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Green's Function Method for Nonlinear Porous Flow and Its Application to Unconventional Reservoirs

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ABSTRACT

Based on the three-parameter nonlinear continuous model, this paper has introduced the concepts of pseudopressure gradient and pseudoflow, presented the pseudotime normalization calculation method, and established Green's source function model; with the adoption of moving boundary method, the rules of nonlinear seepage flows and production decline in staged fracturing horizontal wells in infinite strata have been studied. The outcomes indicate that (1) nonlinear impact can be reduced by adopting a small well spacing as the pressure spread range is small, the pressure gradient is large and nonlinear impact is small in the early stage of production; (2) when the number of fractures increased, the speed of outward propagation at the pressure transmitting zone will be increased, a low-pressure area will be formed within the intervals of fractures and the nonlinear impact will increased. Non-linearity should become an indicator of fracture design optimization; (3) the more sufficient the formation energy is, the larger the producing pressure drop is and the closer it is as to linear seepage flows. The formation energy should be added for unconventional reservoirs, with relatively large producing pressure drop. (4) When the nonlinear parameters are gradually reduced, the nonlinear seepage flows are gradually turned into linear seepage flows; (5) linear seepage flow is a special case of the nonlinear seepage flow model proposed in this paper.

KEYWORDS: Pseudoflow, Normalized pseudotime, Non-linear Green's source function, Moving boundary method, Staged fracturing horizontal well

INTRODUCTION

The point source function was originated from the heat transfer theory in the last half of the 19th century (Lord Kelvin 1880). Gringarten and Ramey (1973) made pioneering contributions to the application of the point source function in seepage mechanics. Kong Xiangyan [6] et al summarized and revised the function systematically. Currently, no analytical theory for the non-linear Green function method is established in the seepage flow domain, and there is little literature on the analytic method for the non-linear Green's source function. Though non-linearity has been involved in many fields such as electromagnetics, hydromechanics and ship engineering [5-8], only approximate approach is applied for solutions. Therefore, further studies should be carried out on the Green's function method for non-linear seepage flows. A new analytic method for Green's source function for non-linear seepage flows is established through the following work in this paper: (1) A straight line source function for non-linear seepage flows is established by introducing pseudopressure gradient, pseudopressure, pseudoflow and pseudotime; (2) As to the new straight line source function, 3D (including 2D) source functions are constructed according to the Newman multiplication method; (3) Based on the definitional equation of the pseudotime described in the paper, normalized pseudotime is derived according to the vectorial addition rule for velocity vectors; (4) The Green's source function for non-linear seepage flows based on pseudoflow and normalized pseudotime is established.

As a calculation example, this paper analyzes the factors affecting the law of production decline in staged fracturing horizontal wells with infinite formation using the non-linear Green's source function method.

THEORY AND METHOD

Non-linear Straight Line Source Function Analytic Method

If proper initial and boundary conditions for three-dimensional problems can be obtained, by using the product of the initial and boundary conditions of three one-dimension seepage flow problems, the Newman's product method is then appropriate [1,6]. Based on the stable successive displacement method, the three directions are regarded as stable straight line sources respectively within each time step. The corresponding non-linear flow equation [3] is as follows:

$$v_l = -\frac{k_l}{\mu}\frac{dp}{dl}\left(1-\frac{1}{a+b\cdot|dp/dl|}\right)(l=x,y,z) \tag{1}$$

And, $a=\frac{\lambda_c-\lambda_a}{\lambda_c}, b=\frac{1}{\lambda_c}$

Where, v_l is seepage velocity, m/s; $\frac{dp}{dl}$ is a directional pressure gradient of l, Pa/m; λ_c is an average starting pressure gradient, Pa/m; λ_a is a starting pressure gradient, Pa/m. λ_a and λ_c have been formulated into a plate [3].

The non-linear straight line source function is expressed as：

$$\frac{\partial}{\partial l}\left(\frac{dp}{dl}\left(1-\frac{1}{a+bdp/dl}\right)\right)=\frac{1}{\eta_l}\frac{\partial p}{\partial t} \tag{2}$$

Where, η_l is a pressure transmitting coefficient, m^2/s. Based on the stable successive displacement method, a pseudopressure gradient is introduced, and

$$\frac{\partial}{\partial l}\left(\frac{dp}{dl}\left(1-\frac{1}{a+bdp/dl}\right)\right)=\frac{1}{\eta_l}\frac{\partial p}{\partial t} \tag{3}$$

In the integral domain with the affecting the boundary L:

$$\frac{dp^*}{dl}=\frac{p_e^*-p_f}{L} \tag{4}$$

Where, p^* is a pseudopressure, Pa ; p_f is a pressure on a side of a fracture, Pa; L is a linear source model affecting the boundary, m, Which is constantly changing for a time-variant seepage. According to (3) and (4), the following is obtained:

$$p_e=p_B+\int_0^L \frac{a-1-b\frac{p_e^*-p_B}{L}+\sqrt{(a-1-b\frac{p_e^*-p_B}{L})^2+4ab\frac{p_e^*-p_B}{L}}}{2b}dL \tag{5}$$

For a linear source model along l direction, comparing the non-linear seepage flow to the linear seepage flow, a pseudotime variation function is introduced:

$$\varepsilon_l(t)=\frac{q_{nl}(t)}{q_{Ll}(t)}=\frac{p_e^*-p_B}{p_e-p_B} \tag{6}$$

Where, $\varepsilon_l(t)$ is pseudotime variation function, dimensionless; p_e^* is a pseudopressure at the moving boundary L of the linear source model, Pa, which is determined by Formula (5); q_{nl} is a non-linear flow at time t, m^3/s, which is determined according to the Darcy's equation.

A pseudotime method is introduced to make the corresponding cumulative volume of the fluid unchanged before and after transformation, namely:

$$\Delta V=q_{nl}dt=q_{Ll}dt_l^* \tag{7}$$

ΔV is the cumulative volume, m^3; t_l^* is the corresponding pseudotime of the linear source model along the direction of l, s. Then, a relationship between the pseudotime infinitesimal element and the actual time infinitesimal element can be established by Formula (6) and (7):

$$\varepsilon_l(t)dt=dt_l^* \tag{8}$$

The new linear source function established using the above method can be expressed as follows:

$$\frac{d^2p}{dl^2}=\frac{1}{\eta_l}\frac{\partial p}{\partial t_l^*} \tag{9}$$

Formula (9) is a new non-linear straight line source function. It can be determined easily using the integral transform method that Newman product conditions[6] can be applied to Formula (9).

Pseudotime Normalization Method

The pseudotime along the direction of l is established according to Formula (9). Considering the two-dimensional or three-dimensional seepage flows, the pseudotime needs to be normalized. For anisotropic homogeneous formation, a proper coordinate system should be selected so that the direction of the coordinate axes can be consistent with the direction of the major permeability. After transformation, the following formula [11] can be obtained:

$$\vec{k} = \begin{bmatrix} k_{xx} & 0 & 0 \\ 0 & k_{yy} & 0 \\ 0 & 0 & k_{zz} \end{bmatrix} \tag{10}$$

where, k_{xx} , k_{yy} and k_{zz} are three major permeability, m^2 . According to the stable successive displacement method, take $\Delta p_x = \Delta p_y = \Delta p_z = \Delta p$ from the direction of various major permeability to determine the active boundary, namely to establish the boundary corresponding to different time infinitesimal elements. In the process of every steady-state seepage, according to the rule of vectorial addition, the actual flow velocity and pseudo velocity of flow can be expressed as follows:

$$v = -\frac{\Delta p}{\mu L}\sqrt{(k_{xx}\varepsilon_x(t))^2 + (k_{yy}\varepsilon_y(t))^2 + (k_{zz}\varepsilon_z(t))^2} \tag{11}$$

$$v^* = -\frac{\Delta p}{\mu L}\sqrt{k_{xx}{}^2 + k_{yy}{}^2 + k_{zz}{}^2} \tag{12}$$

where, v^* is pseudo velocity of flow, m/s , and it is determined with pseudoflow.

According to the definitional equation of pseudotime (7), a normalized pseudotime expression can be derived by (11) and (12) as follows:

$$\frac{dt^*}{dt} = \frac{\sqrt{(k_{xx}\varepsilon_x(t))^2 + (k_{yy}\varepsilon_y(t))^2 + (k_{zz}\varepsilon_z(t))^2}}{\sqrt{k_{xx}{}^2 + k_{yy}{}^2 + k_{zz}{}^2}} \tag{13}$$

Formula (13) shows that the accumulated volume of the pseudo flow velocity on the normalized pseudotime infinitesimal element equals to the accumulated volume of the actual flow velocity on the actual time infinitesimal element.

Non-linear Flow in Staged Fracturing Horizontal Wells

For infinite reservoirs with top and bottom sealed, the staged fracturing horizontal well exploitation mode should be adopted. As the model shown in Figure 1, the radius and length of a horizontal wellbore are r_w and y_d , respectively. The coordinates position of the well in the reservoir is $(x_{01}, y_{01}, z_{01}) \sim (x_{01}, y_{02}, z_{01})$, namely the horizontal wellbore is run parallel with Y-direction. The reservoir is a homogeneous anisotropic reservoir with a height of h. In the diagram, x, y and z correspond to the major permeability directions of xx, yy and zz, respectively. The values of permeability are k_x, k_y and k_z. Multiple fractures parallel with axis direction x are equally spaced and they penetrate the vertical fractures in the horizontal reservoirs. The width of the fractures parallel with axis direction y is d. The formation fluid flows through the fractures first and then into the horizontal borehole. The fluid is a single phase micro-compressible fluid. In the initial time, the pressure values in everywhere of the reservoirs are the same as p_i, which is a constant [14].

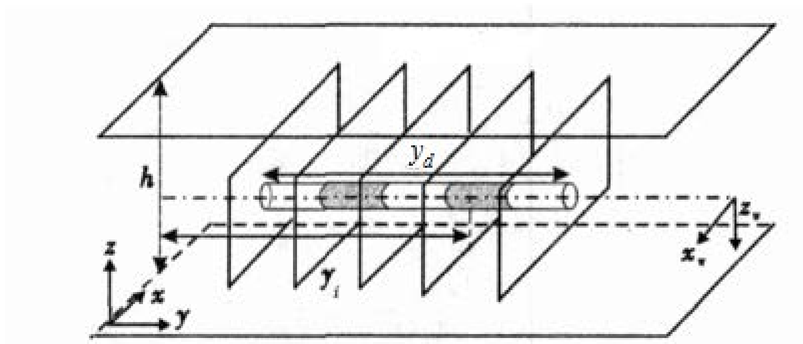

Figure 1: Staged Fracturing Horizontal Well Computation Module (Lian P.Q.2009)

As the derived new nonlinear straight line source function has exactly the same form as which of the existing straight line source function, according to the Newman's product theory, the pressure drop speed equation of the source function model for the 3D non-linear seepage flows established using the normalized pseudotime and pseudoflow can be expressed as:

$$\Delta p_f = p_i - p(x,y,z,t^*) = \frac{\mu}{\alpha x_f hd}\int_0^{t^*} q_f^*(x,y,z,t^*-\tau)S_1S_2S_3 d\tau \tag{14}$$

S_1, S_2 and S_3 are Green's linear source functions for non-linear seepage flows of three directions.

According to reference [1, 12, 13, and 14], the following formula can be obtained:

$$S_1(x,x_0,t^*-\tau) = \frac{1}{2}\left[erf\left[\frac{x_f/2+(x-x_0)}{2\sqrt{k_{xx}(t^*-\tau)/\alpha}}\right]\right] + \frac{1}{2}\left[erf\left[\frac{x_f/2-(x-x_0)}{2\sqrt{k_{xx}(t^*-\tau)/\alpha}}\right]\right] \tag{15}$$

$$S_2(y,y_0,t^*-\tau) = \frac{1}{2}\left[erf\left[\frac{d/2+(y-y_0)}{2\sqrt{k_{yy}(t^*-\tau)/\alpha}}\right]\right] + \frac{1}{2}\left[erf\left[\frac{d/2-(y-y_0)}{2\sqrt{k_{yy}(t^*-\tau)/\alpha}}\right]\right] \tag{16}$$

$$S_3(z,z_0,t^*-\tau) = 1 + \frac{4}{\pi}\sum_{n=1}^{+\infty}\frac{1}{n}\exp\left[-\frac{n^2\pi^2 k_{zz}(t^*-\tau)}{\alpha h^2}\right]\sin\frac{n\pi}{2}\cos\frac{n\pi z_0}{h}\cos\frac{n\pi z}{h} \tag{17}$$

$$\alpha = \phi \mu c_t \tag{18}$$

where, erf is an error function; x_f is fracture length, m; d is fracture width, m. x_0, y_0 and z_0 are spatial location coordinates of the fractures; x, y and z are spatial location coordinates of reservoirs; h is reservoir thickness, m; ϕ is porosity, which is dimensionless; c_t is a system compressibility, Pa^{-1}; q_f is volume flow rate, m^3/s.

Solution

According to the pressure drop rate equation, assuming the flow is constant within the time of Δt^*, and the time t^* is dispersed into $1\Delta t^*$, $2\Delta t^*$, ⋯⋯, and $n\Delta t^*$. As the position coordinates (x_0, y_0, z_0) of the fracture unit is already known, the influence coefficient matrix constituting by the central location of different fracture units in different time can be calculated according to the principle of superposition. The matrix of coefficients can be obtained using the integration within 0~$1\Delta t^*$ of the position coordinates (x_0, y_0, z_0) of different fracture units in different time t^*:

$$\left[A_{1\Delta t^*}\right]\left[A_{2\Delta t^*}\right]\left[A_{3\Delta t^*}\right]\ldots\left[A_{n\Delta t^*}\right] \tag{19}$$

Considering the boundary conditions, a matrix formula as (20) can be formed using the matrix (19) and it is solved as follows:

$$\begin{bmatrix} \left[A_{1\Delta t^*}\right] & & & \\ \left[A_{2\Delta t^*}\right] & \left[A_{1\Delta t^*}\right] & & \\ \left[A_{3\Delta t^*}\right] & \left[A_{2\Delta t^*}\right] & \left[A_{1\Delta t^*}\right] & \\ \cdots\cdots\cdots & & & \\ \left[A_{n\Delta t^*}\right] & \left[A_{n-1\Delta t^*}\right]\ldots & \left[A_{2\Delta t^*}\right] & \left[A_{1\Delta t^*}\right] \end{bmatrix} \begin{bmatrix} \left[Q_{1\Delta t^*}\right] \\ \left[Q_{2\Delta t^*}\right] \\ \left[Q_{3\Delta t^*}\right] \\ \cdots\cdots \\ \left[Q_{n\Delta t^*}\right] \end{bmatrix} = \begin{bmatrix} [b_1] \\ [b_2] \\ [b_3] \\ \cdots\cdots \\ [b_n] \end{bmatrix} \tag{20}$$

If the fractures are considered to have an infinite flow conductivity, various submatrix in the block matrix $b = \left[[b_1]\ [b_2]\ [b_3]\ldots[b_n]\right]^T$ are the same. If the fractures are considered to have limited flow conductivity, the sub matrix needs to be modified gradually in the iterative process. In order not to amend submatrix, where, according to Reference [3, 4], the fractures are equally divided into m sections, the length of fractures of the actual fracture unit can be calculated according to the following formula:

$$\Delta l_c = 0.732\Delta l \tag{21}$$

where $\Delta l = x_f / m$ and m are numbers of elements divided from a fracture. When using such a non-uniform flow model, use Δl_c to replace x_f in Formula (15).

According to Formula (21), it is assumed that the reference points of various fracture units have the same pressure, and the impact of pressure drop on the horizontal wellbore is neglected.

Application

Considering that the vertical permeability is small, the parameters for calculation are selected as follows:

$k_{xx} = 1mD$, $k_{yy} = 3mD$, $k_{zz} = 0.5mD$, $\mu = 1mPa \cdot s$, $h = 10m$, $c_t = 2\times10^{-4} MPa^{-1}$,

$x_f = 80m, d = 0.002m, \phi = 0.12, y_d = 600m$; Where, y_d is the length of the horizontal wellbore. Five schemes are designed to carry out a pilot calculation by selecting different nonlinear parameters as shown in Figure 6. See Table 1 for the nonlinear parameters and the programs.

Table 1 Different Nonlinear Parameter Combination Schemes of Major Permeability Directions

Scheme	λ_{axx} / Pa/m	λ_{cxx} / Pa/m	λ_{ayy} / Pa/m	λ_{cyy} / Pa/m	λ_{azz} / Pa/m	λ_{czz} / Pa/m
1	200	2,000	250	3,000	300	3,500
2	100	1,000	125	1,500	150	1,250
3	20	200	25	300	30	350
4	10	100	12.5	150	15	175
5	4	40	5	60	6	70

In Table 1, λ_{axx}, λ_{ayy} and λ_{azz} are starting pressure gradients along the major permeability direction; λ_{cxx}, λ_{cyy}, λ_{czz} are average starting pressure gradients along the major permeability direction.

As (16), (17) and (18) are two-dimensional space line sources, as well as the three-dimensional space board sources, Production should be considered as the common effect results of every fracture unit in time Δt^*. When there is a non-linear element, the pseudopressure on the boundary along three directions can be determined according to Formula (5). q_f^* in Formula (14) corresponds to the volume density of flow, and it is a flow velocity if converted into a linear density. Where, the inlet area along the direction of xx is $A_x = \Delta l_c \cdot h$; the inlet area along the direction of yy is $A_y = d \cdot h$; and the inlet area along the direction of zz is $A_z = d \cdot \Delta l_c$. According to the concept of pseudoflow, moving boundaries of three board sources can be determined respectively, and thereby the pseudopressure on the outer boundaries of the three board source models can be calculated. The pseudotime transformation function can be calculated according to Formula (6), and then the normalized pseudotime and non-linear flow volume can be calculated. See Figure 2 for the calculations of production changing with time corresponding to different numbers of fractures.

In order to analyze influencing factors of non-linear seepage flows, a relative deviation is introduced:

$$Err = \frac{q_{Ll} - q_{nl}}{q_{Ll}} \times 100\% \tag{22}$$

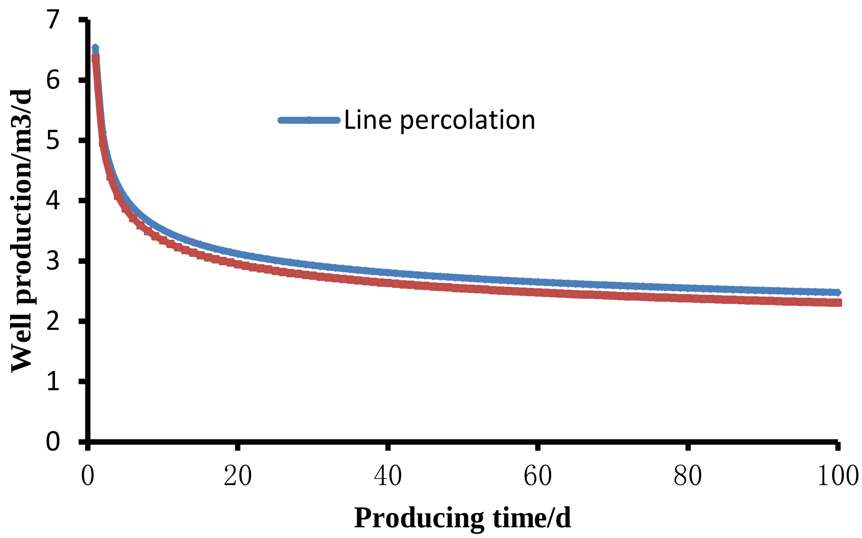

Figure 2: Law of Production Decline (number of fractures n = 1)

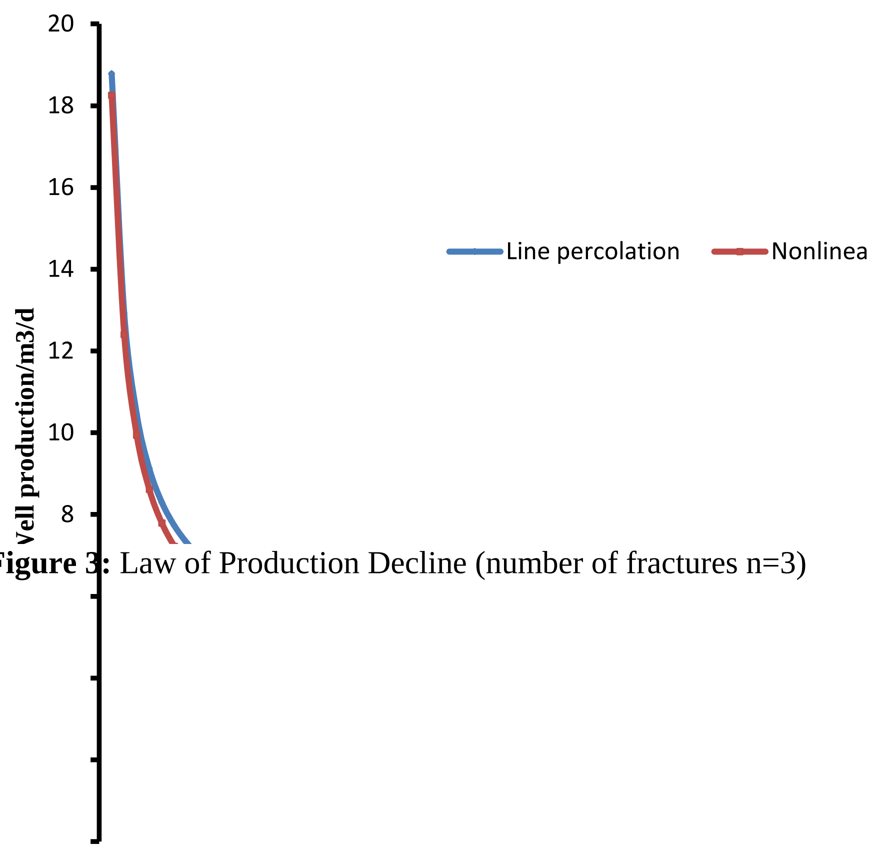

Figure 3: Law of Production Decline (number of fractures n=3)

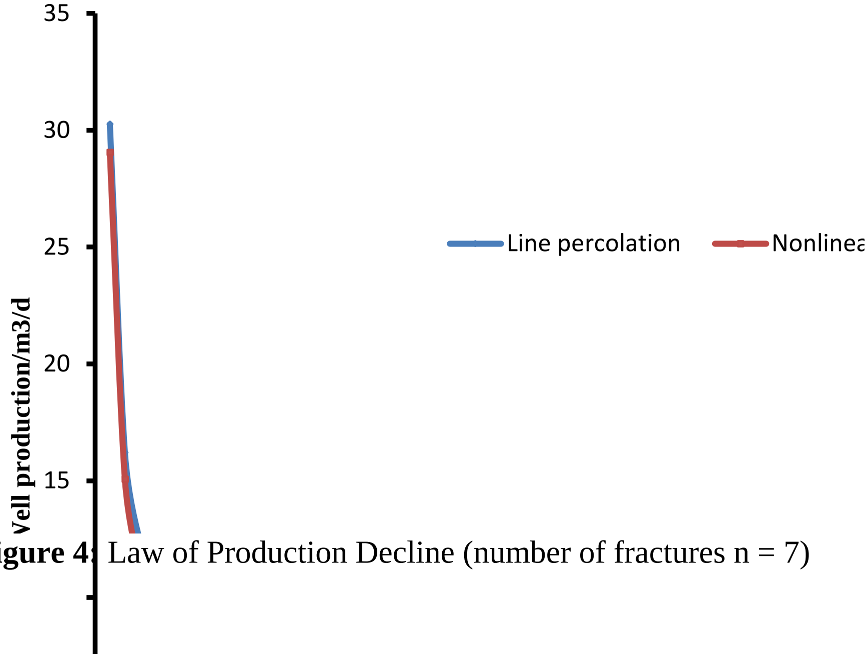

Figure 4: Law of Production Decline (number of fractures n = 7)

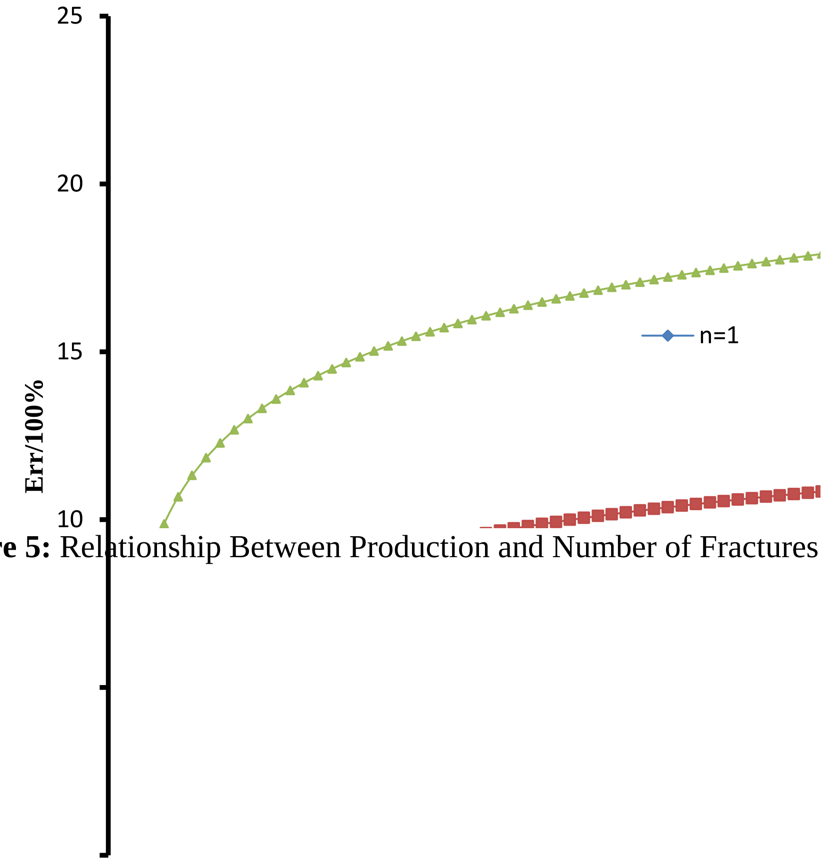

Figure 5: Relationship Between Production and Number of Fractures

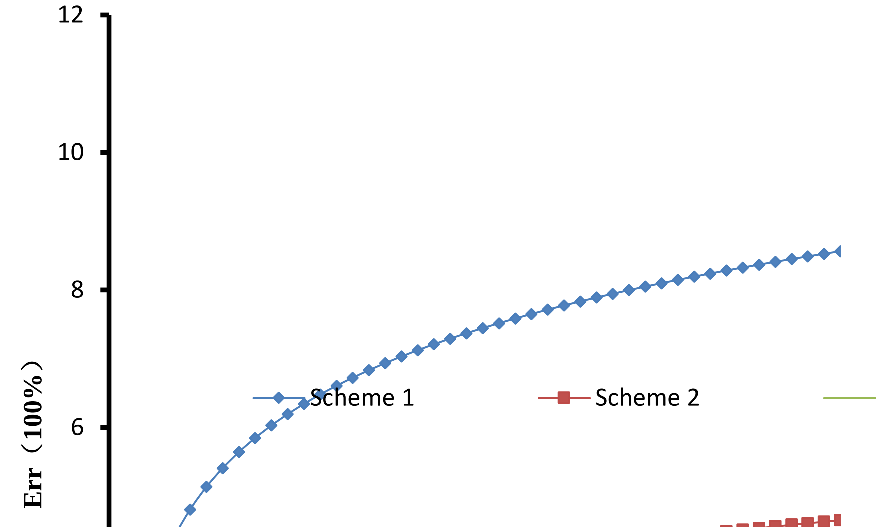

Figure 6: Impact of Nonlinear Parameters on Unsteady-state Production

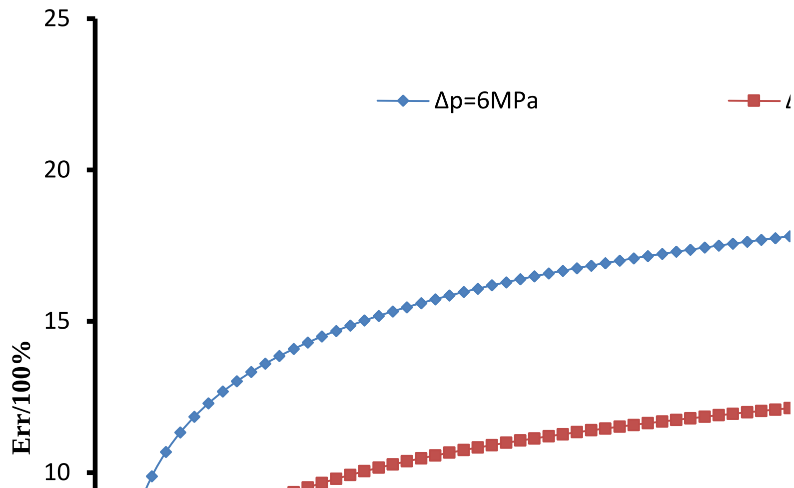

Figure 7: Impact of Producing Pressure Drop on Non-linear Effect

Three kinds of fracture patterns are selected. See Figure 2-Figure 4 for the calculation results of Scheme 1 in Table 1. It is observed that there is only small discrepancy in production in the early stages of linear and non-linear seepage flows. This is because the pressure spread range is small, the pressure gradient is large and the non-linear impact is small in the early stage; when $n=1$, and it is producing for the 100th day, in respect to linear models, the production of non-linear mode is over 6% lower than that of the linear model; when $n=3$,in Figure 3, the 100th day's production of non-linear mode is over 12% lower; when the number of fractures n=7, with the increment of the number of fractures, the impact of non-linearity is increased, and the impact range of non-linearity on long-term production exceeds over 19%. This is because the length of a horizontal wellbore is certain, with the increasing of fracture density, the shielding effect between fractures will also be increased, which results in speeding up of the outward propagation speed of the pressure transmitting zone, and a low pressure area will be formed faster between fractures intervals. Accordingly, the distance passes through by the fluid getting into each fracture unit in the substrate is increased, and the average pressure gradient is reduced, and thereby the impact of nonlinear flows is increased. It also reminds us that the close spacing well pattern should be adopted for developing unconventional permeability reservoirs to reduce the impact of non-linear flows and enhance oil recovery to meet the requirements on production capacity.

It is also found out from Figure 6 that when the nonlinear parameters are calculated according to the five programs listed in Table 1, as the nonlinear parameters are getting smaller, the relative deviation values are getting smaller gradually. When the linear parameters are small enough, the non-linear analytic solution is equal to the linear analytic solution within the range of the required calculation accuracy. This shows that Formula (15) is also suitable for medium & high permeability reservoirs without considering non-linear flows. The medium & high permeability reservoirs can be regarded as one special case of the unconventional reservoir models.

When $n=7$ in Figure 7, the calculation results of scheme 1 are selected from Table 1 with different pressure differences. It indicates from Figure 7 that on the 100th day, when the producing pressure drop is $\Delta p = 6MPa$, $Err = 19.75\%$; When the producing pressure drop is $\Delta p = 10MPa$, $Err = 13.61\%$; when $\Delta p = 15MPa$, $Err = 9.85\%$. Therefore, if formation pressure is increased, keeping a relatively high producing pressure drop can also obviously reduce the impact of non-linearity on production. For on-site applications, formation energy can be supplemented in the early stage and an appropriate pressure difference should be selected for daily production.

CONCLUSION

As to the actual difficulties of constituting a non-linear Green's source function, pseudopressure gradient, pseudoflow and pseudotime are introduced to establish a Green's straight line source function for non-linear seepage flows. The new straight line source function has the same form with that of linear seepage flow source function;

By exporting the normalized pseudotime, a seepage flow model fitting for non-linear flows is established based on the Newman's product theory, and it is calculated and analyzed taking the rule of production decline in staged fracturing horizontal wells in infinite formation as an example;

The analysis shows that fracture spacing has a very clear impact on the production of non-linear flows. The bigger the number of fractures is, the smaller the spacing is and the more obvious shielding effect between fractures is; the longer the production time is, the larger impact the non-linearity has on production. Increasing production pressure drop and keeping a close-spacing well pattern for production can significantly reduce the impact of non-linearity.

The new Green's source function method fitting for non-linear flows proposed in this paper makes the forms of Green's source functions for linear and non-linear seepage flows unified. The calculation results indicate that when the nonlinear parameters are gradually reducing, the non-linear seepage flows are gradually converted to linear seepage flows.

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