationship between D and c , φ

From Figure 8 we can see that when the value range of cohesion and the friction angle of the soil are 10° to 60° and 20kPa to 140kPa, respectively, the distance range between piles is from 1.08m to 7.34m, which is quite close to the empirical value range 5m to 8m (Zhao, 2003). The contour line of the distance between piles is close to horizontal line, indicating that when the cohesion is set as a fixed value, the distance between piles would not change so much with the change of friction angle. On the contrary, the distance between piles is sensitive to cohesion when a fixed value of friction angle is set. On conclusion, the distance between piles is mainly controlled by the cohesion value.

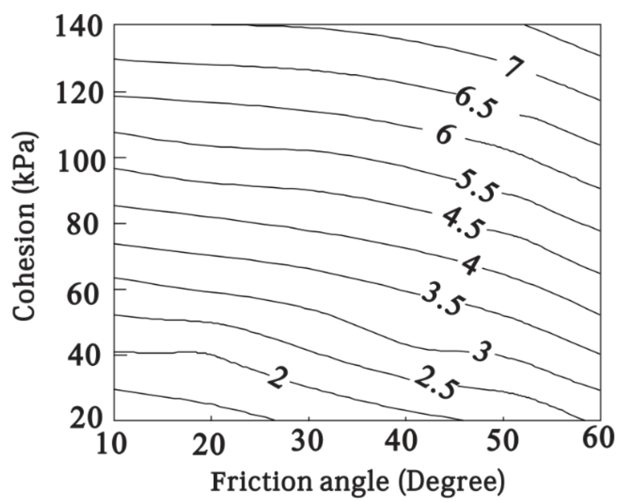

Figure 8: Relationship between *D* and c， φ

Comparison of different calculation method

Based on the same landslide, different scholar adopted different calculating method in obtaining the distance between piles and the results are listed in Table 1 (Zhou and Xiao, 2004; Hu, 2000; Zhao et al., 2010; Wang and Chen, 2001; Hu and Wang, 2007; Zheng et al., 2005).

Table 1: Comparison between several methods of calculation

Calculating method	Zhou method	Hu method	Zhao method	Wang method	Xiaojun method	Zheng method	Method in this paper
Calculating method	2	4.1	5.6	6.9	8.6	9.7	6.04

From the results we can see that the distance between piles varies much from each calculating method. In Zhou method, it mainly concentrates on the supporting arching without consideration of friction arching and the results is too small. However, the Wang method just consider the effect of friction arching, leading to a large distance in results. In this paper, the employment of Ito method makes it possible to consider both friction arching and supporting arching. From the results, the value is smaller than the result from Wang method and bigger than that from Zhou method, and it is more close to real condition.

CONCLUSIONS

(1) A formula for calculating the soil arching is conducted based on Ito plastic theory. The formula takes both the friction arching and supporting arching into consideration. The calculating results is the combination of the consideration of the main effect of bearing the thrust load of friction arching and the sharing effect of the supporting arching, making the results close to real condition.

(2) The soil arching is affected much by distance between piles. The critical distance between piles would increase nearly linearly with the increase of cohesion while the ratio of friction arching would decrease slightly with the increase of cohesion. When the friction angle of soil is increasing, the distance between piles would increase and the increment rate is increasing as well. In the same time, the friction arching is increasing linearly with the growth of friction angle.

(3) The calculation in this paper is based on the assumption that the load is uniformly distributed along pile while the load is actually turning smaller along the pile, which is a three-dimensional problem. More real case study should be made for further revise the calculating results in the near future.

REFERENCES

1. Song Y S, Hong W P, Woo K S. Behavior and analysis of stabilizing piles installed in a cut slope during heavy rainfall[J]. *Engineering Geology*, 2012, 129: 56-67.
2. Salem T N, Mashhour M, Hassan R. Stabilizing Piles of Soft Cohesive Slopes: A Case History[J]. *Electronical Journal of Geotechnical Engineering* (EGJE), 2012, 17: 3803-3820.
3. Lai J, Zheng Y, Liu Y, et al. Seismic design of multiple anti-slide piles by strength reduction dynamics analysis method[J]. *Electronical Journal of Geotechnical Engineering*, 2015, 20(5): 1725.
4. Hu X. L., Wang L. Q., Tang H. M.. Several Key Problems for Optimum Design on Anti-slide Pile of Bank Landslide in Three Gorges Reservoir Region[J]. *Geological Science and Technology Information*, 2005, 24(Z1): 121-125.
5. Jia H. L., Wang C. H., Li J. H.. Discussion on Some Issues in Theory of Soil Arch[J]. Journal of Southwest Jiaotong University, 2003, 38(4): 398-402.
6. Pan J. Z.. The stabilization and stability analysis of structure[M]. Beijing, Water

Resources and Electric Power Press, 1980, 6.

7. Wang S. C.. Anti-slide pile spacing of the lower limit equation[J]. Industrial Construction, 1997, 27(10):32-36. (in Chinese)
8. Wang C. H., Chen Y. B.. Soil arch mechanical character and suitable space between one another anti-sliding pile[J]. *Journal of Mountain Science*, 2001, 19(6):556-559.
9. Chang B. P.. A study on soil arch between stabilizing piles and critical span[C]//Proceeding of landslide (Volume thirteen). Beijing: China Railway Press, 1998, 73-78.
10. Zhou D. P., Xiao S. G.. Discussion on rational spacing between adjacent anti-slide piles in some cutting slope projects[J]. *Chinese Journal of Geotechnical Engineering*, 2004, 26(1): 132-135.
11. Ito T, Tamossum. Methods to estimate lateral force acting on stabilizing piles[J]. Soils and Foundation, 1975, 15(4): 43-59.
12. Ito T, Matsui T, Hong PW. Design method for stabilizing piles against landslide-one row of piles. *Soils and Foundations* 1981, 21(1):21–37.
13. Zhao M. H., Liao B. B., Liu S. S.. Calculation of anti-slide piles spacing based on soil arching effect[J]. *Rock and Soil Mechanics*, 2010, 31(4): 1211-1216.
14. Zhao M. J.. Slope engineering treatment technology[M]. Beijing： China Communications Press. 2003, 10.
15. Hu M. Y.. Estimation of the spacing between retaining piles for excavation[J]. China *Journal of Highway and Transport*, 2000, 14(2): 27-29.
16. Hu X. J., Wang J. G.. Determination of anti-slide pile spacing in side slope consolidation projects[J]. Journal of Hehai University (Natural Sciences), 2007, 35(3): 330-333
17. Zheng L., Yin K. L., Jian W. X.. Analysis on spacing between piles in the design of anti-slide pile[J]. *Hydrogeology and Engineering Geology*, 2005, 32(6): 71-74.

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