Numerical Analysis of the Electro-Osmosis Consolidation for Different Working Conditions in Soft Soil

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ABSTRACT

The paper presents a two-dimensional electro-osmotic consolidation model for numerical analysis. The governing equations coupled with both the electric and seepage fields were solved by using the finite element method software, COMSOL. Then, various cases of different electrode layouts were set for numerical evaluation. The development of the negative excess pore pressure at different locations was discussed, and the influence of electrode layouts on the development of the average degree of consolidation was obtained. The optimum ratios of electro-osmosis permeability and hydraulic permeability were acknowledged with a consideration of the average negative excess pore pressure and average degree of consolidation. The results can provide useful data for in-site electro-osmosis treatment design.

KEYWORDS: Electro-osmosis consolidation; Electrode layouts; Numerical analysis; Negative excess pore pressure; Average degree of consolidation

INTRODUCTION

When an electric field is applied to the soft soil, a water flow is produced, and water will migrate from the anode to the cathode. Generally, a direct current (DC) can be used for soil foundation treatment. During electro-osmosis treatment, the pore water is discharged from soil with an improved shear strength. This phenomenon was first observed by the Russian scientist Reuss. In 1939, Casagrande[1] introduced an electro-osmosis method into practical foundation treatment.

Many studies regarding electro-osmosis have been carried out, and most of them are laboratory tests. Kaniraj & Yee[2] investigated the effects of current intermittence, polarity reversal, and chemical additions on the electro-osmosis of organic soil. Drainage, water content, settlement and the undrained shear strength of soil were measured after the process. Glendinning et al.[3] conducted electro-osmotic reinforcement experiments of cohesive soil by using EKG. The influence of the voltage gradient and duration on the soil shear strength was analyzed, and the variation in the electro-osmotic permeability of soil was also evaluated. Tao et al.[4] studied the effect of electrode materials on electro-osmotic drainage, effective potential evolution, the energy consumption coefficient and final water content; moreover, the advantages and disadvantages of different electrode materials under several conditions were analyzed. Chien et al.[5] studied the effect of the injection of saline solutions on electro-osmosis consolidation, and the relationship between the undrained shear strength of soil and the zeta potential was presented. Beddiar et al.[6] investigated the relationships between pH, electro-osmotic permeability and hydraulic permeability of soil, the results show that the electro-osmotic permeability of soil is a pH-dependent variable.

In theoretical research, Esrig[7] established the one-dimensional electro-osmosis consolidation theory based on the assumption that electro-osmotic flow and water flow could be superimposed. Wan & Mitchell[8] developed a one-dimensional consolidation equation for electro-osmosis combined surcharge based on previous research. Lewis & Humpheson[9] established a two-dimensional electro-osmosis consolidation theory. Furthermore, Su & Wang[10] provided analytical solutions under different boundary conditions. Rittirong & Shang[11] simulated a two-dimensional electro-osmotic process by using the finite difference method and analyzed the settlement. Yuan & Hicks[12] established the large strain consolidation theory of electro-osmosis considering the nonlinear variation in hydraulic permeability, and the changes in pore pressure and settlement during the process were also analyzed. Hu et al.[13] conducted a multifield coupled numerical analysis of electro-osmosis, and the changes in the pore pressure of soil and ground settlement were obtained. Alshawabkeh et al.[14] provided practical evaluations of one-dimensional and two-dimensional electro-osmosis consolidations with the relationship between electrode spacing and energy cost.

In this paper, a numerical model for the two-dimensional electro-osmotic consolidation of soft soil was established. The governing equations coupled with both the electric and seepage field were solved by COMSOL, an FEM software tool. The development of the negative excess pore pressure at different locations was discussed. The influence of the electrodes layouts on the average degree of consolidation was obtained. The optimum ratio for the electro-osmosis permeability and hydraulic permeability was acknowledged with the considerations of the average negative excess pore pressure and average degree of consolidation. The results can be used for in-site engineering.

THEORY OF 2-D ELECTRO-OSMOSIS CONSOLIDATION

Figure 1 provides a simplified calculation model for two-dimensional electro-osmosis consolidation analysis. The left and right boundaries are set with an anode and a cathode. There is a distance L between them. The cathode is open for water drainage. Both the anode and cathode are in direct contact with the soil, and this description is a simplified 2D model of electro-osmosis consolidation.

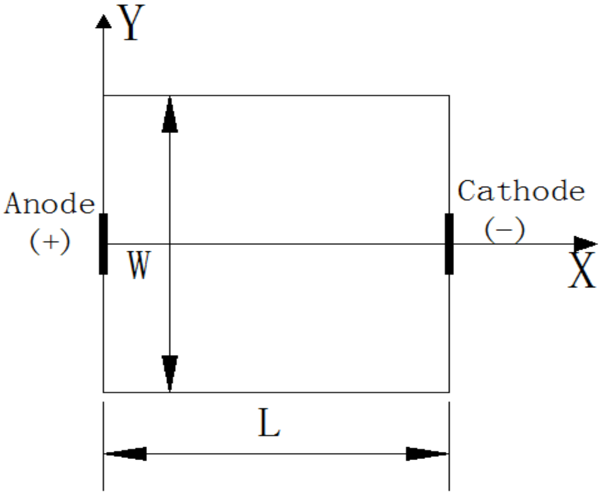


Figure 1: Simplified 2D model of electro-osmosis consolidation

Assuming that a soil body is saturated and isotropic, the value of the horizontal hydraulic permeability (k_{hx}) and the vertical hydraulic permeability (k_{hy}) are equal to the hydraulic permeability (k_h), and the expression of the water flow in soil can be obtained by Darcy's law:

$$v_{hx} = k_{hx} i_h = k_{hx} \frac{\partial H}{\partial x} = \frac{k_h}{\gamma_w} \frac{\partial P}{\partial x} \tag{1}$$

$$v_{hy} = k_{hy} i_h = k_{hy} \frac{\partial H}{\partial y} = \frac{k_h}{\gamma_w} \frac{\partial P}{\partial y} \tag{2}$$

where k_h is the hydraulic permeability, γ_w is the unit weight of pore water, H is the hydraulic head, and P is the excess pore pressure.

Casagrande[1] derived the relationship of the electro-osmotic flow and electric potential gradient based on the Helmholtz-Smoluchowski (H-S) model. Assuming that the values of the horizontal electro-osmotic permeability (k_{ex}) and vertical electro-osmotic permeability (k_{ey}) are equal to k_e, the expression of the electro-osmotic flow rate can be simplified as:

$$v_{ex} = k_{ex}\frac{\partial\varphi}{\partial x} = k_e\frac{\partial\varphi}{\partial x} \tag{3}$$

$$v_{ey} = k_{ey}\frac{\partial\varphi}{\partial y} = k_e\frac{\partial\varphi}{\partial y} \tag{4}$$

where k_e is the electro-osmotic permeability.

According to the assumption that electro-osmotic flow and water flow could be superimposed, the water flow in soil under an electric field and a seepage field can be expressed as:

$$v_x = v_{ex} + v_{hx} \tag{5}$$

$$v_y = v_{ey} + v_{hy} \tag{6}$$

The continuous equation can be obtained as:

$$div(v) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = m_v\frac{\partial P}{\partial t} \tag{7}$$

where m_v is the compressibility coefficient.

Combining equations (5), (6), and (7), the governing equation can be obtained as:

$$\frac{k_h}{\gamma_w}\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right) + k_e\left(\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2}\right) = m_v\frac{\partial P}{\partial t} \tag{8}$$

Assuming the electrical conductivity of the soil body is constant with no free charge in the soil, the governing equation of the electric field in the soil can be obtained according to Gauss's law:

$$\nabla^2\varphi = 0 \tag{9}$$

where φ is the electric potential.

According to the equations (8) and (9), the pore pressure during the entire electro-osmotic process can be obtained. The average degree of consolidation can be expressed as:

$$\overline{U}=\frac{\int_{-\frac{W}{2}}^{\frac{W}{2}}\int_{0}^{L}[p(x,y,0)-p(x,y,t)]\mathrm{d}x\mathrm{d}y}{\int_{-\frac{W}{2}}^{\frac{W}{2}}\int_{0}^{L}[p(x,y,0)-p(x,y,\infty)]\mathrm{d}x\mathrm{d}y} \tag{10}$$

where *p*(x, y, 0) is the initial pore pressure and *p*(x, y, t) is the instantaneous excess pore pressure.

The final excess pore pressure in any location can be expressed as:

$$p(x,y,\infty)=-\frac{k_e}{k_h}\gamma_w\varphi \tag{11}$$

The anode is set as impermeable, and cathode is set as free drainage. Under such boundary conditions, Su & Wang[10] obtained the analytical solution of the following governing equation:

$$\begin{aligned}p\left(x,y,t\right)=-\frac{k_e}{k_h}\varphi\left(x,y,t\right)+\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\left\{B_{mn}+\int_0^t f_{mn}(\tau)e^{\left[\left(\frac{2m-1}{2L}\right)^2+\left(\frac{2n}{W}\right)^2\right]\delta^2a^2\tau}d\tau\right\}\\ \sin\frac{\left(2m-1\right)\delta}{2L}x\cos\frac{2n\delta}{W}y\, e^{-\left[\left(\frac{2m-1}{2}\right)^2+\left(\frac{2nL}{W}\right)^2\right]\delta^2T_H}\end{aligned} \tag{12}$$

where

$$B_{mn}=\frac{8}{LW}\int_0^L\int_0^{\frac{W}{2}}\left[p(x,y,0)+\frac{k_e\gamma_w}{k_h}\varphi(x,y,0)\right]\sin\frac{(2m-1)\delta}{2L}x\cos\frac{2n\delta}{W}y\mathrm{d}x\mathrm{d}y \qquad n\geq 1\,;$$

$$B_{m0}=\frac{4}{LW}\int_0^L\int_0^{\frac{W}{2}}\left[p(x,y,0)+\frac{k_e\gamma_w}{k_h}\varphi(x,y,0)\right]\sin\frac{(2m-1)\delta}{2L}x\mathrm{d}x\mathrm{d}y \qquad n=0\,;$$

$$f(\tau)_{mn}=\frac{8}{LW}\int_0^L\int_0^{\frac{W}{2}}f(x,y,\tau)\sin\frac{(2m-1)\delta}{2L}x\cos\frac{2n\delta}{W}y\mathrm{d}x\mathrm{d}y \qquad n\geq 1\,;$$

$$f(\tau)_{m0}=\frac{4}{LW}\int_0^L\int_0^{\frac{W}{2}}f(x,y,\tau)\sin\frac{(2m-1)\delta}{2L}x\mathrm{d}x\mathrm{d}y \qquad n=0\,;$$

$$T_H=\frac{k_h t}{m_v\gamma_w L^2}\,.$$

The numerical analysis is conducted by COMSOL. The boundary conditions are set for an impermeable anode and free drainage cathode. The length and width of the model are 20 cm. The soil parameters in equations (8) and (10) are shown in Table 1, which are typical indexes of saturated soil and commonly used in-site. The calculated cases under different conditions are shown in Table 2.

Table 1: Soil parameters

Variable	Value
Hydraulic permeability, k_h ($m \cdot s^{-1}$)	1.1E-9
Electro-osmotic permeability, k_e ($m^2 \cdot s^{-1} \cdot v^{-1}$)	1.1E-9
Coefficient of compressibility, m_v (Pa^{-1})	1E-6
Initial excess pore pressure, q (Pa)	0
Unit weight of water, γ_w ($kN \cdot m^{-1}$)	10

Table 2: Calculated cases

Case	Voltage (V)	Width of anode plate (cm)	Width of cathode plate (cm)
E1	15	4	4
E2	15	12	4
E3	15	20	4
E4	15	4	12
E5	15	4	20

NUMERICAL RESULTS

Development of excess pore pressure

Case E1 is chosen to demonstrate the development of the negative pore pressure at different locations during the electro-osmosis process. Because the numerical model is symmetrical, the points for the excess pore pressure analysis are shown in Figure 2. The analysis points are located in front of anode and cathode and in the middle of the model. The space between each point is 2 cm. Figure 3 shows the relationship between the excess pore pressure and time during the entire electro-osmotic process.

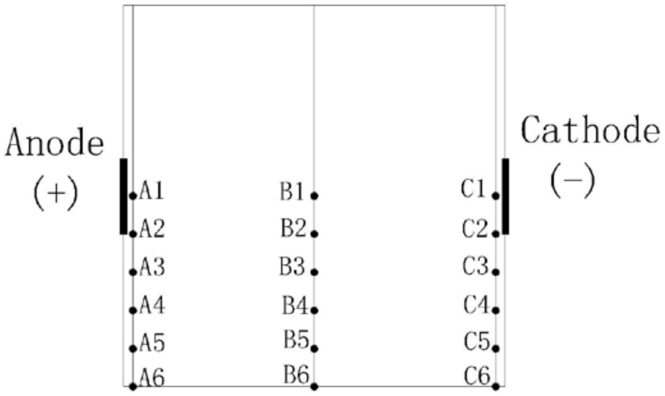
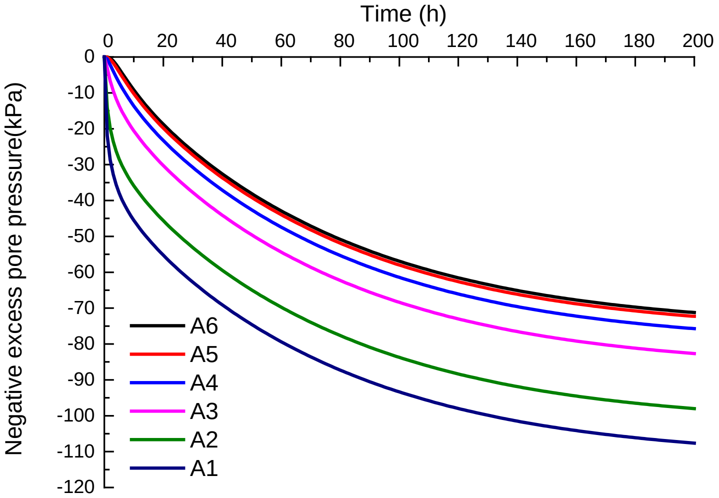
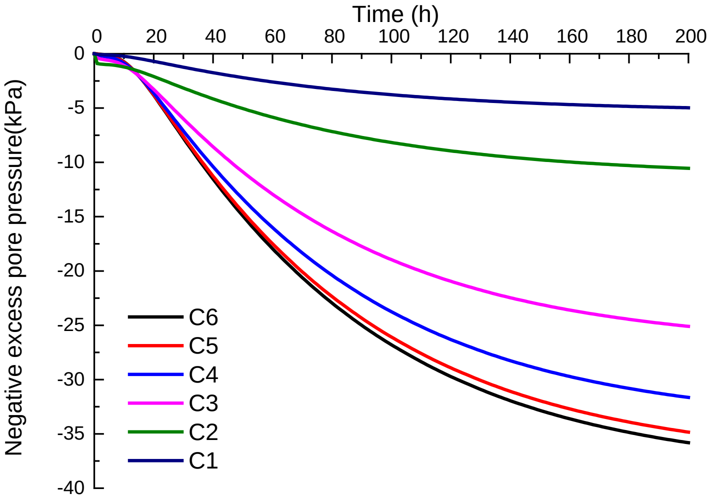


Figure 2: Points for negative excess pore pressure analysis

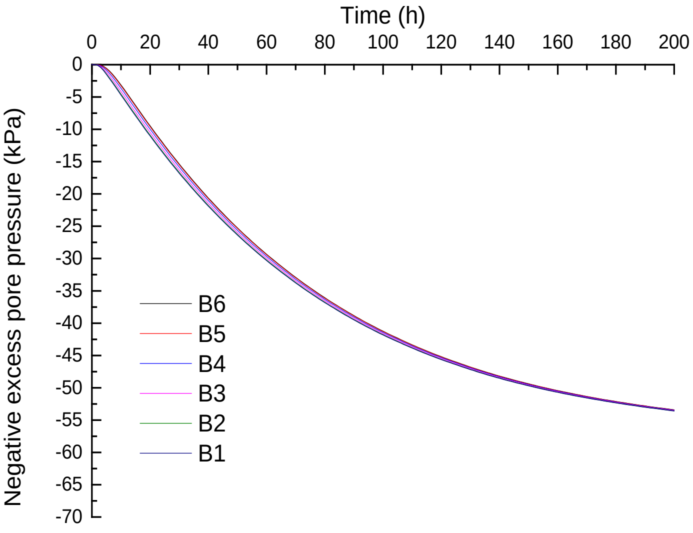


(a)



(b)

Figure 3:*Continues on the next page*



(c)

Figure 3: Negative excess pore pressure-time curves of E1

Figure 3(a) and (b) shows the development of negative excess pore pressure near the anode and cathode. It appears that the distribution of the pore pressure distribution is not uniform and the value of the negative excess pore pressure near the plate differs from that far away from the plate greatly. Figure 3(c) shows the development of the negative excess pore pressure in the middle. It is shown that the development rate of the negative excess pore pressure at each analysis point is almost the same during the entire process. The negative excess pore pressure is more uniform in the middle of the model than in the vicinity of the anode and cathode after electro-osmotic treatment.

It can be concluded from equation (11) that the distribution of the final negative excess pore pressure depends on the electric potential, and the distribution of the electric potential E1 is shown in Figure 4. It appears that the distribution of the electric potential is uneven in the vicinity of the anode and cathode and more uniform in the middle, which result in the differences in the distribution of negative excess pore pressure.

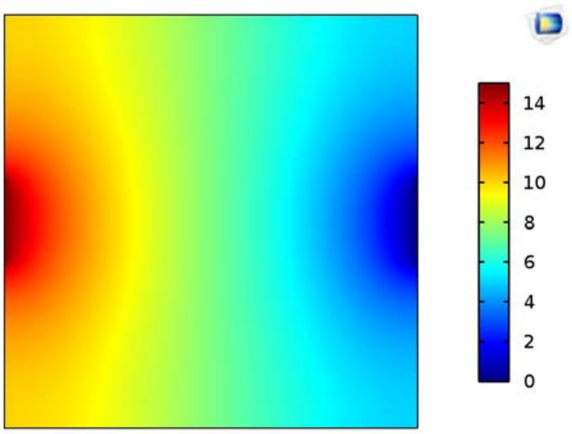


Figure 4: The distribution of the electric potential for E1

Average degree of consolidation

The electrodes layouts can influence the electro-osmosis consolidation. A comparison of average degree of consolidation with changing anode and cathode areas is presented in this section, and the results are shown in Figures 5 and 6.

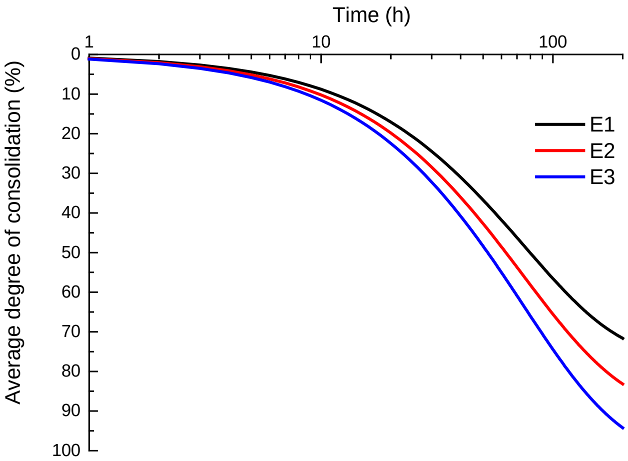


Figure 5: Average degree of consolidation-time curves for E1, E2 and E3

The relationship between average degree of consolidation and time with an expanding anode plate is shown in Figure 5. The final average degrees of consolidation for E1, E2 and E3 are 71.6%, 83.1%, and 94.2%, respectively. It is clear that the average degree of consolidation increases with an enlargement in the anode plate, when cathode plate remains unchanged. During the entire electro-osmotic process, average degree of consolidation always maintains the following relation E3>E2>E1. Under the same processing time, an increasing anode area is beneficial for an improvement in the average degree of consolidation of the treated soft soil.

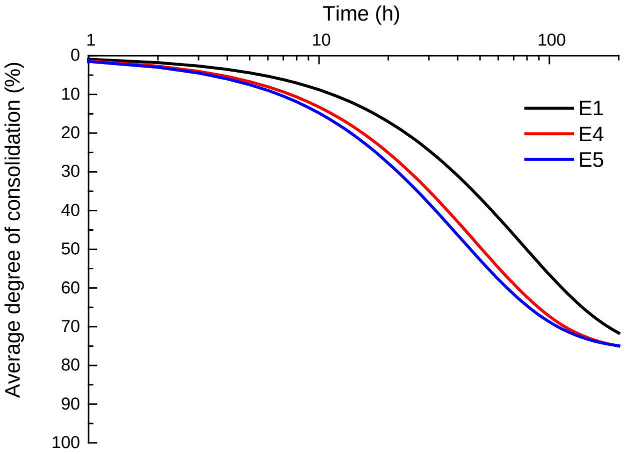


Figure 6: Average degree of consolidation-time curves for E1, E4 and E5

The relationship between the average degree of consolidation and time with an increasing cathode plate is shown in Figure 6. The final average degrees of consolidation for E1, E4 and E5 are 71.6%, 75.0% and 74.9%, respectively. An enlarged cathode area with an unchanged anode cannot effectively improve the average consolidation degree of the treated soil. During the entire electro-osmotic process, the increasing rate of the average degree of consolidation for E4 and E5 is very close and larger than E1. It takes only 142 hours for E4 and E5 to reach the final average degree of consolidation of E1. The electro-osmotic duration is sharply reduced by 34%. Compared with an increasing anode area, an expanding cathode area can increase the rate of average degree of consolidation but cannot improve the final average degree of consolidation like the enlarging anode area does.

Influence of k_e/k_h

Because the hydraulic permeability and electro-osmosis permeability are often different in practical engineering, the ratio of electro-osmosis permeability and hydraulic permeability k_e/k_h can be a significant factor in electro-osmosis consolidation. In this section, the layout type of the electrodes is the same as that of E1. The electro-osmosis permeability k_e=1.1E-9 $m^2 \cdot s^{-1} \cdot v^{-1}$, the coefficient of compressibility m_v=1E-6 Pa^{-1}, the initial excess pore pressure q=0 kPa and the unit weight of pore water γ_w=10 $kN \cdot m^{-1}$ are kept constant. The ratios of k_e/k_h, namely, 0.1, 1, 2, 5, 10, were studied, and the differences of the average negative excess pore pressure and average degree of consolidation were compared. The results are shown in Figures 7 and 8.

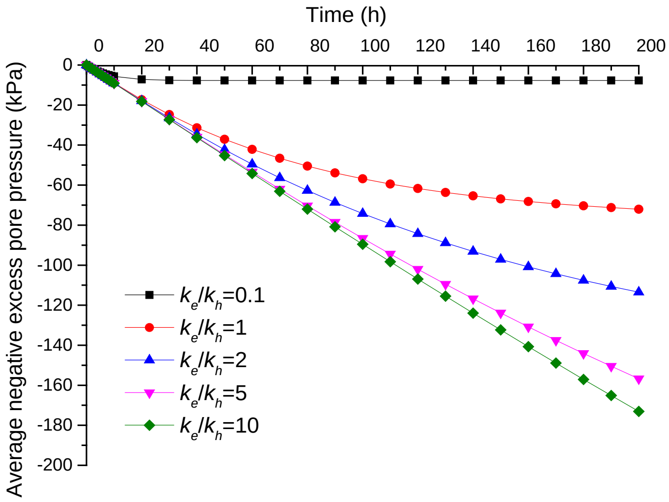


Figure 7: Average negative excess pore pressure-time relationship of the different ratios of k_e/k_h (0.1, 1, 2, 5, and 10)

Figure 7 shows the relationship between the average negative excess pore pressure and time during the process. The development of average negative excess pore pressure increases with an increasing ratio of k_e/k_h. The electro-osmosis drainage is improved with a higher ratio of k_e/k_h.

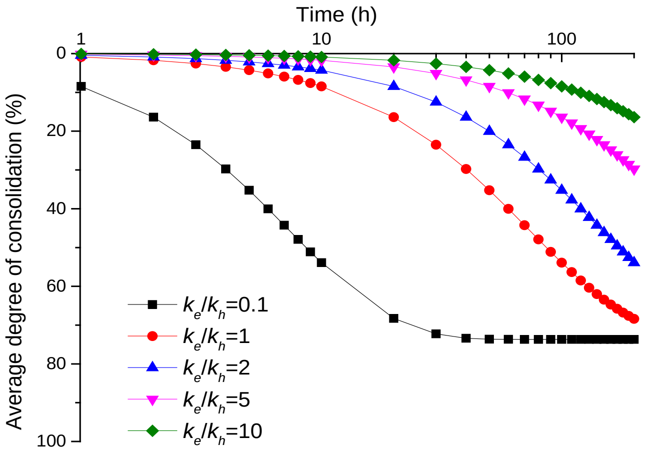


Figure 8: Average degree of consolidation-time relationship of the different ratios of k_e/k_h (0.1, 1, 2, 5, and 10)

Figure 8 shows the relationship between the average degree of consolidation and time during the process. The rate of the average degree of consolidation increases with a decreasing ratio of k_e/k_h. If the hydraulic permeability coefficient is too small, then the rate of the average degree of consolidation will slow down.

With considerations of the average negative excess pore pressure and the rate of average degree of consolidation, the optimum ratio of k_e/k_h is between 1 and 5. Therefore, the electro-osmosis permeability k_e and the hydraulic permeability k_h should be tested before in-site work to determine whether the soil is suitable for electro-osmotic treatment.

CONCLUSION

(1) The negative excess pore pressure is not the same at different locations after an electro-osmosis treatment. In the vicinity of the anode and cathode, the value of the negative pore pressure near the plate differs from that far away from the plate greatly. However, in the middle of the model, the negative excess pore pressure is more uniform. This phenomenon is mainly caused by the uneven distribution of electric potential.

(2) An enlargement in the anode plate area is beneficial for an improvement in the average degree of consolidation of the treated soft soil. However, enlarging the cathode plate area can improve the increasing rate of the average degree of consolidation with a shorter duration, but it cannot effectively improve the final average consolidation degree of the treated soil.

(3) The ratio of k_e/k_h can be a significant factor in electro-osmosis consolidation. An increase in the ratio of k_e/k_h can increase the development of the average negative excess pore pressure but can slow down the rate of the average degree of consolidation. With considerations of the average negative excess pore pressure and average degree of consolidation, the optimum ratio of k_e/k_h is between 1 and 5. The numerical values of k_e and k_h should be tested carefully before electro-osmotic treatment.

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